

Demonstrating worker quality through strategic absenteeism.

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March 27, 2003

Abstract

Determining the productivity of individual workers engaged in team production is difficult. Monitoring expenses may be high, or the observable output of the entire team may be some single product. One way to collect information about individual productivity is to observe how total output changes when the composition of the team changes. While some employers may explicitly shift workers from team to team for exactly this reason, the most common reasons for changes in team composition are at least partly voluntary: vacation time and sick days. In this paper, we develop a model of optimal absenteeism by employees which accounts for strategic interactions between employees. We assume the employer uses both observed changes in output and the strategies of the employees to form beliefs about a given worker's type. We argue that the model we develop is applicable to a variety of workplace situations where signaling models are not, because it allows a worker's decisions to provide information about other workers, as well as about himself.

JEL codes C72, J22. Keywords: Absenteeism, Non-cooperative Games, Signaling, Teamwork.

The authors wish to thank Rick Harbaugh, John Hillas, Van Kolpin, Michael Maschler, Henk Norde, and Dries Vermeulen for their helpful comments and suggestions.

1 Introduction

In 1996 the Board of Governors of the Federal Reserve, responding to a string of fraud cases, told banks that

”One of the many basic tenets of internal control is that a banking organization ensure that employees in sensitive positions be absent from their duties for a minimum of two consecutive weeks. Such a requirement enhances the viability of a sound internal control environment because most frauds or embezzlements require the continual presence of the wrongdoer.”

Member banks were then instructed that

”...a minimum of two consecutive weeks absence be required of employees in sensitive positions.” (Board of Governors, 1996).

This is an extreme example - one where productivity is actually negative - of a situation where an employer can learn something about an individual worker’s job performance by observing what happens during his absence from work. Other examples include productive workers who deliberately take time off to show the boss how vital they are to the firm, and not-so-productive workers who are afraid to take a sick day for fear that their employer will realize that the business does just fine without them.

Such situations often involve team production. With team production, determining the productivity of individual workers can be difficult either because monitoring expenses are high, or because the observable output of the entire team is some single product. In many situations, employers will want to know workers’ individual productivities so that they can pay the more productive workers wages high enough to keep them, while paying lower wages

to the less productive ones. At the same time, high-quality workers will want to convince the employer of their type, while low types will want to hide theirs. One way that the employer can get information about individual productivity is to watch how output changes when the composition of the team changes. While some employers may require workers to take time off from the team, as the Fed recommends above, changes in team composition are also made voluntarily by workers when they absent themselves from work using vacation time and sick days. In this paper we analyze the strategic behavior of employees in such situations and use this analysis to provide new explanations for some aspects of vacation behavior and vacation policies.

There is a large literature covering many forms of signaling and screening in employment situations, beginning with Spence (1973). Several papers consider the problem of an employer trying to screen out low-quality workers by setting the terms of employment. For example, Rebitzer and Taylor (1995) argue that employers may require workers to work long hours to prove that they have a low taste for leisure, and Landers et al. (1996) show that a signaling model along these lines can explain why associates in law firms work long hours. Our model differs from these in three ways. First, none of these models address the phenomena of paid or mandatory vacations. Second, our model incorporates two types of information - information from observations of productivity changes, if the composition of the team changes, and information from vacation-taking decisions. Third, as we will show, our model allows for strategic interactions between the workers. One worker's decision to take a vacation, or not take one, reveals information about the types of both workers.

2 The Model

The basic model in this paper has one employer with two workers producing some joint product. The situation we have in mind is one where two workers are generally employed because of increasing returns to scale, but one worker can still produce alone. The employer can measure team output periodically, but cannot observe individual output, which is random. The high-type worker is more likely to produce high output than the low-type worker is. Workers know their own type and their co-worker's type, while the employer only knows that he has one worker of each type. We only consider the case where the employer has one of each type of worker, rather than a random draw of workers, because after enough observations of team output he can use his knowledge of the output probabilities and the high and low output levels to determine with high probability whether his team consists of two low-type workers, one high-type worker and one low-type worker, or two high-type workers. Note that this provides the employer with information on the composition of the team only and not on the types of individual workers, as he does not observe their individual output but only team output. Only when the employer knows he has one worker of each type will the employees be concerned with conveying further information to him.

Each employee can choose to work continuously or take a fixed period of time off. For exposition, we use the case of vacations. We set the vacation length at two weeks, and for simplicity we also assume that output is observed at the end of every two week period. Employees must make their vacation decision without knowing the other's decision, and they cannot take vacations simultaneously.¹ If both workers work continuously, then the employer gets no new information on their types. If one worker takes time off, then the employer has

¹There are many situations in which employees have to decide early in the year whether or not they are going to take a vacation and the employer does not allow them to take vacations at the same time.

one observation of the individual output of the other worker and can use this observation to form an expectation about the type of that worker accordingly. Note that this also enables the employer to form an expectation about the type of the worker who takes time off, since the employer already knows one of them is the low type and the other one is the high type. If both workers take vacations, the employer has an observation of the individual output of each worker and forms an expectation of the type of each worker accordingly.² After observing the employees' actions and measuring output, the employer sets wages for the workers based on his estimates of the types of the workers, and then pays those wages for a one year period, including the vacations, if any. The employer can use both vacation behavior and observations on output when determining wages for the individual workers. Note that this implies that it is not automatically optimal for the high-type worker to take a vacation, as the employer might take the fact that he is taking a vacation as a signal that he is the low type, irrespective of the output observation. As the beliefs of the employer play an important role, we will consider sequential equilibria, to obtain reasonable restrictions on beliefs in information sets off the equilibrium path. We now set up the formal model, starting with the workers.

2.1 The workers

Each worker is one of a team of two workers, A and B , one of whom is high type and the other low type. In each period each worker produces either low output q_l or high output q_h , where $q_l < q_h$.³ Output is subject to a shock, so that the high-type worker produces q_h with probability $p_h \in (0, 1)$ (and q_l with probability $1 - p_h$) and the low-type worker produces

²The case in which the employer may get multiple output observations is addressed in section 3.

³Increasing returns to scale would imply that team output is greater than the sum of individual outputs. Since the employer only updates based on the information he gets when workers work by themselves, we can make the simplifying assumption of constant returns without affecting the results of the model.

q_h with probability $p_l \in (0, 1)$, where $p_l < p_h$. We assume output is determined by ability, so that the workers cannot change the probability of high output by changing effort. The workers know their type and their co-worker's type. The employer knows that he employs one high-type worker and one low-type worker, but not whether A is the high type and B the low one or vice versa. The workers can try to convey information about their types to the employer by taking a vacation and thereby changing the composition of the team. A vacation is always one period (two weeks) long. The employer prohibits both workers from taking vacations during the same period, and the workers must decide whether or not to take a vacation at the beginning of the year, without knowledge of the other worker's decision. The payoff of a worker consists of two components: his wage (which depends on the employer's estimate about his type), and the utility from a vacation, $v > 0$, if the worker takes one.

