An Examination of the Time Series Evidence on AK-Style Endogenous Growth Models

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Abstract: In "Time Series Tests of Endogenous Growth Models" (1995)[1], Charles Jones concludes that the two textbook endogenous growth models — AK-models and R&D based models —are inconsistent with time series evidence. Showing that there is little or no persistent increase in growth rates of OECD countries but large persistent movements in investment, Jones argues that either the persistent movements in growth determinant variables have been offsetting, or "the hallmark of the endogenous growth models, that permanent changes in policy variables have permanent effects on growth rates, is misleading." This paper replicates his time series test of AK models and extends its original data to 2004[2]. The extended data exhibit negative trends or no trend at all in total investment rates, but in general still support the conclusions of the original paper.

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[1] Jones, Charles I. (1995). "Time series tests of endogenous growth models". *Quarterly Journal of Economics* 110, 495-525

[2] For the detailed explanations of data, see Appendix C.

I. Introduction

The major difference between neoclassical growth models and endogenous growth models is long-run effects of initial conditions. The endogenous growth models show that permanent changes in variables that are potentially affected by government policies, such as subsidies to research or taxes on investment, lead to permanent changes in growth rates. In contrast, neoclassical growth models, such as Solow (1956), show that the main engine of long-run growth is exogenous technological progress and the permanent changes in government policies only cause level effects, but no long-run growth effects. Since Jones (1995), there have been numerous attempts to examine endogenous growth theory in a time-series context in order to distinguish the short-run and long-run growth effects[3]. However, as Durlauf (2005) points out, there had not been much agreement on the methods that should be used to distinguish neoclassical and endogenous growth theory empirically. The underlying problem is the insufficient amount of available data which span at most 140 years, and consequently, the difficulty of separating the effects of policy on the transition from the steady-state behavior of a stochastic process.

In his simple time series analysis, Jones (1995) shows that per capita real GDP growth rates in OECD countries have little or no persistent upward trend, whereas investment rates, that is, the ratio of investment to GDP, contain significant and persistent movements for most of OECD countries (the upward trends are sufficiently observed

[3] Not only time-series, but also dynamic panel methods with fiscal policy variables have been used as in Evans (1998), Caselli et. al. (1996). Also, static panel methods on five year averaged data are commonly used to capture those long-run policy effects; however, according to Bleaney et. al. (2001), whether five year lags are enough length to capture the long-run policy effects is under-discussion. Also there had been several counterarguments to Jones (1995). For example, Mcgrattan (1998) argues that time-series with longer horizon of data and cross-country analysis supports the AK growth models.

especially in producer durables investment rates). Based on these results, Jones then argues that time-series evidence is inconsistent with the restriction of AK model, which states the permanent changes in investment rates lead to permanent changes in growth. Furthermore, he allows the AK model to have both contemporaneous and non-contemporaneous interaction between growth and investment rates, and constructs the dynamic responses of growth rates to a permanent increase in investment rate. The dynamic responses exhibit strong evidence against the AK model, due to the fact that the effects from a permanent increase in investment rate disappear after about 6 years and hence movements in growth caused by a permanent change in investment rate seem transitory. Jones warns that this short duration of effect on growth may also provide a misleading view of long-run growth.

In this paper, the author follows the methods used by Jones (1995) and analyzes the original data and the data extended to 2004. Specifically, this paper will examine the unit root process and time trend of two key variables in this model: growth rate and investment rate. After examining the time series properties of these two variables, the author constructs dynamic responses of growth to permanent increase in the investment rate. The time series properties of growth rates are examined in Section II, and the AK model and its time series test results are explained in Section III.

II. Movements in GDP Growth Rates

Figure 1 plots per capita GDP growth rates of fifteen OECD countries[4] from 1880 to 2004. Although there are some visible changes in stochastic properties associated with

[4] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, the United Kingdom, and the United States

WWI and WWII, one can immediately see that growth rates seem to fluctuate around the constant mean in those advanced economies. The formal analysis of time series properties of growth rates are given in Table 1. The null hypothesis of a unit root is rejected at 1% level in every country by the augmented Dickey-Fuller (ADF) tests, and time trend tests show a positive trend in eight countries in the overall sample and a downward trend after WWII in seven countries. For the extended period after WWII (1950-2004), time trend tests show a negative trend in about one half of the sample. These results support and somewhat augment the trend seen in the original results of Jones (1995) [5], which indicates positive mean shifts and a downward trend after WWII in several countries. Jones explains these movements as transition dynamics due to the change in the marginal product of the decimated resources, and these changes in stochastic properties can be accounted for by only considering the sample in the postwar period.

In summation, according to this simple time series analysis, growth rates in selected OECD countries exhibit little or no persistent movements, and possibly some contain negative trends after 1950.

III. AK models

A. Time Series Specification of AK model

Expounded early in Romer (1987) and Rebelo (1991), the "AK" model[6] is one of the simplest endogenous growth models. Here, we consider the case in which the dynamic interaction over time between investment and growth is allowed.

[5] See Appendix D for the original results of Jones (1995).
[6] See Appendix A for its derivation.

From Appendix A equation (c):

$$g_y = -\delta + \tilde{A} i^k .$$

where g_y is the growth rate of output, δ is the rate of depreciation, $\tilde{A}$ is a constant, and i^k is the investment rates for physical capital.

Reinterpreting this equation, we have:

(1) $$g_t = A(L) g_{t-1} + B(L) i_t + \varepsilon_t ,$$

where $A(L)$ and $B(L)$ are lag polynomials with roots outside the unit circle. Then equation (1) becomes:

(2) $$g_t = A(L) g_{t-1} + B(1) i_t + C(L) \Delta i_t + \varepsilon_t$$

where $C(L)$ is a $(p-1)$th-order lag polynomial such that $c_j = -\sum_{i=j+1}^{p} b_i$ for $j = 1,...,p-1$. Adding a country-specific intercept and a country-specific time trend, and imposing the restriction $B(1) = 0$, that is, a permanent shock to investment will have no effect on the growth rate, we now have:

(3) $$g_{it} = \alpha_i + b_i t_i + A(L) g_{it-1} + C(L) \Delta i_{it} + \varepsilon_{it} .$$

The time trends are included to capture any exogenous movements in growth rates (ex. changes in labor population) that are omitted from the specification[7].

[7] However, Jones (1995) argues that the first difference in investment rates will be stationary and should be uncorrelated with the time trend. Although the stochastic process of investment rates cannot be a pure unit root process, in a relevant range, investment rates are well characterized by a unit root process. Also, this leads to the validity of the ADF tests for this model –from equation (a) and (c) in Appendix A, we can also expect a unit root process to characterize growth rates to the extent that a unit root process characterizes investment rates.

B. Results and Interpretations

1) Time Series Properties

Given that growth rates exhibit little or no persistent increase, the model will be contradicted by empirical evidence if investment rates show significant persistent upward trends. Table 2 and 3 show the results from the ADF tests and time trend tests of total investment and producer durables investment rates[8] for the postwar period (1950-1988 and 1950-2004). We can see the overall (statistically significant) upward deterministic trend for producer durables investment in both periods, but total investment shows mixed results. Although the original result of Jones (Table IV) is more in favor of a positive trend, the results from the replication (Table 2) are ambiguous[9]. In particular, the result from the extended period (Table 3) seems to strongly contradict the argument of Jones (1995). Six countries reject the null hypothesis of a unit root, and only one out of those six shows a positive time trend. Moreover, significantly negative time trends are shown in one half of the entire sample[10]. However, if we follow the suggestion of De Long and Summers (1991) and Jones (1994), total gross investment is not as important as producer durables investment for growth, and therefore the results from total investment don't provide as good of a test as the durables investment data.

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[8] Since the composition of investment has shifted from structures to producer durables, using producer durables investment can well address the issue of differences in depreciation rates. Also, as discussed in De Long and Summers (1991), there is a view of machinery investment as a principle driving force of economic growth among the components of investment, since cross-country regressions exhibit the strong correlation between machinery investment rates and growth rates but no correlation between non-machinery investment rates and growth rates. Note that machinery investment only differs from producer durables investment in the exclusion of investment for transportation.

[9] The apparent differences in the ADF test results between the original (Table IV) and the replication (Table 2) might seem surprising. Since the author used the same data set as Jones (1995), the most likely explanation would be the difference in lag length chosen by the Schwartz information criteria. Estimations and tests in this paper are performed in EViews 6.

[10] For the visual representation of the movements of investment rates, see Figure 2 and 3.