2.2 The employer

A priori the employer assigns a probability of $\frac{1}{2}$ to A being the high type and B the low one, or vice versa. If the employer knew the workers' types with certainty, he would pay the high type wage w_h and the low type wage w_l with $w_l < w_h$, where wages are assumed to be paid (or at least determined) annually. Not knowing the types of his individual workers with certainty, the employer chooses an $\alpha \in [0, 1]$ according to which he distributes the total wage pie, which is interpreted as “pay A wage $(1 - \alpha)w_l + \alpha w_h$ and pay B wage $\alpha w_l + (1 - \alpha)w_h$ ”. Hence, the employer's strategy space is $[0, 1]$. The employer can re-distribute wages among his employees, each employee getting a wage between w_l and w_h , but we do not allow him to, for example, cut both employees' wages. Note that the employer would choose $\alpha = 1$ if he knew that A is the high type and that he would choose $\alpha = 0$ if he knew that A is the

low type. We define the payoffs to the employer so as to capture the fact that he wants to pay wages that reflect the actual productivity of the workers, and the farther he is from this target the worse off he is. Specifically, if the employer chooses $\alpha \in [0, 1]$, then his payoff is $-(1 - \alpha)^2$ if A is the high type (and B the low type) and it is $-\alpha^2$ if A is the low type (and B the high type). Later we will extend the scope of our study by replacing this specific payoff function with some general assumptions about how the employer will use the information he obtains to set wages. Such an approach has the advantage of producing results that apply to a variety of different employer environments. It turns out that the workers' strategies in sequential equilibria are quite robust to variations in the employer's strategy.

2.3 The game

Figure 1 represents a part of the extensive-form game. First, nature determines whether worker A is the high type and worker B the low type or the other way around. Each of these events has a probability of $\frac{1}{2}$. Then, the workers learn their types and independently choose whether to take a vacation (denoted by the Y of "Yes, I am going to take a vacation") or not (denoted by the N of "No, I am not going to take a vacation"). Note the information sets for worker B that indicate that he is not aware of worker A 's choice when making the decision to take a vacation or not. After both workers have made their decisions, nature determines, with the appropriate probabilities, whether A and/or B produce high or low output if and when the other worker is taking a vacation. For example, if A is the high type and decides to take a vacation, while B (who then is the low type) decides not to take a vacation, then the employer has one observation of B working alone and none of A working alone. In the period when the low type B is working alone, he produces high output (denoted by q_h^B) with probability p_l and low output (denoted by q_l^B) with probability $1 - p_l$. Note that if neither of

the workers takes a vacation, then the employer has no observations of any worker working alone. After observing the workers' outputs while working alone, the employer chooses the parameter α which determines the redistribution of wages. In order to not clutter the figure too much, we exclude this last stage. So, the right-most nodes in the figure are not the end nodes of the game, but they are the decision nodes of the employer, and they are grouped in information sets, each of which consists of two nodes to indicate that the employer cannot observe which one of his workers is the low type and which one is the high type. After the employer has chosen an $\alpha \in [0, 1]$, the game reaches an end node in which the payoffs to both workers and the employer are as described above.

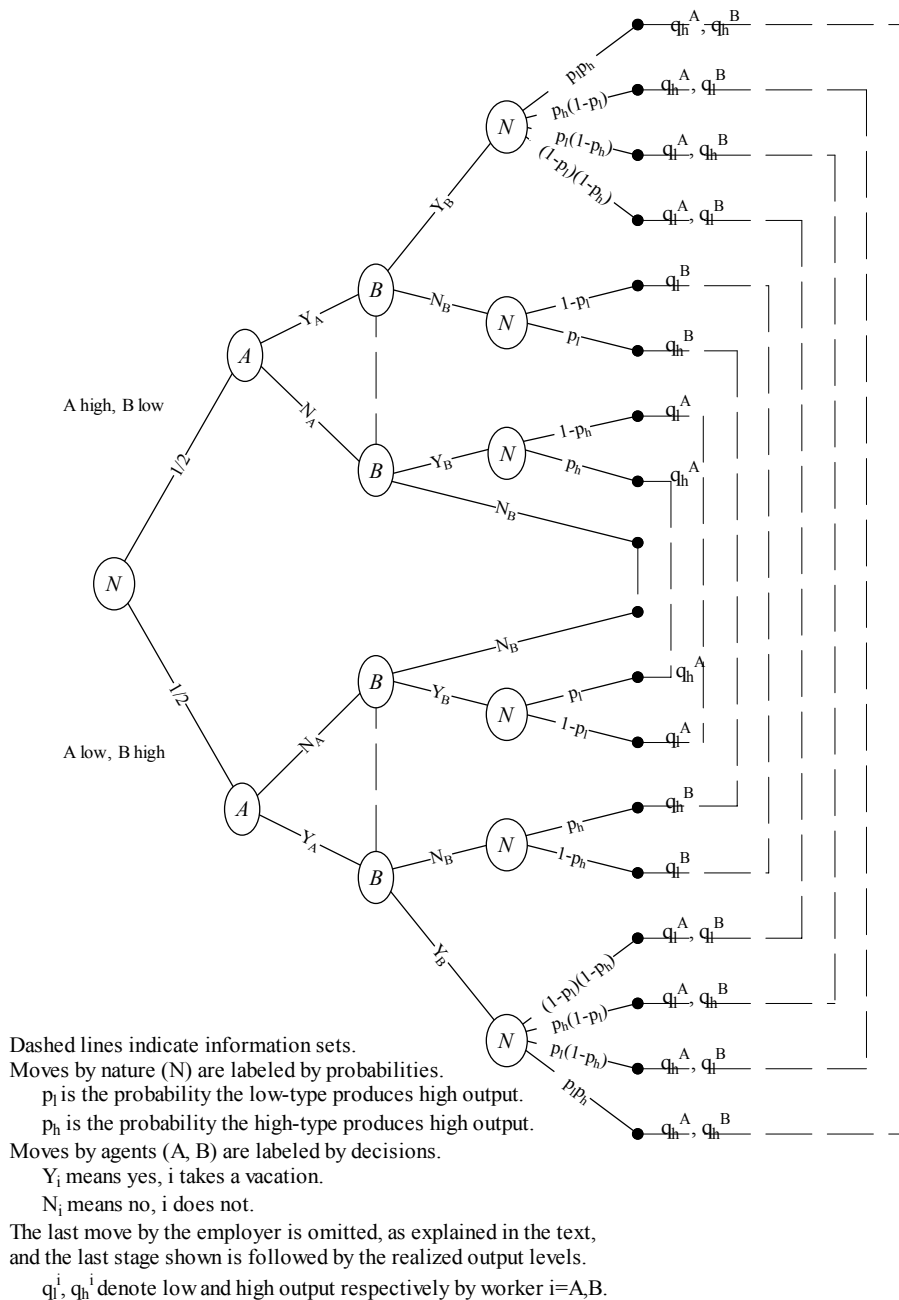


Figure 1: Abbreviated extensive form game.

2.4 Analysis of the game

We consider two equilibrium definitions in the analysis of this game. First, a *Nash equilibrium* of the game is a strategy profile σ that specifies a strategy for each of the players (workers A and B and the employer), such that each player's strategy maximizes his payoff given the strategies of the remaining players. The game has nine information sets for the employer. Hence, a strategy for the employer consists of nine components, each of which prescribes the action to be taken by the employer in one of the nine information sets. Given two strategies for the workers, if a certain information set of the employer is reached with positive probability, then the employer can use Bayes' rule to update his beliefs about the workers' types and base his choice of action on these beliefs. The actions chosen by the employer in information sets that are not reached when the workers follow their strategies, have no influence on the employer's (expected) payoff and are therefore not limited by the employer's payoff maximization requirement. Note, however, that these parts of the employer's strategy might influence the workers' (expected) payoffs for various strategies. For example, the employer might threaten a worker with some severe retaliation if the worker deviates from his equilibrium strategy, and so give the worker an incentive to indeed follow that strategy.

We use the *sequential equilibrium* of Kreps and Wilson (1982) to impose restrictions on beliefs in information sets off the equilibrium path. A sequential equilibrium consists of beliefs π and a strategy profile σ such that the following two conditions hold.

1. π is fully consistent with σ : there exists a sequence of completely mixed strategies that converge to σ such that the beliefs corresponding to these completely mixed strategies converge to π . Note that the beliefs corresponding to a completely mixed strategy are completely determined by Bayes' rule.
2. Each player's strategy in σ is sequentially rational at every information set with beliefs

π : σ maximizes the players' payoffs given π at every information set.

A strategy profile σ is called a *sequential equilibrium profile* if there exist beliefs π such that π and σ together form a sequential equilibrium. The sequential equilibrium restriction is sensible because it requires that the employer's beliefs (and therefore his payoff-maximizing actions) in information sets that are not reached if the workers follow their equilibrium strategies must still be reasonable, in the sense that they can be derived from mixed strategies that are arbitrarily close to the strategies actually played. In information sets that are on the equilibrium path, the beliefs are still those obtained by Bayes' rule.

Note that every sequential equilibrium profile is a Nash equilibrium, but that the converse does not necessarily hold because the Nash equilibrium definition places no constraints on the employer's actions in information sets that are off the equilibrium path. Hence, if a strategy profile is a Nash equilibrium but not a sequential equilibrium profile, then it can be sustained as an equilibrium only by choices of actions that are not supported by beliefs that are fully consistent with the strategies in information sets that are off the equilibrium path. This implies that even if a certain strategy profile is a Nash equilibrium, it is worth investigating whether it is a sequential equilibrium profile as well. In what follows, we will mainly focus on sequential equilibria. However, when appropriate, we will also discuss Nash equilibria.

We start by considering the employer's choice in each one of his information sets. In a sequential equilibrium, the employer must maximize his payoff given his beliefs about the workers' types. Denote by β the employer's belief that A is the high type (and, consequently, that B is the low type). Then his expected payoff from choosing a wage distribution $\alpha \in [0, 1]$ is $-(1 - \alpha)^2 \beta - \alpha^2 (1 - \beta)$. Taking the derivative with respect to α and setting it equal to 0, we find that the employer's payoff-maximizing action is to choose $\alpha = \beta$.

At this point, we are going to take a little detour. The payoff function that we chose for

the employer is rather arbitrary and the reader might get the impression that the results that we obtain later will only hold for this specific function. However, the results are quite robust. To substantiate this statement, consider wage-distribution strategies by the employer that are more general than the one that we found above. Specifically, assume that the wage distribution depends on the employer's beliefs according to some function $f : [0, 1] \rightarrow [0, 1]$. If the employer believes that A is the high-type worker with probability β (and, consequently, that B is the high-type worker with probability $1 - \beta$), then he will pay A a wage $(1 - f(\beta))w_l + f(\beta)w_h$. That is, he will choose the action $\alpha = f(\beta)$.

We assume that f satisfies $f(1 - \beta) = 1 - f(\beta)$ for all $\beta \in [0, 1]$. This is a very mild symmetry assumption: it amounts to saying that it is the employer's beliefs about his workers' types, and not their names, that determine his choice of wage distribution. This form of symmetry implies that the wage paid to A under beliefs β should be the same as the wage paid to B under beliefs $1 - \beta$. By definition of the strategy of the employer, the first of these wages equals $(1 - f(\beta))w_l + f(\beta)w_h$ and the second one equals $f(1 - \beta)w_l + (1 - f(1 - \beta))w_h$. Since $w_l < w_h$, these two wages are equal only if $f(1 - \beta) = 1 - f(\beta)$. We will later use the result that $f(\frac{1}{2}) = \frac{1}{2}$, which is easily found by applying $f(1 - \beta) = 1 - f(\beta)$ to $\beta = \frac{1}{2}$. A second assumption on f is $f(1) = 1$. This is basically a re-iteration of the definition of the wage w_h , since it means that the employer pays a worker wage w_h if he believes that this worker is the high-type worker with probability 1. Note that the first and second assumptions on f together imply that $f(0) = 0$. Our third and last assumption on f is that it is non-decreasing and that $f(\beta) > \frac{1}{2}$ if $\beta > \frac{1}{2}$. Again, we believe this is a pretty weak assumption. It means that the employer will not decrease a worker's wage if his belief that this worker is the high-type worker increases, and that he will pay his two workers different wages if he assigns different probabilities to each being the high-type worker.

One function that satisfies all the properties discussed above is $f(\beta) = \beta$. According

to the wage-distribution strategy described by this function, the employer pays A wage $(1 - \beta)w_l + \beta w_h$ if he believes that worker A is the high type with probability β . This is the strategy that we found above, which is therefore included in the general analysis that follows. Another interesting function satisfying all the properties is given by $f(\beta) = 0$ for $\beta < \frac{1}{2}$, $f(\frac{1}{2}) = \frac{1}{2}$, and $f(\beta) = 1$ for $\beta > \frac{1}{2}$. This function represents a situation in which a worker is paid a wage w_l if he is believed to be low type with probability more than $\frac{1}{2}$, a wage $\frac{1}{2}w_l + \frac{1}{2}w_h$ if he is believed to be high type with probability $\frac{1}{2}$, and a wage w_h if he is believed to be high type with probability more than $\frac{1}{2}$. In any realistic environment, the optimal behavior of the employer can surely be represented by some $f(\beta)$ that satisfies the assumptions above, which amount to little more than requiring that the employer does not pay less to a worker when he gets some information that increases his belief that this worker is the high type.

The next step is to consider the workers' choices in a sequential equilibrium. As the workers are symmetric in the game, we will restrict attention to sequential equilibria in which the workers play symmetric strategies. We investigate the four possible symmetric pure strategy combinations of the workers to see if these, together with a strategy $f(\beta)$ for the employer, are sequential equilibrium profiles. As our results turn out to be insensitive to variations in the employer's optimal strategy $f(\beta)$ (within the parameters explained above), we will suppress it and concentrate on the workers' strategy choices. Note that we have set the game up with two workers, labeled A and B , one assigned by nature to be low type and the other high type. Therefore the strategies we consider are properly given in terms of decisions by the workers A and B . However, it seems more intuitive to think of the workers as the low type and the high type, and so we also discuss the results from that angle.

We first investigate the strategy in which both types of workers take a vacation. In the following theorem we show that this strategy is part of a sequential equilibrium profile.

Theorem 1 *For any strategy $f(\cdot)$ of the employer, there is a sequential equilibrium in which A and B both decide to take a vacation, irrespective of their types.*

Proof. Denote the strategy of worker A to always take a vacation by Y^A and the strategy of B to always take a vacation by Y^B . To prove that these strategies are part of a sequential equilibrium profile we have to find beliefs π such that $(Y^A, Y^B, f(\cdot))$ and π satisfy conditions 1 and 2 for a sequential equilibrium.