Thus, assuming producer durables investment explains growth rate better than total investment, the positive trend in producer durables investment indicates that the model of equation (a) is not supported by the data, and hence we can still support the argument of Jones (1995), that is, AK models do not provide a good description of the driving forces behind growth in developed countries. An argument can be made that the positive trend in producer durables investment could be offset by omitted variables. Jones (1995) explains that the major elements of those would be human capital investment and openness which certainly trend upward in the postwar period; there might exist relevant omitted variables which could offset the effect of investment at least for the sample countries which exhibit the significant upward trend of investment, but none seems to readily apparent[11].

2) Dynamic Response

Although the time series properties of total investment do not support AK models, we can still say that it is a good approximation if the effects on growth last in the long-run –say, 25 to 30 years. Alternatively, the model is misleading if the effects only last for medium to short-run. For testing the horizon over which the effects on growth are important, we use the equation 3 with the restriction $B(1)=0$.

Table 4 shows the result of OLS estimation of equation 3:

$$g_{it} = \alpha_i + b_i t_i + A(L)\, g_{it-1} + C(L)\,\Delta i_{it} + \varepsilon_{it}.$$

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[11] There had been numerous researches to identify the engine of long-run economic growth. The major explanatory variables include physical and human capital accumulations, population growth, discovery of new ideas, and institutions. "Indeed, the problem now confronting growth economists is how to choose among the abundance of competing explanations. Empirical work provides some guidance, but a number of difficulties such as the accurate measurement of ideas or human capital or even growth itself lead this research to be less than conclusive" (Jones, 2003).

Using the estimated coefficients of growth rates and first differences of investment rates, the author calculated the dynamic response of growth rates and output to a one-percentage-point permanent increase in the investment rate. The results for the period 1950-1988 and 1950-2004 are reported in Table 5 and 6, and its graphical representation is shown in Figure 4.

From those results, we can see the effects on growth disappear only after 6 years, which confirms the results of Jones (1995). For producer durables investment, the effects are negligible even earlier. The cumulative effect after 20 years shows that the long-run effect of a one-percentage-point permanent increase in the investment rate is to raise output per capita by 0.880 for total investment, and 0.969 for producer durables investment for the period 1950-1988. To see its validity, we can examine these numbers in terms of the returns to capital, α.

The basic Solow model gives us:

$$y = k^{\alpha}, \quad \alpha < 1$$
$$\dot{k} = iy - (n + g + \delta)k$$

where n is population growth and g is exogenous productivity growth, y, k and i are measured in per unit of effective labor. Now we can show that the level of technology is held constant as follows:

$$\frac{\partial \ln y^{ss}}{\partial i} = \frac{\alpha}{1-\alpha}\frac{1}{i}$$

By assuming i=0.25, which is the average value of the investment rate from our sample for the period 1950-1988, we have α =0.18 for total investment and α =0.20 for producer durables investment. By the same procedure with i=0.26 for the extended data, we have α =0.15 for total investment and α =0.36 for producer durables investment.

Comparing with the standard value $\alpha = \frac{1}{3}$, these estimates of long-run effects seem plausible but underestimated[12]. The author also calculated the dynamic responses by taking the sample of group which failed to reject a unit root process in Table 2 and 3, but the results are similar (but closer to the standard value)[13].

It might appear at first glance that the effect on growth is biased by the endogeneity of investment, since the contemporaneous term of investment rate is present in equation 3. For controlling for the endogeneity of investment in the equation 3, see Appendix B. The result shows that the effect on growth only lasts for 8 to 9 years at most. This relatively short horizon of AK models would make a prediction of the models misleading.

IV. Concluding Remarks

This paper concludes that the extended data, especially producer durables investment, support the conclusion of Jones (1995). However, it is important to note that total investment rates exhibit either negative trends or no trends at all in the majority of sample countries. Further careful inspection of the investment component is needed, as well as a more complete inspection of the possibility of omitted variables. Also, as has often happened in this literature, the estimation in the section III, B (2) is conditional on the assumptions of the model being correct. To examine the growth models in a more accurate manner, the author expects that Bayesian approaches would be desired in the growth literature.

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[12] This is expected, especially for total investment since depreciation from structural investment is not removed.

[13] The results are shown in Table 7 and 8. It indicates that, for the period 1950-1988, α =0.19 for total investment and α =0.24 for producer durables investment; for the period 1950-2004, α =0.15 for total investment and α =0.33 for producer durables investment.

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Appendix

A. The derivation of AK model

Consider a simple growth model such as:

(a) $$\max_{i_t^k, i_t^h} \int_{t=0}^{\infty} e^{-\rho t} u(c_t)\, dt$$

where

$$c_t = \left(1 - i_t^k - i_t^h\right) y_t$$
$$y_t = A k_t^{\alpha} h_t^{1-\alpha}$$
$$\dot{k}_t = i_t^k y_t - \delta k_t$$
$$\dot{h}_t = i_t^h y_t - \delta h_t$$

with the following standard notations: k is physical capital, h is human capital, $u(\cdot)$ is a CRRA utility function, c is consumption, y is output, δ is the rate of depreciation (same for both k and h), ρ is the rate of time preference, and i^k and i^h are the investment rates for k and h.

Solving these equations, it is easily shown that the ratio h/k is constant and equal to $(1-\alpha)/\alpha$. Thus these two types of capital k and h accumulate together (in lockstep with a presence of adjustment costs). Then we can rewrite a production function in reduced-form production technology:

(b) $$y_t = \tilde{A} k_t, \text{ where } \tilde{A} = A\psi^{1-\alpha} \text{ and } \psi = h/k.$$

Finally, by taking logs and differentiating the equation (b), we have:

(c) $$g_y = -\delta + \tilde{A} i^k.$$

In this model[14], we can see the movement in the investment rates will be matched with the movement in the steady state growth rate of output g_y, that is, a permanent increase in the investment rate generates a permanent increase in growth.

B. Controlling for Endogeneity of Investment

According to Jones (1995), we can reasonably assume that the lower bound of the true contemporaneous response is zero and the upper bound is the OLS estimate of c_0, the coefficient on the change in contemporaneous investment rate. Under these assumptions, we can see the bounds of the OLS by calculating the dynamic responses for the N different values of the contemporaneous response within the bounds. Following the method used by Jones (1995), the author calculated the dynamic responses for each of the $N = 11$ contemporaneous response bounded between zero and c_0. The results are shown in Figure 5 (a) through (d).

C. Data Sources for the Replication and Extension

For the replication, the data file from Jones website was used:

http://elsa.berkeley.edu/~chad/TimeEGM.asc

Specifically, the data on per capita GDP is constructed from Maddison (1982, 1989) by Bernard (1991), and total gross investment rates are from Summers and Heston (1991). The data on producer durables investment rate was provided by Robert Summers.

For extending the data to 2004, the data from Penn World Table (PWT) and OECD.Stat extracted on 2008/11/05 were used.

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[14] Jones (1995) explains that time series tests of the restriction given in this equation will represent a test of an entire class of models in the literature, since the movement of the two types of capital adjusting together is likely to be robust to a number of changes and interpretations of the two-types model.

Specifically, per capita GDP growth rate was extended with the growth rate of "rgdpch" series (real GDP per capita chain series) in PWT6.1, and total gross investment rate was replaced or extended with "ci" series (investment share of real gross domestic product per capita) in PWT 6.1. Using the OECD.Stat data, producer durables investment rate was first approximated by the share of non-construction components under gross fixed capital formation (P51) to the GDP expenditure approach (B1_GE). The non-construction components used are: products of agriculture, forestry, fisheries and aquaculture (P51PI61), metal products and machinery (P51PI62), transport equipment (P51PI63), and other products (P51PI66). Although there is a gap between this approximated producer durables investment rate and the original rate, the series captures the movements in original data fairly well. Thus, the author extended the producer durables investment rate by adding averaged gaps of the data in overlapped years to the series, country by country.