The beliefs that 'do the job' are as follows: if neither worker takes a vacation, then the employer's posterior beliefs are equal to his prior beliefs $\beta = \frac{1}{2}$. If one worker takes a vacation and the other does not, then the posterior belief of the employer is that the worker taking vacation is the high-type worker, no matter what output the other worker produced when working alone. If both workers take a vacation, then the posterior beliefs are based on the output observations of the two workers working alone. If both A and B take a vacation, then the employer has an observation of A working alone and an observation of B working alone. Obviously, if both A and B produce the same output while working alone, then the employer gets no new information about their types and his posterior beliefs are equal to his prior beliefs, so $\beta = \frac{1}{2}$. But suppose A produces q_h while B is gone and B produces q_l while A is gone. This happens with probability $p_h(1 - p_l)$ if A is the high-type worker and B the low-type worker and it happens with probability $p_l(1 - p_h)$ if A is the low-type worker and B the high-type worker. Hence the posterior belief of the employer that A is the high-type worker is $\beta = \frac{p_h(1-p_l)}{p_h(1-p_l)+p_l(1-p_h)} > \frac{1}{2}$. Or, suppose A produces q_l while B is gone and B produces q_h while A is gone. Then the employer's posterior beliefs will be $\beta = \frac{(1-p_h)p_l}{(1-p_h)p_l+(1-p_l)p_h} < \frac{1}{2}$. We refer to the set of beliefs described above as π .

With these beliefs, and given that the other worker takes a vacation, both the high-type worker and the low-type worker want to take a vacation, because not taking a vacation will lead the employer to believe that he is the low-type worker and result in wage w_l . But

taking a vacation too will lead to an expected wage no lower than w_l and in addition the vacation itself brings a utility of v . We conclude that condition 2 for a sequential equilibrium is satisfied: $(Y^A, Y^B, f(\cdot))$ is sequentially rational with beliefs π .

To prove that π is fully consistent with $(Y^A, Y^B, f(\cdot))$, consider the sequence of completely mixed strategies $(\sigma_t)_{t=1}^\infty$ for the workers: according to σ_t both A and B choose to take a vacation with probability $1 - \frac{1}{t}$ if they are the low-type worker and with probability $1 - \frac{1}{t^2}$ if they are the high-type worker. Clearly, the strategies σ_t converge to (Y^A, Y^B) . The posterior beliefs that are obtained from the σ_t 's using Bayes's rule converge to the beliefs π . Checking this is straightforward, and we will only show one case. Suppose worker A takes a vacation and worker B doesn't and worker B produces q_h while working alone. According to strategy σ_t this happens with probability $(1 - \frac{1}{t^2}) \frac{1}{t} p_l$ if B is the low-type worker and with probability $(1 - \frac{1}{t}) \frac{1}{t^2} p_h$ if B is the high-type worker. Therefore, the posterior belief of the employer that A is the high-type worker is $\pi_t = \frac{(1 - \frac{1}{t^2}) \frac{1}{t} p_l}{(1 - \frac{1}{t^2}) \frac{1}{t} p_l + (1 - \frac{1}{t}) \frac{1}{t^2} p_h}$. Taking the limit gives the desired result: $\lim_{t \rightarrow \infty} \pi_t = \lim_{t \rightarrow \infty} \frac{(1 - \frac{1}{t^2}) p_l}{(1 - \frac{1}{t^2}) p_l + (1 - \frac{1}{t}) \frac{1}{t^2} p_h} = \frac{p_l}{p_l} = 1$. Note that this is the posterior belief according to π . ■

In the following theorem we show that the strategy where the high-type worker takes a vacation and the low-type does not is not part of a Nash equilibrium profile. This, of course, immediately implies that there is no sequential equilibrium in which the high-type worker takes a vacation and the low-type does not.

Theorem 2 *For any strategy $f(\cdot)$ of the employer, there is no Nash equilibrium (and, hence, no sequential equilibrium) in which both A and B decide to take a vacation if they are the high-type worker and not to take a vacation if they are the low-type worker.*

Proof. Denote the strategy of worker A to take a vacation if he is the high-type worker and not to take a vacation if he is the low-type worker by (Y_h^A, N_l^A) and the same strategy of

B by (Y_h^B, N_l^B) . Consider a strategy profile $((Y_h^A, N_l^A), (Y_h^B, N_l^B), f(\cdot))$. Then, if one worker takes a vacation and the other doesn't, it follows from Bayes' law that the posterior belief of the employer is that the worker taking the vacation is the high-type worker, no matter what output the other worker produced when working alone. Hence, in this case an employer who is giving a best response to the strategies of the workers will pay the vacationing worker a wage w_h and the other worker a wage w_l . Now suppose A is the high-type worker and B the low-type worker. Then A will take a vacation and B should not take a vacation according to his strategy (Y_h^B, N_l^B) . However, this will result in a wage w_l for B . If, however, B diverts from his strategy and takes a vacation, then he will not have a lower expected wage, no matter what action α the employer chooses in this case, and on top of that B will derive utility from having a vacation. This shows that (Y_h^B, N_l^B) is not a best response for B . ■

The intuition behind theorem 2 is that if the employer uses vacation behavior as a signal, then the low-type worker reveals himself to be low type if he does not take a vacation and the high-type worker does. Hence, not taking a vacation brings about a low wage and also deprives the worker of the utility that he would derive from taking a vacation. A worker's output does not matter to the employer's posterior beliefs and therefore to wages, because it provides only probabilistic information about his type which will never be enough to offset the firm's deterministic belief based on the vacation behavior.

We now turn our attention to the strategy in which the low-type worker takes a vacation and the high-type worker does not. The following example illustrates that this strategy can be part of a Nash equilibrium.

Example: Suppose that $v = \frac{1}{2}(w_h - w_l)$. Denote the strategy of worker A to take a vacation if he is the low-type worker and not to take a vacation if he is the high-type worker by (Y_l^A, N_h^A) and the similar strategy of B by (Y_l^B, N_h^B) . Suppose the employer plays the following strategy. If exactly one worker takes a vacation, then pay the vacationing worker a

wage w_l and the other worker a wage w_h , no matter what output is observed. If no worker takes a vacation or if both workers take a vacation, then pay each a wage $\frac{1}{2}w_l + \frac{1}{2}w_h$. This strategy could be used by an employer who bases his decisions on the vacation behavior of the workers solely and who does not use any information on output. This strategy for the employer is a best response on his part, as the only information sets reached with positive probability are those in which exactly one worker takes a vacation and in these information sets the posterior beliefs derived using Bayes' rule are that the vacationing worker is the low-type worker. Now, let us demonstrate that the strategies (Y_l^A, N_h^A) and (Y_l^B, N_h^B) are best responses for workers A and B, respectively.

Suppose A is the high-type worker and B the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and does not take a vacation. Then B should take a vacation according to his strategy (Y_l^B, N_h^B) . This will result in a wage w_l for B and the utility v from having a vacation. If, however, B diverts from his strategy and does not take a vacation, then he will have a higher wage, namely $\frac{1}{2}w_l + \frac{1}{2}w_h$, but he will lose the utility from having a vacation. Taking a vacation is optimal for B because $w_l + v = \frac{1}{2}w_l + \frac{1}{2}w_h$.

Now, suppose B is the high-type worker and A the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and takes a vacation. If B sticks with his strategy (Y_l^B, N_h^B) and does not take a vacation he gets wage w_h . If he does take a vacation, he will gain the utility v from having a vacation, but he gets a lower wage $\frac{1}{2}w_l + \frac{1}{2}w_h$. Not taking a vacation is optimal for B because $w_h = \frac{1}{2}w_l + \frac{1}{2}w_h + v$.

We have shown that (Y_l^B, N_h^B) is a best response for worker B. Using symmetry, we easily see that (Y_l^A, N_h^A) is a best response for worker A as well. This establishes that the strategy profile considered is a Nash equilibrium.

We show in the following theorem that, even though the strategy in which the low-type worker takes a vacation and the high-type worker does not can be part of a Nash equilibrium,

it is not part of any sequential equilibrium profile.