D. Results from Jones (1995)

Table II, pp.500

TIME SERIES PROPERTIES OF SELECT OECD GROWTH RATES

Country	ADF test 1900–1987	Time trend 1900–1987	Difference in means	Time trend 1950–1988
Australia	0.29	0.028	1.834	−0.010
	(−6.46)***	(1.61)	(2.85)***	(−0.15)
Austria	0.07	0.052	2.974	−0.110
	(−8.59)***	(1.62)	(2.71)**	(−2.53)**
Belgium	0.23	0.035	1.740	−0.032
	(−7.26)***	(1.34)	(1.44)	(−0.68)
Canada	0.37	0.015	0.617	0.020
	(−6.25)***	(0.54)	(0.56)	(0.38)
Denmark	0.04	0.016	0.772	−0.029
	(−8.83)***	(0.93)	(0.93)	(−0.41)
Finland	0.23	0.033	1.823	−0.036
	(−7.27)***	(1.24)	(1.48)	(−0.63)
France	0.24	0.036	1.472	−0.087
	(−7.18)***	(1.19)	(1.06)	(−2.38)**
Germany	0.02	0.033	2.242	−0.153
	(−9.05)***	(1.16)	(1.79)*	(−3.26)***
Italy	0.27	0.031	2.166	−0.095
	(−6.93)***	(1.31)	(2.17)**	(−2.63)**
Japan	0.12	0.055	3.989	−0.182
	(−8.10)***	(1.90)*	(3.90)***	(−3.07)***
Netherlands	0.19	0.026	1.003	−0.075
	(−7.57)***	(1.16)	(1.05)	(−1.40)
Norway	−0.00	0.028	1.282	0.025
	(−9.20)***	(1.75)	(1.42)	(0.73)
Sweden	0.22	0.020	1.190	−0.033
	(−7.39)***	(0.94)	(1.48)	(−1.00)
United	0.24	0.025	1.639	0.002
Kingdom	(−7.19)***	(1.38)	(1.88)*	(0.06)

Notes. Test-statistics are reported in parentheses. See the notes to Table I, except note that the Difference in means in this table refers to 1900–1929 versus 1950–1987. Significance levels are denoted by (*) for 10 percent, (**) for 5 percent, and (***) for 1 percent.

Table IV, pp.507

Time Series Properties of Select OECD Investment Rates 1950–1988

Country	Total investment		Producer durables investment	
	ADF test	Time trend	ADF test	Time trend
Australia	0.559	−0.083	0.805	0.030
	(−2.27)	(−1.50)	(−1.71)	(1.60)
Austria	0.748	0.279	0.420	0.071
	(−1.72)	(4.46)***	(−3.59)*	(3.82)***
Belgium	0.794	0.034	. . .	. . .
	(−2.06)	(0.41)		
Canada	0.531	0.083	0.810	0.077
	(−2.95)	(1.91)*	(−1.71)	(3.85)***
Denmark	0.882	−0.018	0.651	0.096
	(−1.41)	(−0.11)	(−2.66)	(5.55)***
Finland	0.618	−0.068	0.677	0.042
	(−2.57)	(−0.69)	(−2.84)	(1.22)
France	0.916	0.166	0.902	0.113
	(−1.17)	(1.68)	(−1.28)	(5.69)***
Germany	0.769	−0.146	0.659	0.086
	(−2.27)	(−2.12)**	(−3.48)*	(6.18)***
Italy	0.797	−0.095	0.374	0.037
	(−2.53)	(−0.85)	(−4.30)**	(3.69)***
Japan	0.899	0.426	0.820	0.159
	(−1.41)	(2.84)***	(−1.56)	(7.76)***
Netherlands	0.823	−0.140	0.854	0.008
	(−2.21)	(−1.36)	(−1.69)	(0.21)
Norway	0.573	−0.036	0.666	−0.155
	(−3.02)	(−0.64)	(−2.73)	(−2.62)**
Sweden	0.819	−0.033	0.443	0.052
	(−1.82)	(−0.43)	(−3.61)**	(6.08)***
United Kingdom	0.723	0.158	0.605	0.066
	(−2.62)	(2.71)**	(−2.73)	(5.48)***
United States	0.028	0.068	0.712	0.080
	(−5.74)***	(2.18)**	(−2.43)	(5.90)***

Notes. Data on total investment are taken from Summers and Heston [1991]. Data on producer durable investment is unpublished data provided by Robert Summers. The ADF tests in this table include a time trend in the regression. The Time trend columns report the coefficient on a time trend in a simple regression, as in Table I.

Table V, pp.511

DYNAMIC RESPONSE OF GROWTH RATES AND OUTPUT TO A ONE-PERCENTAGE-POINT PERMANENT INCREASE IN THE INVESTMENT RATE

	Total investment		Producer durables investment	
Period (year)	OLS dynamic response	OLS cumulative response	OLS dynamic response	OLS cumulative response
0	0.802	0.802	1.020	1.020
1	−0.013	0.789	0.167	1.186
2	0.030	0.819	−0.222	0.965
3	0.055	0.874	0.263	1.228
4	0.081	0.955	−0.012	1.216
5	0.133	1.088	−0.015	1.201
6	0.008	1.096	−0.026	1.175
7	−0.008	1.088	0.009	1.184
8	−0.014	1.074	0.005	1.189
9	−0.007	1.067	0.001	1.190
10	−0.009	1.058	−0.002	1.188
15	0.001	1.062	−0.000	1.188
20	−0.000	1.061	−0.000	1.188

Notes. The dynamic responses are calculated using regressions of growth rates on a country effect, a country-specific time trend, lagged growth rates, and current and lagged changes in the investment rates. For the specification with total investment, five lags of both growth and investment are used. For the specification with producer durables investment, four lags of growth and investment are used. The results are robust to the choice of lag length.

Figure III, pp.512

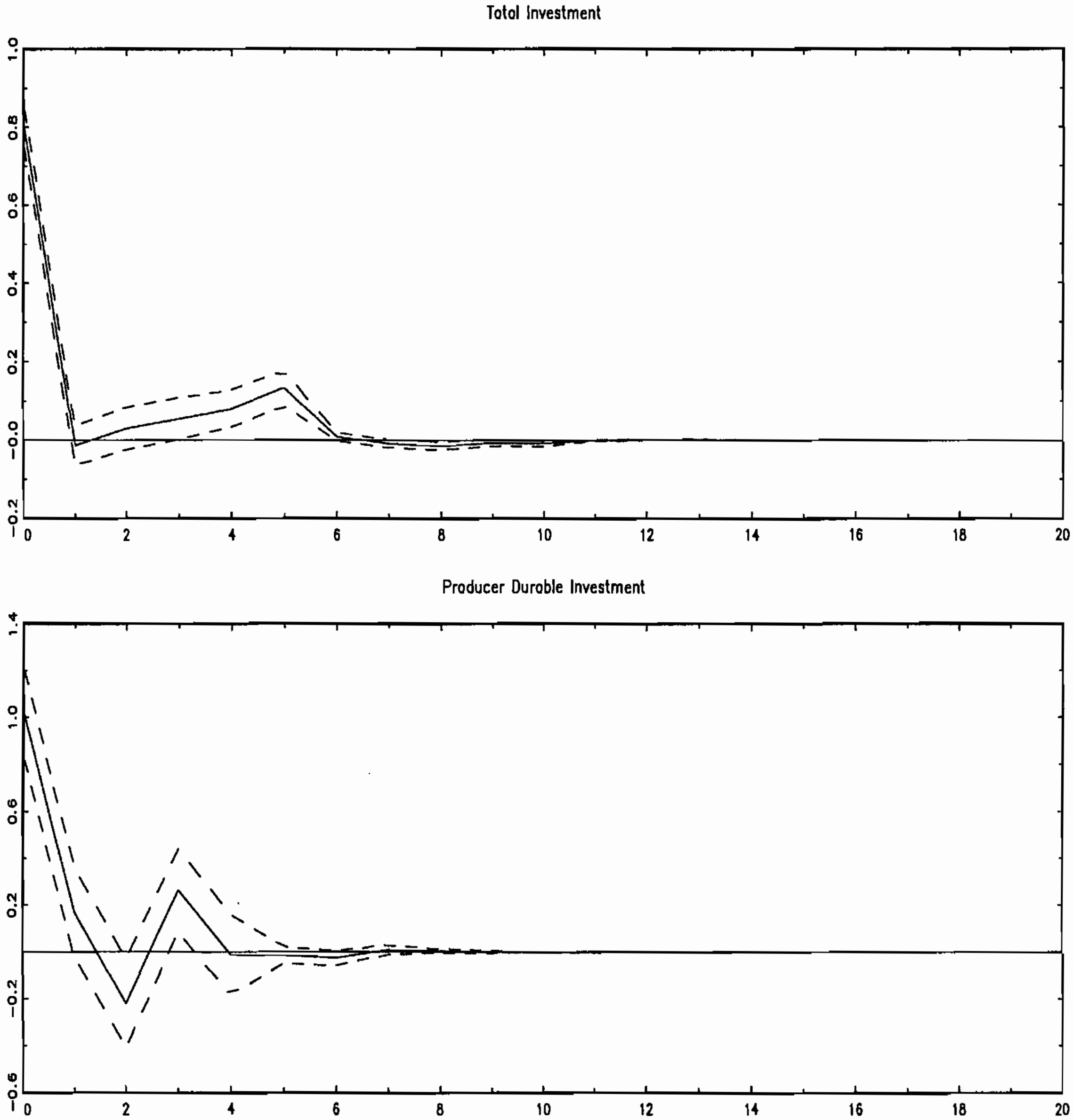


FIGURE III
Dynamic Response of Growth Rates to a Permanent One-Percentage-Point Increase in the Investment Rate

Source. Author's calculations. Dotted lines represent one standard error deviations computed using the delta method. See notes to Table V.