Theorem 3 *For any strategy $f(\cdot)$ of the employer, there is no sequential equilibrium in which both A and B decide to take a vacation if they are the low-type worker and not to take a vacation if they are the high-type worker.*

Proof. Denote the strategy of worker A to take a vacation if he is the low-type worker and not to take a vacation if he is the high-type worker by (Y_l^A, N_h^A) and the similar strategy of B by (Y_l^B, N_h^B) . To prove that these strategies are not part of a sequential equilibrium profile we show that for any beliefs π that are fully consistent with $((Y_l^A, N_h^A), (Y_l^B, N_h^B), f(\cdot))$ condition 2 for a sequential equilibrium is violated.

Choose any beliefs π that are fully consistent with $((Y_l^A, N_h^A), (Y_l^B, N_h^B), f(\cdot))$. Let $(\sigma_t^A, \sigma_t^B)_{t=1}^\infty$ be a sequence of completely mixed strategies for the workers that converges to $((Y_l^A, N_h^A), (Y_l^B, N_h^B))$: according to σ_t^A worker A chooses to take a vacation with probability $1 - \gamma_t^A$ if he is the low-type worker and with probability δ_t^A if he is the high-type worker, whereas according to σ_t^B worker B chooses to take a vacation with probability $1 - \gamma_t^B$ if he is the low-type worker and with probability δ_t^B if he is the high-type worker, where $\lim_{t \rightarrow \infty} \gamma_t^A = \lim_{t \rightarrow \infty} \delta_t^A = \lim_{t \rightarrow \infty} \gamma_t^B = \lim_{t \rightarrow \infty} \delta_t^B = 0$.

It follows fairly easily that if one worker takes a vacation and the other doesn't, then the posterior belief of the employer according to π is that the worker taking the vacation is the low-type worker, no matter what output the other worker produced when working alone. Suppose, for example, that A takes a vacation and B does not and, moreover, B produces high output while working alone. According to strategies (σ_t^A, σ_t^B) this happens with probability $\delta_t^A \gamma_t^B p_l$ if A is the high-type worker and B the low-type worker and with probability $(1 - \gamma_t^A)(1 - \delta_t^B) p_h$ if A is the low-type worker and B the high-type worker. Therefore, the posterior belief of the employer that A is the high-type worker is $\pi_t = \frac{\delta_t^A \gamma_t^B p_l}{\delta_t^A \gamma_t^B p_l + (1 - \gamma_t^A)(1 - \delta_t^B) p_h}$.

Taking the limit gives the desired result: $\lim_{t \rightarrow \infty} \pi_t = \frac{0}{0+1p_h} = 0$.

If both workers do not take a vacation, then the employer's posterior belief that A is the high-type worker is $\pi(N^A, N^B) = \lim_{t \rightarrow \infty} \frac{(1-\delta_t^A)\gamma_t^B}{(1-\delta_t^A)\gamma_t^B + \gamma_t^A(1-\delta_t^B)}$. It is impossible to determine what exactly the beliefs π of the employer are, because these can be practically anything ranging from 0 to 1, depending on how fast γ_t^A , δ_t^A , γ_t^B , and δ_t^B approach zero relative to one another.

If both workers take a vacation, then the employer has two observations of output. We will compute the posterior belief according to π that A is the high-type worker. Suppose A produces high output while working alone and B produces low output while working alone. According to strategies (σ_t^A, σ_t^B) this happens with probability $\delta_t^A(1-\gamma_t^B)p_h(1-p_l)$ if A is the high-type worker and B the low-type worker and with probability $(1-\gamma_t^A)\delta_t^B p_l(1-p_h)$ if A is the low-type worker and B the high-type worker. Therefore, the posterior belief of the employer that A is the high-type worker is

$$\pi(q_h^A, q_l^B) = \lim_{t \rightarrow \infty} \pi_t(q_h^A, q_l^B) = \lim_{t \rightarrow \infty} \frac{\delta_t^A(1-\gamma_t^B)p_h(1-p_l)}{\delta_t^A(1-\gamma_t^B)p_h(1-p_l) + (1-\gamma_t^A)\delta_t^B p_l(1-p_h)}.$$

Similar computations show that

$$\begin{aligned} \pi(q_l^A, q_h^B) &= \lim_{t \rightarrow \infty} \pi_t(q_l^A, q_h^B) = \lim_{t \rightarrow \infty} \frac{\delta_t^A(1-\gamma_t^B)p_l(1-p_h)}{\delta_t^A(1-\gamma_t^B)p_l(1-p_h) + (1-\gamma_t^A)\delta_t^B p_h(1-p_l)}, \\ \pi(q_h^A, q_h^B) &= \lim_{t \rightarrow \infty} \pi_t(q_h^A, q_h^B) = \lim_{t \rightarrow \infty} \frac{\delta_t^A(1-\gamma_t^B)p_h p_l}{\delta_t^A(1-\gamma_t^B)p_h p_l + (1-\gamma_t^A)\delta_t^B p_l p_h}, \text{ and} \\ \pi(q_l^A, q_l^B) &= \lim_{t \rightarrow \infty} \pi_t(q_l^A, q_l^B) = \lim_{t \rightarrow \infty} \frac{\delta_t^A(1-\gamma_t^B)(1-p_h)(1-p_l)}{\delta_t^A(1-\gamma_t^B)(1-p_h)(1-p_l) + (1-\gamma_t^A)\delta_t^B(1-p_l)(1-p_h)}. \end{aligned}$$

To simplify these expressions somewhat, it is helpful to introduce $\varepsilon_t := \frac{\delta_t^A(1-\gamma_t^B)}{\delta_t^A(1-\gamma_t^B) + (1-\gamma_t^A)\delta_t^B}$.

Then, $\frac{1-\varepsilon_t}{\varepsilon_t} = \frac{(1-\gamma_t^A)\delta_t^B}{\delta_t^A(1-\gamma_t^B)}$ and, moreover,

$$\begin{aligned}\pi_t(q_h^A, q_h^B) &= \pi_t(q_l^A, q_l^B) = \varepsilon_t, \\ \pi_t(q_h^A, q_l^B) &= \frac{1}{1 + \frac{1-\varepsilon_t}{\varepsilon_t} \frac{p_l(1-p_h)}{p_h(1-p_l)}}, \\ \pi_t(q_l^A, q_h^B) &= \frac{1}{1 + \frac{1-\varepsilon_t}{\varepsilon_t} \frac{p_h(1-p_l)}{p_l(1-p_h)}}.\end{aligned}$$

Note that $\frac{p_l(1-p_h)}{p_h(1-p_l)} < \frac{p_h(1-p_l)}{p_l(1-p_h)}$, so that $\pi_t(q_h^A, q_l^B) > \pi_t(q_l^A, q_h^B)$. We conclude that

$$\pi(q_h^A, q_l^B) \geq \pi(q_l^A, q_h^B),$$

which we will use later.

Now, we will prove that the strategies (Y_l^A, N_h^A) and (Y_l^B, N_h^B) are not sequentially rational with beliefs π . Suppose A is the high-type worker and B the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and does not take a vacation. Then B should take a vacation according to his strategy (Y_l^B, N_h^B) . This will result in a wage w_l for B and the utility v from having a vacation. If, however, B diverts from his strategy and does not take a vacation, then he will have a possibly higher expected wage, namely $f(\pi(N^A, N^B))w_l + (1 - f(\pi(N^A, N^B)))w_h$, but he will lose the utility from having a vacation. Hence, it is optimal for B to take a vacation if $w_l + v \geq f(\pi(N^A, N^B))w_l + (1 - f(\pi(N^A, N^B)))w_h$, or $v \geq (1 - f(\pi(N^A, N^B)))(w_h - w_l)$.