E. Figures and Tables

Table 1
Time Series Properties of Growth Rates, with time trend extended to 2004

Country	ADF test 1900-1987	Time Trend 1900-1987	Time Trend 1950-1988	Time Trend 1950-2004
AUSTRALIA	0.400	0.025	-0.010	0.002
	(-6.09)***	(2.14)**	(-0.45)	(0.09)
AUSTRIA	0.066	0.052	-0.145	-0.094
	(-8.68)***	(2.27)**	(-4.99) ***	(-5.24)***
BELGIUM	0.229	0.034	-0.048	-0.038
	(-7.34)***	(1.86)*	(-1.64)	(-2.43)**
CANADA	0.368	0.015	-0.005	-0.011
	(-6.25)***	(0.74)	(-0.15)	(-0.49)
DENMARK	0.035	0.015	-0.037	-0.043
	(-8.93)***	(1.36)	(-1.10)	(-2.12)**
FINLAND	0.030	0.033	-0.034	-0.053
	(-5.45)***	(1.54)	(-1.39)	(-1.59)
FRANCE	0.241	0.034	-0.088	-0.064
	(-7.26)***	(1.75)*	(-3.73) ***	(-4.20)***
GERMANY	0.015	0.032	-0.185	-0.118
	(-9.13)***	(1.64)	(-4.87) ***	(-5.00)***
ITALY	0.274	0.027	-0.118	-0.096
	(-7.01)***	(1.54)	(-5.66) ***	(-7.27)***
JAPAN	0.121	0.054	-0.175	-0.166
	(-8.29)***	(2.80)***	(-4.63) ***	(-6.73)***
NETHERLANDS	0.191	0.026	-0.069	-0.033
	(-7.57)***	(1.67)*	(-1.90) *	(-1.16)
NORWAY	-0.001	0.029	0.013	-0.024
	(-9.28)***	(2.99)***	(0.58)	(-1.42)
SWEDEN	0.221	0.020	-0.055	-0.030
	(-7.39)***	(1.31)	(-2.11) **	(-1.64)
U.K.	0.237	0.027	-0.019	-0.002
	(-7.28)***	(1.88)*	(-0.71)	(-0.20)
U.S.A.	0.308	0.006	-0.025	-0.007
	(-6.77)***	(0.43)	(-0.53)	(-0.34)

Notes: Growth rates are approximated by the first difference of natural logarithm of per capita real GDP, multiplied by 100. The ADF tests include an intercept, and reports the estimate of ρ (the coefficient of one-lagged growth rate) from the regression below.

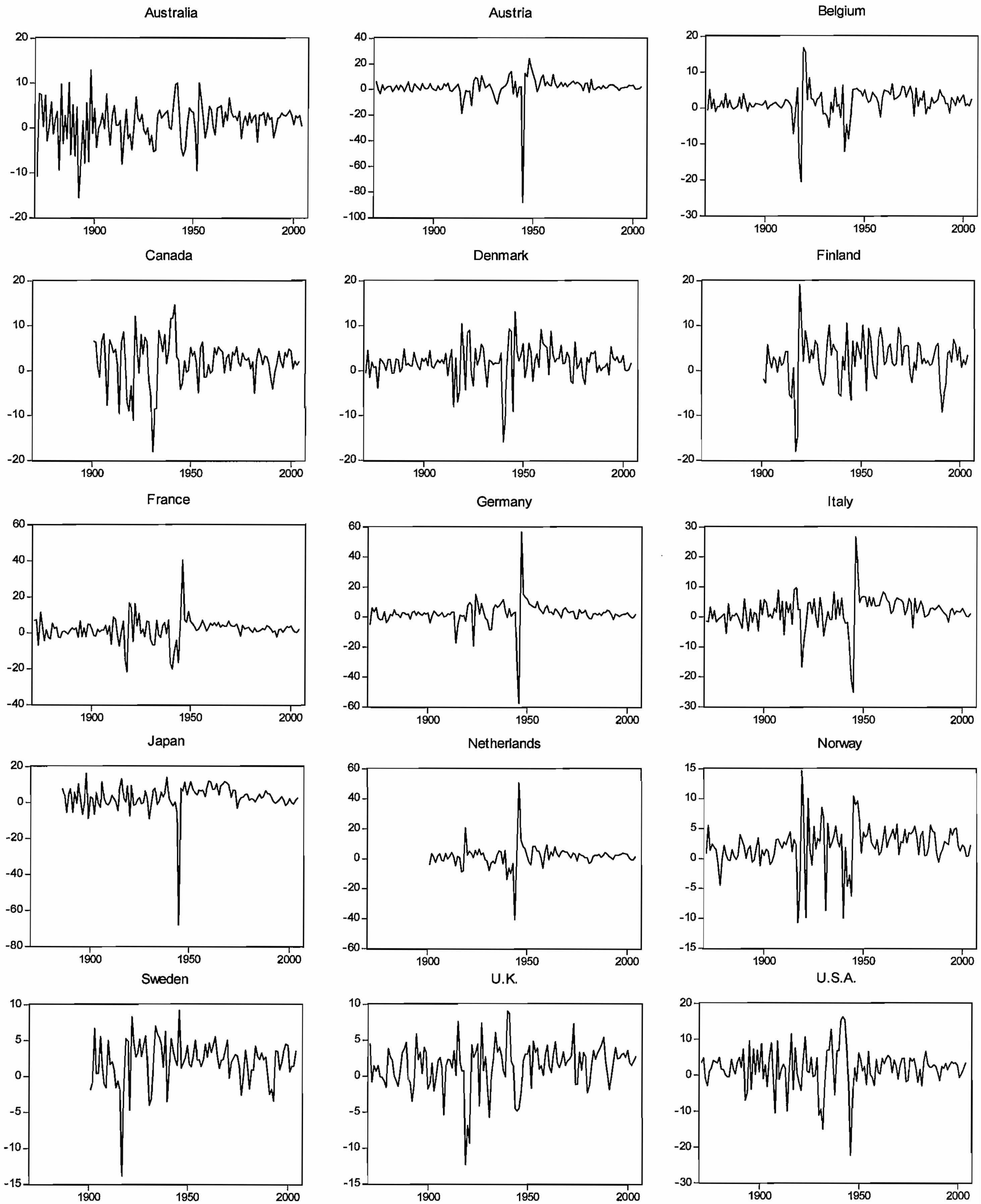
$$g_t = \mu + \rho g_{t-1} + \theta(L)\Delta g_{t-1} + \varepsilon_t$$

The lag length of $\theta(L)$ is chosen by the Schwartz information criteria. The t-statistics are in parenthesis.

The time trend test reports the coefficient of a time trend estimated with OLS, and Newey-West corrected t-statistics are in parenthesis.

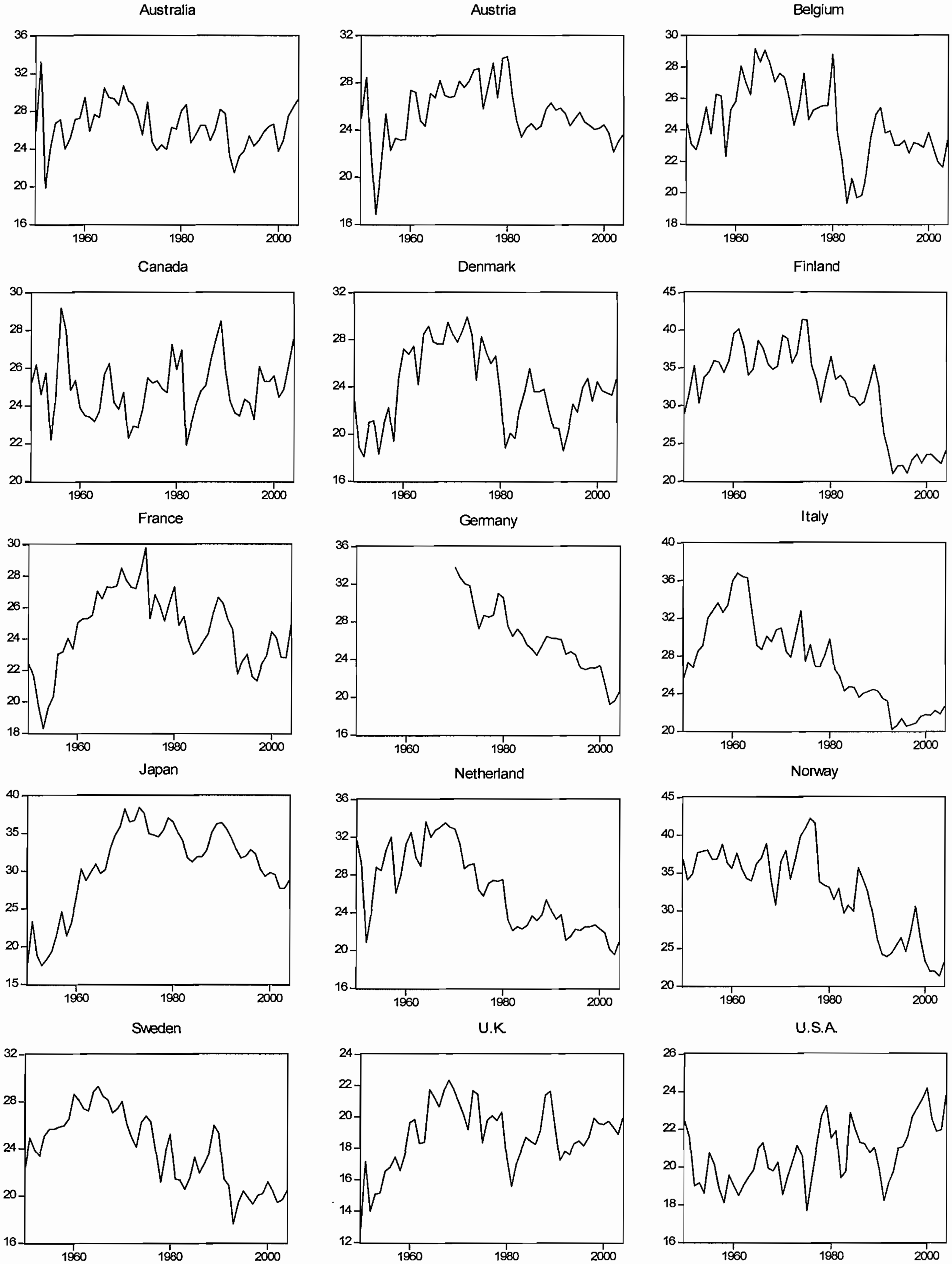
Significance levels are denoted by (*) for 10 percent, (**) for 5 percent, and (***) for 1 percent. The data starts from 1900 in order to avoid the problems associated with border changes, as in Bernard [1991].

Figure 1
Per Capita GDP Growth Rates, 1870-2004



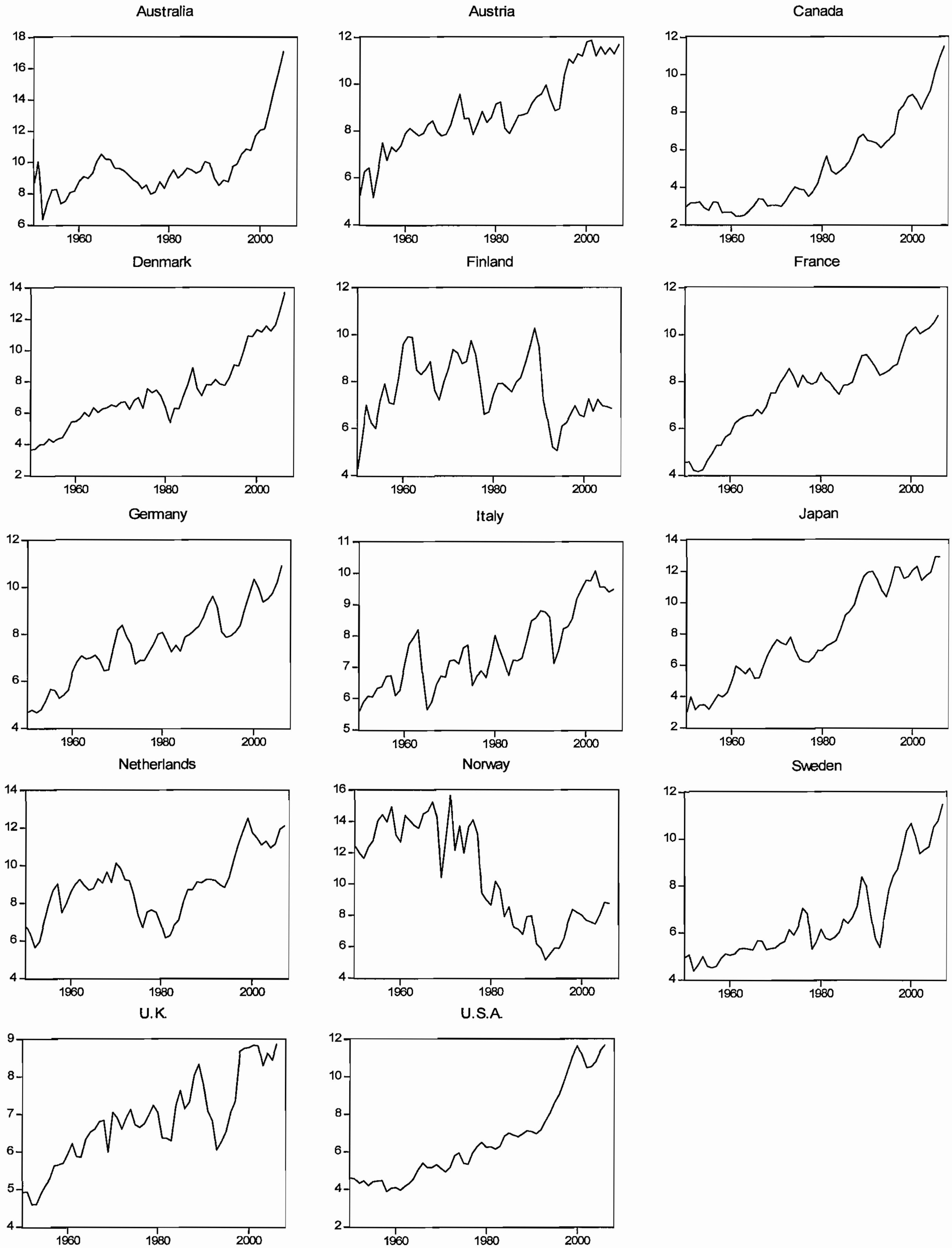
Source. Per capita GDP data for the period 1870-1987 are constructed from Maddison [1982, 1989] by Bernard [1991]. The author extended the growth rates using real GDP per capita (Constant Prices: Chain series) from PWT 6.1 for the period 1950-2004.

Figure 2
Total Investment Rates, 1950-2004



Sourse. PWT 6.1

Figure 3
Producer Durables Investment Rates, 1950-2006



Source. The data for the period 1950-1988 are from Robert Summers.
The author extended data for the period after 1988 using the data from the OECD Department of Economic and Statistics Analytic Database.