Now, suppose B is the high-type worker and A the low-type worker. Also suppose A plays according to his strategy (Y_l^A, N_h^A) and takes a vacation. If B sticks with his strategy (Y_l^B, N_h^B) and does not take a vacation he gets wage w_h . If he does take a vacation, he will

gain the utility v from having a vacation, but he gets a possibly lower expected wage

$$\begin{aligned}
& p_l p_h [f(\pi(q_h^A, q_h^B)) w_l + (1 - f(\pi(q_h^A, q_h^B))) w_h] \\
& + p_l (1 - p_h) [f(\pi(q_h^A, q_l^B)) w_l + (1 - f(\pi(q_h^A, q_l^B))) w_h] \\
& + (1 - p_l) p_h [f(\pi(q_l^A, q_h^B)) w_l + (1 - f(\pi(q_l^A, q_h^B))) w_h] \\
& + (1 - p_l) (1 - p_h) [f(\pi(q_l^A, q_l^B)) w_l + (1 - f(\pi(q_l^A, q_l^B))) w_h] \\
= & c w_l + (1 - c) w_h,
\end{aligned}$$

where $c := f(\pi(q_h^A, q_h^B)) p_l p_h + f(\pi(q_h^A, q_l^B)) p_l (1 - p_h) + f(\pi(q_l^A, q_h^B)) (1 - p_l) p_h + f(\pi(q_l^A, q_l^B)) (1 - p_l) (1 - p_h)$. Hence, it is optimal for B not to take a vacation if $w_h \geq c w_l + (1 - c) w_h + v$, or $v \leq c(w_h - w_l)$.

We have shown that for (Y_l^B, N_h^B) to be sequentially rational for B with beliefs π it must hold that

$$v \geq (1 - f(\pi(N^A, N^B))) (w_h - w_l) \text{ and } v \leq c(w_h - w_l).$$

In a similar fashion we derive that for (Y_l^A, N_h^A) to be sequentially rational for A with beliefs π it must hold that

$$v \geq f(\pi(N^A, N^B)) (w_h - w_l) \text{ and } v \leq (1 - d) (w_h - w_l),$$

where $d := f(\pi(q_h^A, q_h^B)) p_l p_h + f(\pi(q_h^A, q_l^B)) p_h (1 - p_l) + f(\pi(q_l^A, q_h^B)) (1 - p_h) p_l + f(\pi(q_l^A, q_l^B)) (1 - p_l) (1 - p_h)$.

We proceed by showing that it is impossible for the four conditions

$$\begin{aligned} v &\geq (1 - f(\pi(N^A, N^B)))(w_h - w_l), \\ v &\leq c(w_h - w_l), \\ v &\geq f(\pi(N^A, N^B))(w_h - w_l) \text{ and} \\ v &\leq (1 - d)(w_h - w_l) \end{aligned}$$

to all hold at the same time. Suppose that these conditions all hold. Then it follows that

$$\begin{aligned} 1 - f(\pi(N^A, N^B)) &\leq c, \\ 1 - f(\pi(N^A, N^B)) &\leq 1 - d, \\ f(\pi(N^A, N^B)) &\leq c \text{ and} \\ f(\pi(N^A, N^B)) &\leq 1 - d \end{aligned}$$

all have to hold, from which we derive that $c \geq \frac{1}{2}$ and $d \leq \frac{1}{2}$. However,

$$d - c = (f(\pi(q_h^A, q_l^B)) - f(\pi(q_l^A, q_h^B)))(p_h(1 - p_l) - p_l(1 - p_h)) \geq 0,$$

where the inequality follows because $p_h > p_l$ and $\pi(q_h^A, q_l^B) \geq \pi(q_l^A, q_h^B)$. Hence, the only possibility is that $c = d = \frac{1}{2}$ and $f(\pi(q_h^A, q_l^B)) = f(\pi(q_l^A, q_h^B))$. In view of the conditions on $f(\cdot)$, $f(\pi(q_h^A, q_l^B)) = f(\pi(q_l^A, q_h^B))$ can only hold if either $\pi(q_h^A, q_l^B) = \pi(q_l^A, q_h^B) = \frac{1}{2}$, or if $\pi(q_h^A, q_l^B)$ and $\pi(q_l^A, q_h^B)$ are both strictly larger than $\frac{1}{2}$, or if $\pi(q_h^A, q_l^B)$ and $\pi(q_l^A, q_h^B)$ are both strictly smaller than $\frac{1}{2}$. Clearly, $\pi(q_h^A, q_l^B) = \pi(q_l^A, q_h^B) = \frac{1}{2}$ is impossible as this would imply that $\lim_{t \rightarrow \infty} \frac{1 - \varepsilon_t}{\varepsilon_t} = \frac{p_h(1 - p_l)}{p_l(1 - p_h)}$ and at the same time $\lim_{t \rightarrow \infty} \frac{1 - \varepsilon_t}{\varepsilon_t} = \frac{p_l(1 - p_h)}{p_h(1 - p_l)}$. So, suppose that $\pi(q_h^A, q_l^B)$ and $\pi(q_l^A, q_h^B)$ are both strictly larger than $\frac{1}{2}$ (the case in which both are strictly

smaller than $\frac{1}{2}$ can be handled in a similar manner). This means that $\lim_{t \rightarrow \infty} \frac{1-\varepsilon_t}{\varepsilon_t} < 1$ has to hold, or $\lim_{t \rightarrow \infty} \varepsilon_t > \frac{1}{2}$. Then also $\pi(q_h^A, q_h^B) = \pi(q_l^A, q_l^B) = \lim_{t \rightarrow \infty} \varepsilon_t > \frac{1}{2}$. Hence, $f(\pi(q_h^A, q_h^B))$, $f(\pi(q_h^A, q_l^B))$, $f(\pi(q_l^A, q_h^B))$, and $f(\pi(q_l^A, q_l^B))$ are all strictly larger than $\frac{1}{2}$, so that both c and d are strictly larger than $\frac{1}{2}$. This gives us a contradiction. ■

The preceding proof is so complicated because it is possible to find beliefs that are fully consistent with the proposed strategies such that the strategy for worker A is sequentially rational or such that the strategy for worker B is sequentially rational. Therefore we have to show that it is impossible to find beliefs that are fully consistent with the strategies such that *simultaneously* the strategies for workers A and B are sequentially rational. Considered from a different angle, the proposed strategies for the workers provide restrictions on beliefs that are fully consistent with these strategies, and these conditions guarantee that *either* the prescribed strategy for worker A does not maximize his payoff in one of his information sets *or* the prescribed strategy for worker B does not maximize his payoff in one his information sets.

The last strategies we consider are those in which neither the high type nor the low type decides to take a vacation. We find that these strategies are part of a sequential equilibrium profile if the utility from taking a vacation is low enough.

Theorem 4 *For any strategy $f(\cdot)$ of the employer, there is a sequential equilibrium in which A and B both decide not to take a vacation, irrespective of their types, iff $v \leq \frac{1}{2}(w_h - w_l)$.*

Proof. Denote the strategy of worker A to never take a vacation by N^A and the strategy of B to never take a vacation by N^B . Assume that $v \leq \frac{1}{2}(w_h - w_l)$. To prove that the strategies $(N^A, N^B, f(\cdot))$ form a sequential equilibrium profile we have to find beliefs π such that $(N^A, N^B, f(\cdot))$ and π satisfy conditions 1 and 2 for a sequential equilibrium.