Table 2
ADF and Time Trend Tests of Investment Rates, 1950-1988

Country	Total investment		Producer durables investment	
	ADF test	Time trend	ADF test	Time trend
AUSTRALIA	0.171	-0.083	0.534	0.030
	(-4.98)***	(-2.36)	(-3.24)*	(2.28)**
AUSTRIA	0.589	0.279	0.507	0.071
	(-3.02)	(5.82)***	(-3.12)	(4.98)***
BELGIUM	0.794	0.034	-	-
	(-2.06)	(0.51)	-	-
CANADA	0.531	0.083	0.844	0.077
	(-2.95)	(2.41)**	(-1.23)	(4.88)***
DENMARK	0.885	-0.018	0.651	0.096
	(-1.36)	(-0.15)	(-2.66)	(6.86)***
FINLAND	0.540	-0.068	0.646	0.042
	(-3.31)*	(-0.89)	(-3.39)*	(1.57)
FRANCE	0.916	0.166	0.902	0.113
	(-1.17)	(2.12)**	(-1.29)	(7.13)***
GERMANY	0.769	-0.146	0.607	0.086
	(-2.27)	(-2.63)**	(-3.44)*	(7.62)***
ITALY	0.797	-0.095	0.086	0.037
	(-2.53)	(-1.06)	(-4.35)***	(4.50)***
JAPAN	0.899	0.426	0.821	0.159
	(-1.41)	(3.62)***	(-1.56)	(9.60)***
NETHERLANDS	0.843	-0.140	0.854	0.008
	(-1.72)	(-1.72)*	(-1.70)	(0.25)
NORWAY	0.573	-0.036	0.667	-0.155
	(-3.02)	(-0.82)	(-2.73)	(-3.27)***
SWEDEN	0.819	-0.033	0.444	0.052
	(-1.82)	(-0.55)	(-3.61)**	(7.80)***
U.K.	0.723	0.158	0.606	0.066
	(-2.62)	(3.49)***	(-2.73)	(6.96)***
U.S.A.	0.028	0.068	0.591	0.080
	(-5.74)***	(2.85)***	(-3.16)	(7.34)***

Notes: See the notes to Table I, except note that the ADF tests in this table include a time trend.

Table 3
ADF and Time Trend Tests of Investment Rates, 1950-2004

	Total investment		Producer durables investment	
Country	ADF test	Time trend	ADF test	Time trend
AUSTRALIA	0.270	-0.030	0.992	0.079
	(-5.29)***	(-1.18)	(-0.09)	(3.13)***
AUSTRIA	0.649	-0.005	0.629	0.090
	(-3.27)*	(-0.14)	(-3.65)**	(12.06)***
BELGIUM	0.688	-0.071	-	-
	(-3.11)	(-2.79)***	-	-
CANADA	0.457	0.009	0.962	0.133
	(-4.23)***	(0.59)	(-0.55)	(8.29)***
DENMARK	0.797	-0.018	0.912	0.132
	(-2.40)	(-0.38)	(-1.05)	(10.10)***
FINLAND	0.801	-0.266	0.718	-0.012
	(-2.24)	(-4.10)***	(-3.83)**	(-0.63)
FRANCE	0.853	0.004	0.849	0.100
	(-2.06)	(0.09)	(-2.33)	(13.19)***
GERMANY	0.247	-0.331	0.626	0.085
	(-2.51)	(-11.36)***	(-4.76)***	(11.77)***
ITALY	0.744	-0.234	0.587	0.061
	(-3.41)*	(-4.69)***	(-4.04)**	(7.97)***
JAPAN	0.920	0.178	0.754	0.181
	(-1.58)	(2.07)**	(-3.13)	(25.49)***
NETHERLANDS	0.710	-0.195	0.867	0.067
	(-2.94)	(-4.70)***	(-2.21)	(4.67)***
NORWAY	0.725	-0.281	0.755	-0.155
	(-2.89)	(-6.98)***	(-2.70)	(-6.22)***
SWEDEN	0.699	-0.141	0.772	0.099
	(-3.10)	(-4.34)***	(-2.92)	(7.39)***
U.K.	0.668	0.038	0.737	0.060
	(-3.68)**	(1.37)	(-2.82)	(7.79)***
U.S.A.	0.536	0.054	0.924	0.130
	(-4.16)***	(3.19)***	(-1.54)	(8.44)***

Notes: See the notes to Table II.

Table 4
Pooled OLS for the Estimation of Dynamic Responses

	Total Investment		Producer Durables Investment	
Variables	1950-1988	1950-2004	1950-1988	1950-2004
Constant	3.614	2.978	4.403	3.863
CTREND_AUS	-0.017	-0.009	-0.031	-0.026
CTREND_AUT	-0.094	-0.054	-0.135	-0.087
CTREND_BEL	-0.034	-0.030	-	-
CTREND_CAN	-0.010	-0.011	-0.026	-0.027
CTREND_DNK	-0.022	-0.039	-0.054	-0.052
CTREND_FIN	-0.012	-0.020	-0.032	-0.033
CTREND_FRA	-0.068	-0.049	-0.099	-0.064
CTREND_DEU	-0.086	-0.040	-0.130	-0.081
CTREND_ITA	-0.089	-0.075	-0.128	-0.097
CTREND_JPN	-0.151	-0.115	-0.214	-0.170
CTREND_NLD	-0.038	-0.022	-0.085	-0.041
CTREND_NOR	0.020	-0.011	0.053	-0.015
CTREND_SWE	-0.065	-0.020	-0.082	-0.035
CTREND_GBR	0.010	0.011	-0.010	0.004
CTREND_USA	0.002	-0.001	0.001	-0.016
DCI	0.640	0.909	-	-
DCI(-1)	0.044	-0.199	-	-
DCI(-2)	0.004	0.011	-	-
DCI(-3)	-0.008	-0.003	-	-
DCI(-4)	-0.016	0.024	-	-
DCI(-5)	0.135	0.036	-	-
DPDCI	-	-	0.935	1.664
DPDCI(-1)	-	-	0.125	0.106
DPDCI(-2)	-	-	-0.141	0.051
DPDCI(-3)	-	-	0.161	0.345
DPDCI(-4)	-	-	-0.104	-0.147
GRGDP(-1)	0.054	0.254	0.066	0.141
GRGDP(-2)	-0.026	0.006	-0.132	-0.100
GRGDP(-3)	0.103	0.009	0.053	-0.020
GRGDP(-4)	0.006	-0.046	0.026	0.029
GRGDP(-5)	-0.045	-0.003	-	-
Country Fixed Effects	yes	yes	yes	yes
Period Fixed Effects	no	no	no	no
Adjusted R-squared	0.452	0.624	0.275	0.379
nob	480	715	462	700

Notes: Dependent variable is the log change in real per capita GDP (GRGDP). DCI and DPDCI (Δi in the equation below) are the first differences in total investment and in producer durables investment respectively. The regression is estimated by the pooled OLS with cross-section fixed effects.

$$g_{it} = \alpha_i + b_i t_i + A(L) g_{it-1} + C(L) \Delta i_{it} + \varepsilon_{it}$$

As for the length of lag polynomials $A(L)$ and $C(L)$, five lags are used for the specification with total investment, and four lags are used for the specification with producer durables investment. Due to the data availability, the specification with producer durables investment does not include Belgium.

"CTREND__" are the country-specific time trends for each country, where AUS is Australia, AUT is Austria, BEL is Belgium, CAN is Canada, DNK is Denmark, FIN is Finland, FRA is France, DEU is Germany, ITA is Italy, JPN is Japan, NLD is Netherlands, NOR is Norway, SWE is Sweden, GBR is the United Kingdom, and finally, USA is the United States.

t-statistics are omitted from the results, since these are trivial for the estimation of dynamic responses.

Table 5
Dynamic Response of Growth Rates and Output to a One-Percentage-Point Permanent Increase in the Investment Rate (based on the original data: 1950-1988)

	Total investment		Producer Durable Investment	
Period (year)	OLS dynamic response	OLS cumulative response	OLS dynamic response	OLS cumulative response
0	0.640	0.640	0.935	0.935
1	0.079	0.719	0.002	0.937
2	-0.008	0.710	-0.092	0.845
3	0.055	0.766	0.197	1.042
4	-0.001	0.765	-0.054	0.988
5	0.104	0.869	-0.034	0.953
6	0.008	0.877	0.013	0.966
7	-0.002	0.875	0.008	0.974
8	0.008	0.883	-0.004	0.970
9	0.002	0.885	-0.002	0.968
10	-0.005	0.880	0.001	0.969
15	0.000	0.880	0.000	0.969
20	0.000	0.880	0.000	0.969