Beliefs that 'do the job' are as follows: if both workers take a vacation, and the employer

has two observations, then the posterior beliefs are the beliefs β based on output that we have computed in the proof of theorem 1. If one worker takes a vacation and the other does not, then the posterior belief of the employer is that the worker taking vacation is the low-type worker, no matter what output the other worker produced when working alone. If neither worker takes a vacation, then the employer's posterior beliefs are equal to his prior beliefs. We refer to this set of beliefs as π .

With these beliefs, and given that the other worker does not take a vacation, neither the high-type worker nor the low-type worker wants to take a vacation. This can be seen as follows: taking a vacation will lead the employer to believe that he is the low-type worker and result in wage w_l and the vacation itself brings a utility of v . But taking no vacation will lead to the higher expected wage $(1 - f(\frac{1}{2}))w_l + f(\frac{1}{2})w_h = \frac{1}{2}w_l + \frac{1}{2}w_h$. So, taking no vacation is sequentially rational iff $\frac{1}{2}w_l + \frac{1}{2}w_h \geq w_l + v$, or $v \leq \frac{1}{2}(w_h - w_l)$.

To prove that π is fully consistent with $(N^A, N^B, f(\cdot))$, consider the sequence of completely mixed strategies $(\sigma_t^A, \sigma_t^B)_{t=1}^\infty$ for the two workers: according to (σ_t^A, σ_t^B) both A and B choose to take a vacation with probability $\frac{1}{t}$ if they are the low-type worker and with probability $\frac{1}{t^2}$ if they are the high-type worker. Clearly, the strategies σ_t converge to (N^A, N^B) . The posterior beliefs that are obtained from the (σ_t^A, σ_t^B) 's using Bayes' rule converge to the beliefs π . Checking this is a straightforward exercise so we will only do so for one case, as an example.

Suppose worker A takes a vacation and worker B doesn't and worker B produces q_h while working alone. According to strategy (σ_t^A, σ_t^B) this happens with probability $\frac{1}{t^2}(1 - \frac{1}{t})p_l$ if B is the low-type worker and with probability $\frac{1}{t}(1 - \frac{1}{t^2})p_h$ if B is the high-type worker. Therefore, the posterior belief of the employer is that A is the high-type worker is $\pi_t = \frac{\frac{1}{t^2}(1 - \frac{1}{t})p_l}{\frac{1}{t^2}(1 - \frac{1}{t})p_l + \frac{1}{t}(1 - \frac{1}{t^2})p_h}$. Taking the limit gives the desired result: $\lim_{t \rightarrow \infty} \pi_t =$

$\lim_{t \rightarrow \infty} \frac{\frac{1}{t}(1-\frac{1}{t})^{p_l}}{\frac{1}{t}(1-\frac{1}{t})^{p_l} + (1-\frac{1}{t^2})^{p_h}} = \frac{0}{0+p_h} = 0$. Note that this is the posterior belief according to π .

We have chosen a specific set of beliefs π that are fully consistent with $(N^A, N^B, f(\cdot))$ and for this set of beliefs we obtained the condition $v \leq \frac{1}{2}(w_h - w_l)$ necessary for (N^A, N^B) to be sequentially rational. We could have chosen other sets of beliefs that are fully consistent with $(N^A, N^B, f(\cdot))$ and then we could have found another upper bound on v . However, the upper bound so obtained can be no larger than $\frac{1}{2}(w_h - w_l)$. This can be seen as follows. It is not hard to show that the posterior beliefs of the employer if none of the workers takes a vacation must be equal to his prior beliefs if the beliefs are to be fully consistent with $(N^A, N^B, f(\cdot))$. The beliefs after observing one worker taking a vacation can be practically anything. However, they will always lead to a worker taking a vacation receiving an expected wage $w \geq w_l$ and a utility v from the vacation. Hence, taking no vacation is sequentially rational if $\frac{1}{2}w_l + \frac{1}{2}w_h \geq w + v$, or $v \leq \frac{1}{2}w_h - w + \frac{1}{2}w_l \leq \frac{1}{2}(w_h - w_l)$. ■

The beliefs used in this proof are such that if only one worker takes a vacation, the employer believes that this person is the low-type worker with probability one. This might seem strange, since *ex ante* the only worker who could benefit from the firm observing the output change in his absence is the high-type worker. However, as we have discussed before, the firm also gains information from a worker's vacation choice itself, and the worker also gains utility from the vacation, so this belief is not implausible. For example, suppose that the firm has the above belief. Now suppose that only one worker takes a vacation, gaining him the utility from vacation time, and the firm then observes that output falls by more than one-half during this worker's vacation. Is the firm's belief that this worker is the low type unreasonable? Not necessarily - since there is a random component to output, it is quite possible that the worker who took the vacation is low-type. Given this, the firm might well put more weight on the vacation choice itself than on the observed output, so the firm's

belief that this worker is low-type is not unreasonable. In addition, while the probability-one belief is easiest to work with, we show in the proof that it is also possible to use beliefs that assign a probability strictly less than one to the vacationing worker being the low type.

In theorems 1 through 4 we addressed the question of whether the 4 possible symmetric pure strategy combinations for the workers are part of sequential equilibrium profiles. We did not address mixed strategies in those theorems. We were unable to solve for mixed strategy sequential equilibrium profiles analytically, so we checked for the existence of such profiles for specific parameter values, setting $f(\beta) = \beta$. For all the parameter values we examined, when a mixed equilibrium existed it was unique.

3 Discussion

We have shown that we can support two different symmetric pure strategies for the workers in sequential equilibria. One equilibrium profile is where both workers take vacations, another is where neither worker takes a vacation. The cases where only one type of worker takes a vacation are *not* supportable in sequential equilibrium under *any* consistent beliefs. We begin this section of the paper by considering if one of the two pure strategy sequential equilibrium profiles is more plausible. We then ask whether the employer might prefer other outcomes, and consider what sorts of changes in vacation policy might produce results that are more in the employer's interests.

Which pure strategy equilibrium profile is achieved depends first on the employer's beliefs, and in the case of the (N^A, N^B) equilibrium profile there is also the necessary requirement that $v \leq \frac{1}{2}(w_h - w_l)$. Since if this requirement is not satisfied, the (N^A, N^B) equilibrium profile is not supportable by *any* consistent beliefs, it is worth asking if this condition is likely to hold. Suppose that w_l and w_h denote annual wages, that a vacation is two weeks

long, and that $w_h = 2w_l$. Then the condition $v \leq \frac{1}{2}(w_h - w_l)$ translates to $v \leq \frac{w_l}{2}$. As a year consists of 26 two-week periods, this amounts to saying that the utility from a two-week vacation cannot be more than 13 times what a worker positively identified as the low type would have received in wages for that two-week period. Looking at it the other way, if we assume that the utility from a two-week vacation is equal to the average wage for two weeks work, the above condition implies $\frac{1}{2}(\frac{w_l}{26} + \frac{w_h}{26}) \leq \frac{1}{2}(w_h - w_l)$, or that w_h must be at least 8% greater than w_l . Either way, it seems quite possible that this condition may be satisfied in practice, and therefore that, given the appropriate beliefs, the equilibrium profile may be one where neither worker wants to take a vacation.

Given this possibility, we ask which equilibrium profile the employer will prefer, and how he might attempt to ensure that it is achieved. In the model developed above we have assumed that the employer will pay the workers w_l and w_h if he knows their types with certainty, and that if he does not he will pay each worker a weighted average of those wages. This captures the idea that the employer gets a benefit from rewarding high-type workers and penalizing low-type ones, but bears a cost when he gives these rewards and penalties to the wrong types. However, this wage function does not say anything about why or by how much the employer might gain from better information about the workers' types: it just specifies how he will use the information he has.