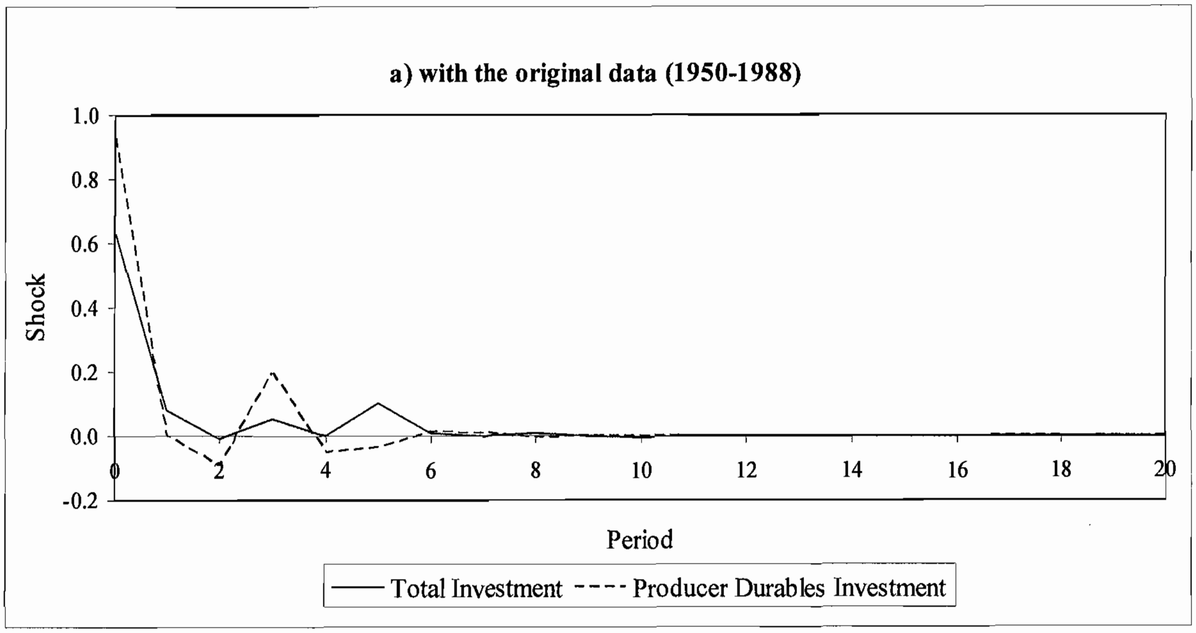
Notes: The OLS dynamic responses are calculated using the estimated coefficients of Δi and g from Table IV.

Table 6
Dynamic Response of Growth Rates and Output to a One-Percentage-Point Permanent Increase in the Investment Rate (updated: 1950-2004)

	Total investment		Producer durable investment	
Period (year)	OLS dynamic response	OLS cumulative response	OLS dynamic response	OLS cumulative response
0	0.909	0.909	1.664	1.664
1	-0.194	0.715	0.341	2.005
2	0.018	0.733	-0.067	1.937
3	-0.046	0.687	0.268	2.205
4	-0.031	0.656	-0.061	2.144
5	0.034	0.690	-0.024	2.120
6	0.008	0.698	-0.005	2.116
7	0.004	0.702	0.011	2.126
8	0.003	0.705	0.001	2.127
9	-0.001	0.704	-0.002	2.125
10	-0.001	0.703	-0.001	2.125
15	-0.001	0.703	0.000	2.125
20	0.000	0.703	0.000	2.125

Notes: See the notes for Table V.

Figure 4
Dynamic Response of Growth Rates and Output to a One-Percentage-Point Permanent Increase in the Investment Rate



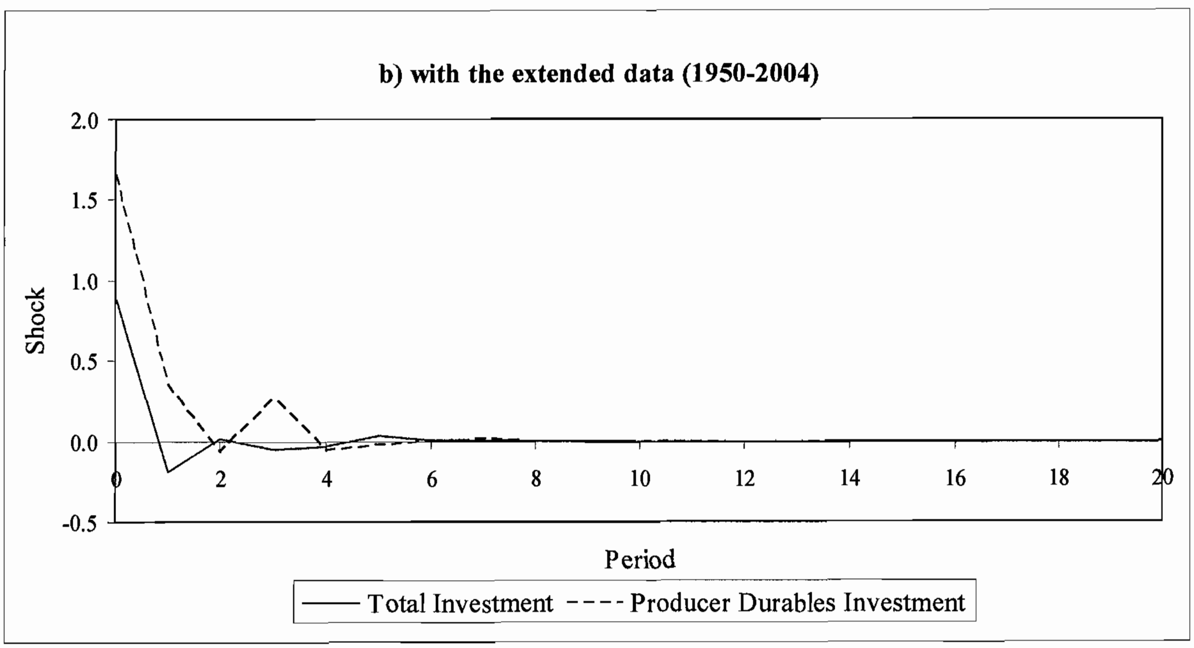


Table 7

Samples with non-stationarity, 1950-1988

Dynamic Response of Growth Rates and Output to a One-Percentage- Point Permanent Increase in the Investment Rate

	Total investment		Producer durable investment	
Period (year)	OLS dynamic response	OLS cumulative response	OLS dynamic response	OLS cumulative response
0	0.674	0.674	0.795	0.795
1	0.086	0.759	0.096	0.890
2	-0.003	0.756	-0.020	0.871
3	0.080	0.837	0.367	1.238
4	0.005	0.842	-0.013	1.225
5	0.100	0.942	-0.021	1.204
6	0.011	0.953	0.016	1.220
7	0.000	0.952	0.013	1.233
8	0.009	0.961	-0.002	1.231
9	0.002	0.963	-0.001	1.230
10	0.000	0.963	0.001	1.232
15	0.001	0.964	0.000	1.232
20	0.000	0.964	0.000	1.232

Notes: See the text for the explanation.

Table 8

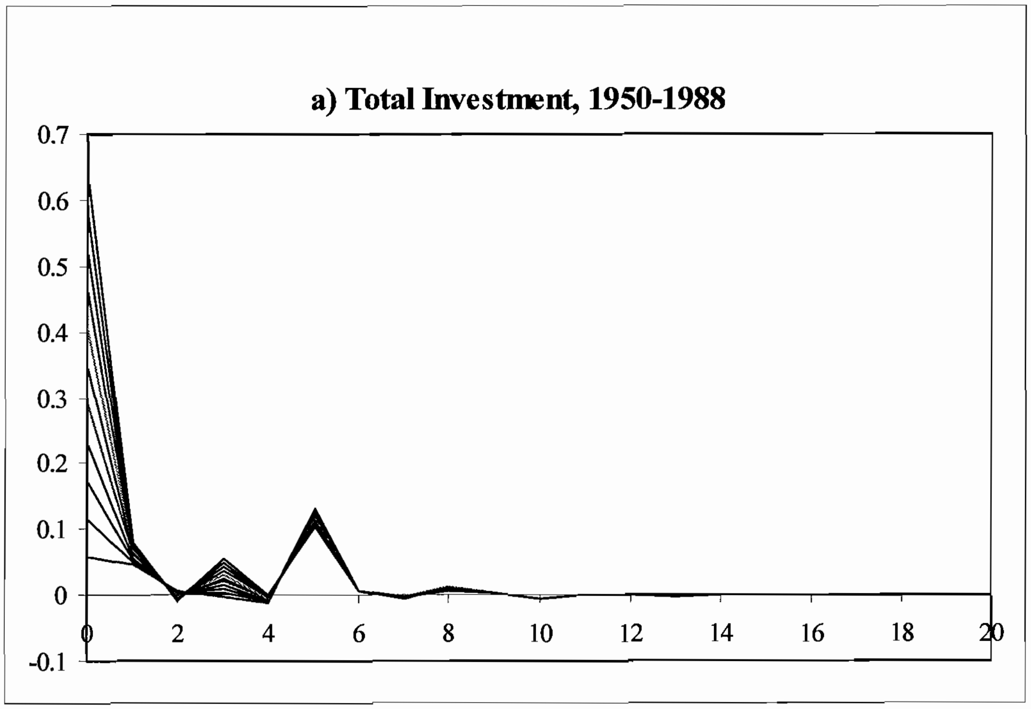
Samples with non-stationarity, 1950-2004

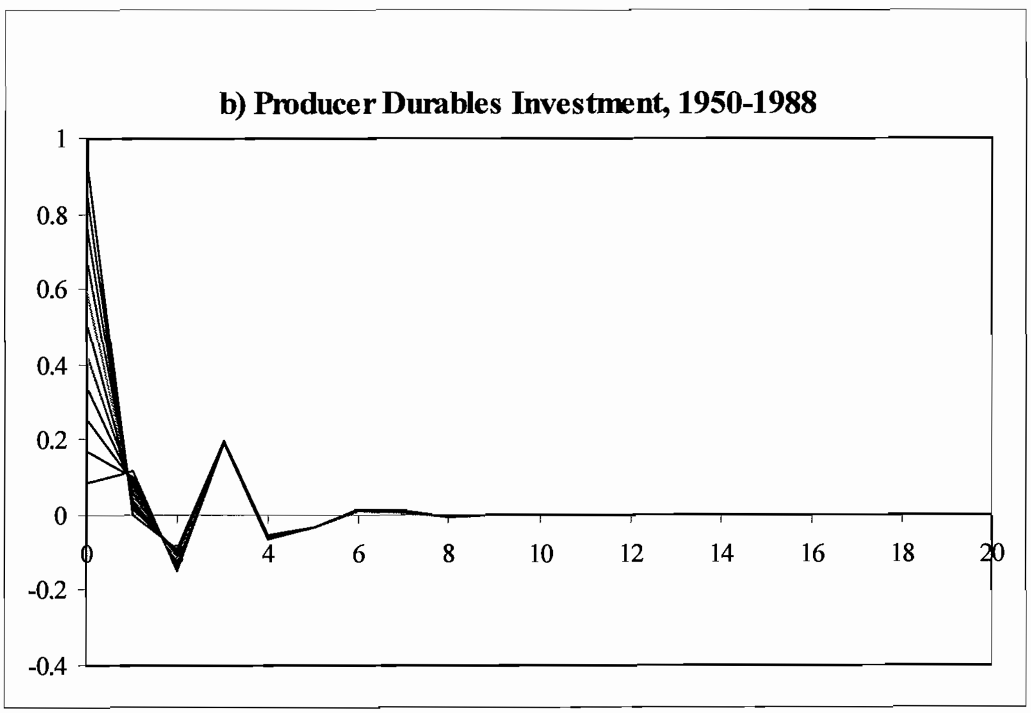
Dynamic Response of Growth Rates and Output to a One-Percentage- Point Permanent Increase in the Investment Rate

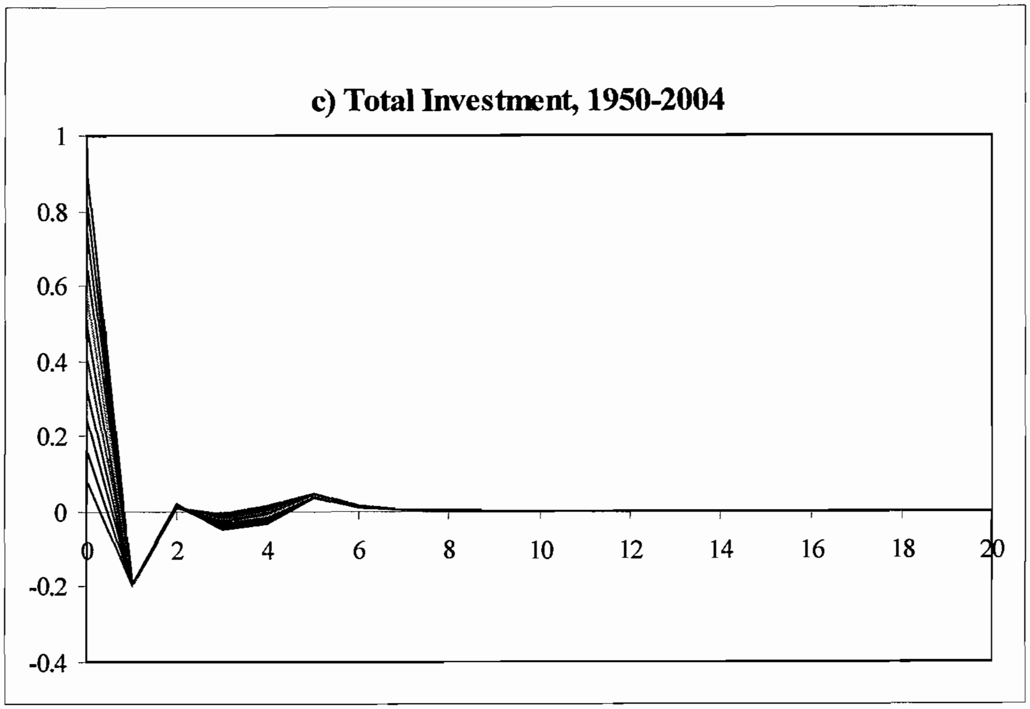
	Total investment		Producer durable investment	
Period (year)	OLS dynamic response	OLS cumulative response	OLS dynamic response	OLS cumulative response
0	0.842	0.842	1.371	1.371
1	-0.201	0.640	0.258	1.629
2	0.082	0.722	-0.026	1.603
3	-0.040	0.683	0.301	1.904
4	-0.046	0.637	0.020	1.924
5	0.040	0.677	-0.013	1.911
6	-0.001	0.676	-0.006	1.904
7	0.003	0.679	0.009	1.914
8	0.004	0.683	0.003	1.916
9	-0.003	0.680	-0.001	1.915
10	0.000	0.680	-0.001	1.915
15	0.000	0.680	0.000	1.915
20	0.000	0.680	0.000	1.915

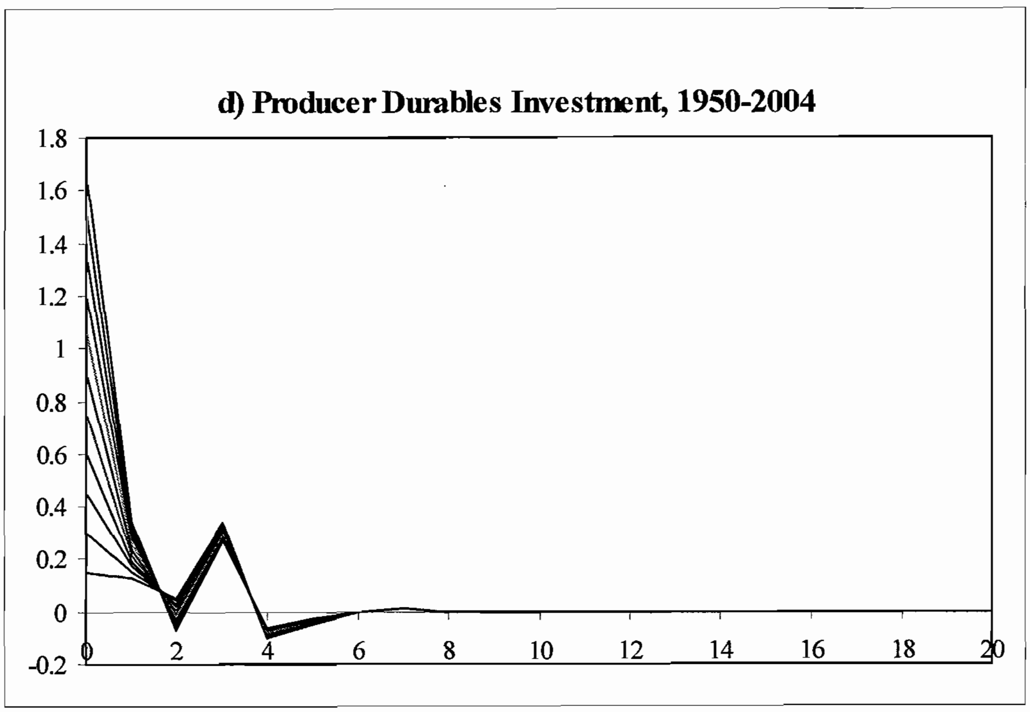
Notes: See the text for the explanation.

Figure 5 (a) - (d)
OLS Bounds






c) Total Investment, 1950-2004
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
2
4
6
8
10
12
14
16
18
20


d) Producer Durables Investment, 1950-2004
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20

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