We assume that the employer gets some benefit from better information about types and that he bears some cost if workers take paid vacations. Strictly speaking in this paper we have only formally modeled a one shot game, while the motivation for why the employer cares about the workers' types comes most easily when thinking about dynamic settings, where the employer applies what he learns in one period to make decisions in the next. A formal dynamic model is beyond the scope of this paper, but we will discuss how some future options would affect current incentives to get information about the workers.

First, the larger the difference between p_h and p_l , the more important it will be for the employer to figure out types. It will also be more important for the firm to know types when it is easier to hire higher quality replacement workers, or if the employer wants to promote the higher types, say if he has some other production process where high-types are more productive. Increasing returns to worker ability will also give the employer a stronger incentive to determine types. An employer with some degree of market power in setting wages may also want to determine whether a worker is high or low type in order to best exercise that power. On the other hand, having to pay workers while they are on vacation, and losing their output will make it expensive for the employer to use vacations to determine types. Depending on the relative weights of these two effects, the employer may clearly prefer either equilibrium profile, or may even prefer that the workers take more vacation time than in the (Y^A, Y^B) outcome, in order to obtain still more information about types.

If he does not like one equilibrium profile, the employer can try promote another by altering wages, by not paying wages during vacations, by altering the length of the vacation period, or simply by removing the element of choice and either prohibiting vacations or making them mandatory. These actions can be expected not only to alter how much knowledge the employer derives about his workers' types, but also to alter the kinds of workers who will tend to apply to the firm.

First we will consider policies that simply attempt to encourage workers to adopt a particular sequential equilibrium profile, rather than force them to take vacation time. Wages are one such policy tool. (Though there will certainly be other motives for setting particular wages, which may limit the employer's use of them for this purpose.)

We have shown that theorems 1 through 4 hold for very general $f(\beta)$ functions. Therefore altering the rule used to divide wages will not be a very useful tool for the employer. On the other hand, from theorem 4, decreasing the gap between w_l and w_h will make it less likely

that the (N^A, N^B) equilibrium profile can be sustained. The intuition is that the smaller wage gap lowers the penalty to taking a vacation and having the employer believe you are low type, which is the belief that sustains that equilibrium profile. So, employers who want to promote the (Y^A, Y^B) equilibrium profile, to get more information about types, might want to equalize wages. But this benefit may be mitigated by the fact that equalizing wages will also tend to encourage low types to apply in the first place.

Interestingly, the net effect of a decreased wage gap on the expected wage of workers is uncertain. It reduces the premium to being identified as the high type, but as explained above, it also makes it more likely that the high-type worker will be identified as such. So, paradoxically, the high-type worker may well prefer a lower wage premium, because of the higher probability that he will then be identified as high.

Another variable under the employer's control is the length of vacations. Longer vacations mean that, in the (Y^A, Y^B) pooling equilibrium profile, the employer will get more observations of output and therefore more information about worker types. In the model we have developed above longer vacations will never be enough to destroy the (Y^A, Y^B) equilibrium profile, because they only provide probabilistic information about types that will not offset the employers deterministic beliefs from vacation behavior, when only one worker vacations. However, in a model where the employer's beliefs from vacation behavior were uncertain, or in which workers made random errors in their strategies, longer vacations might conceivably destroy the (Y^A, Y^B) equilibrium profile, by reducing the advantages to the low-type worker of mimicking a high-type worker who took a vacation.

Longer vacations should also make it less likely that the (N^A, N^B) equilibrium can be sustained, because longer vacations will presumably have higher v 's, raising the benefit to deviating from the equilibrium profile by taking a paid vacation. These longer vacations will in general be more expensive to the employer, though, to the extent that workers get utility

from time off, the cost may be mitigated by their willingness to accept lower overall annual wages in return. Still, this cost may well limit the employer's desire to use longer vacation periods as a way of collecting information about types.

This raises the possibility that the firm may prefer not to pay workers during vacations. While we do not derive optimal strategies in this situation, it seems possible that this might produce very different results from those derived from the model above. For example, if the lost wages for the low type are enough larger than v , (Y^A, Y^B) might not be an equilibrium profile: the low type could prefer to work, even though by doing so the employer identifies him as the low type. On the other hand, the resulting lower wage then reduces the cost of a vacation, so the net effect is uncertain.

The policies discussed above leave open the possibility that the employer will be dissatisfied with the amount of information about types that is revealed in an equilibrium profile, or that the employer may find that the cost of encouraging the employees to reveal this information voluntarily exceeds its value to him. As an alternative, the employer could either scramble the teams, or simply require the employees to take some fixed amount of vacation time. This is in fact the policy that the Federal Reserve has implemented for banks, and one that many other firms also follow.

4 Conclusion

In this paper we have developed a simple model of vacation behavior in a strategic setting with two workers and an employer. The workers make decisions about taking time off, knowing that the employer will use both variations in output during their absence, and the decision to take or not to take time off itself, to try to determine their productivity. While our model allows for signaling behavior by the employees it is different from what

are usually known as signaling models. In the extensive form of these models, Nature first determines the worker's type, and knowing his type the worker then decides how much, say, education to obtain. When deciding this, he knows that his wage will depend on the employer's beliefs about what a worker of a different type would do, in his place. *However, this other type worker is not actually in the game.* The worker's payoff is determined solely by the employer's beliefs and by *his own* strategy, which in turn depends on those beliefs. This model is a reasonable description of situations where what an employer believes about one worker does not affect what he believes about another worker.

However, it is *not* appropriate in situations where what an employer believes about one particular worker's type depends on his beliefs about another's type. In these situations a given worker can, by altering his strategy, affect not only the employer's beliefs about his type, but also the employer's beliefs about the type of another worker. In turn, a worker must worry not only about what *his* actions tell the employer about his type, but also about how his actions affect the incentives of the *other* worker to alter his behavior, and thus indirectly affect the employer's beliefs about the first worker. Our model accounts for this possibility by having each worker play a strategic game with the employer *and* with the other worker. The employer's beliefs (and thus the workers' payoffs) depend on the strategies of *each* worker.

We believe this is an important improvement, and that models of the sort we develop in this paper are appropriate for studying a host of games of workplace intrigue. Convincing the boss that you are a productive worker is vital to career advancement, and efforts to do so are always complicated by the fact that your co-workers are simultaneously trying to prove that they are the ones making the business succeed, not you. Still, the model is a stylized one, and care should be exercised before treating its conclusions as the final word on the role of absenteeism. However, we do believe this model provides an interesting starting point

for a new way of looking at this behavior.

The model in this paper addresses the simplest interesting case - teams with two workers. While extending the model to allow for larger teams is straightforward, it leads to considerably more complexity and might well result in more pure-strategy equilibria. Another extension to the model we have developed in this paper would be to allow output to depend on effort, rather than ability. While there are many situations where workers can influence their co-workers' productivity, as described in Lazear (1989), the simplest case is where their effort only affects their own output. In this situation, vacation choices are intertwined with choices of effort. Each worker will want output to drop when he is on vacation, but not when their co-worker is. If output depends on effort, then it would also seem reasonable to consider the possibility that vacations rejuvenate the workers, perhaps in the sense of lowering the disutility of effort. Employers must therefore consider the lost output from a vacation, and balance it against the possibility of increased output from a rested employee. We leave these ideas for future research.

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