BEHAVIORAL BIASES IN GENERAL EQUILIBRIUM: IMPLICATIONS FOR WEALTH INEQUALITY AND HUMAN CAPITAL FORMATION

by

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DISSERTATION ABSTRACT

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My research focuses on the integration of behavioral economics into well understood general equilibrium macroeconomic models populated by overlapping generations of heterogeneous agents. Specifically, I analyze the implications of populating model economies with present-biased agents who are finitely lived, subject to idiosyncratic labor income shocks, and heterogeneous in both exponential and present-biased discount factors. My primary goal is characterizing the contribution of behavioral biases towards resolving several issues in the literature pertaining to human capital investment and aggregate wealth inequality. Further, the inclusion of present bias in carefully calibrated model economies allows me to rationalize empirical differences in consumption, wealth, and education that arise between observationally similar households that models of homogeneous, exponential discounters are unable to match.

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CHAPTER I

INTRODUCTION

Present bias was first brought into the field of economics by David Laibson (1997) after he reintroduced the preference paradigm outlined by Strotz (1956) and Phelps and Pollak (1968). Present-biased agents, or quasi-hyperbolic (geometric) discounters, display time inconsistent preferences characterized by an additional discount factor applied between the current period and all future periods. This additional discount factor is different than the discount factor applied to trade-offs between any two dates further in the future. Present bias leads agents to a) make decisions that they systematically regret and b) create consumption-savings plans for themselves that they systematically ignore. Without the availability of commitment devices, even in the absence of earnings uncertainty or borrowing constraints, a present-biased agent is unable to construct a consumption-savings plan that she will follow through with in future periods.

In the second chapter, I embed agents in a three period overlapping generations model. I then ask rational and present-biased agents to allocate their time between earning a wage in the unskilled labor market and accumulating human capital when young. Their human capital decision maps directly into a deterministic wage that each agent receives for inelastically supplying labor when middle-aged and agents retire and consume out of their savings when old.

I consider three different allocations of tax revenue; completely wasteful government spending, funding a social security system, and splitting funding between social security and providing education incentives. Under each spending regime, I find that present-biased agents develop less human capital, consume more when young, and save less for retirement than exponential discounters. Dedicating tax revenue to education incentive programs increases human capital and consumption in all periods of life for both rational and present-biased agents. Further, the human capital attainment gap between present-biased and rational agents is decreasing in the degree to which education incentive programs are funded in lieu of social security funding.

The third chapter is dedicated to understanding the role of present bias and preference heterogeneity in an overlapping generations economy modeled after the well known Huggett (1996) model. A hallmark shortcoming of this class of models is the inability of the equilibrium wealth distribution to match the empirical wealth distribution of US households, measured using PSID or SCF data. Specifically, these models predict lower inequality, a lower concentration of wealth in the hands of the richest 1% of households, and a higher concentration of wealth for the poorest 40% of households than empirical studies support. I find that a society populated by present-biased agents displays nearly identical wealth inequality when compared to a society of homogeneous exponential discounters. This result depends crucially on embedding agents in a general equilibrium framework in which the interest rate responds to the aggregate savings decisions of households. Further, I find that if a mixture of present-biased and exponential discounters occupy the same model economy, wealth in equality in the model is increased significantly. This increase in inequality occurs as exponential discounters benefit greatly from the increased market interest rate they have available due to their mis-optimizing peers.

The fourth chapter embeds an education/ low wage labor trade-off for young agents (as in my second chapter) into a carefully calibrated multi period overlapping generations model (as in my third chapter). Agents are tasked with making a formal education investments in the first 6 periods of life. They will then receive a stochastic income profile based on their level highest degree completed, calibrated to match the income process of college educated and non college educated workers in PSID data. This model will be populated with both exponentially discounting and present-biased agents and policy interventions aimed at reducing drop-out rates and wealth inequality will be considered.

CHAPTER II

HUMAN CAPITAL FORMATION AND QUASI-HYPERBOLIC DISCOUNTING

Introduction

In the United States and many other developed countries, the decision to attend school beyond the state (or nationally) mandated age rests in the hands of young individuals. The level of investment in education chosen by these young individuals will play a large role in determining their lifetime human capital and, consequently, labor earnings. I aim to contribute to the literature examining the implications of choosing non-mandatory education when young, particularly when young agents may suffer from behavioral biases. Thus, I study non-mandatory schooling choices in societies populated by rational exponential discounters (RE) and for societies populated by quasi-hyperbolic discounters (QH). Results of this simplified three period overlapping generations (OLG) model are compared to features of U.S. data in order to evaluate the likelihood that optimizing agents are discounting quasi-hyperbolically. Several policy interventions are explored and their effectiveness for improving the welfare of both RE and QH optimizers is evaluated. Policy interventions that improve welfare for both RE and QH societies are targeted so as to provide guidance for a policy maker who is unsure if agents in her society are RE discounters or QH discounters.

I find that present-biased individuals invest less time in schooling when young than rational individuals. This leads to a reduction in lifetime earnings and a steep drop off in retirement consumption for QH optimizers relative to their RE peers. If the government provides social security in an attempt to increase old age consumption, there is a reduction in schooling for both RE and QH optimizers as well as a decrease in consumption at every age for both types of optimizing agent. However, if tax revenue is dedicated to funding an education incentives program (outlined in Section 4-c), I find that both RE and QH discounters invest more time in schooling when young and are able to increase consumption in every period relative to a regime in which no taxes are levied.

This chapter is the first attempt to address the education investment decision of present-biased individuals in an overlapping generations framework. Further, I am the first author to establish equilibrium distinctions in schooling between QH and RE discounters, both analytically and numerically. Outside of an empirical result established in Cadena and Keys (2015), I am the first author to comment on the role of present bias for human capital accumulation in any setting. I also consider a unique education incentive program not analyzed elsewhere in the literature as a means of increasing societal human capital and consumption for both RE and QH agents in every period of life. To the best of my knowledge, this paper marks the first attempt to analyze the direct trade-off between dedicating tax revenue to social security and dedicating tax revenue towards funding education incentive pay.

Review of the Literature

Human capital accumulation has been integrated into a number of macroeconomic models in order to better understand the role of skill upgrading on economic activity (see Lucas (1988)). Early contributions to this literature typically speak to the optimizing decisions made by parents for their children instead of focusing on young agents who must decide how much to consume and save while simultaneously deciding to invest time in furthering their education (clear exceptions being Ben-Porath (1967)). There is a vast empirical literature that focuses on the long run financial (Mincer (1974), Katz and Autor (1999)), professional (Bates 1990), and even health (Case and Deaton (2015)) benefits of increasing one's education beyond the mandatory level.

I contribute to the literature examining the role of human capital in a model of overlapping generations of agents in two distinct ways. First, by focusing on an investment in non-mandatory education, I am able to model the decision to attend school from the perspective of a young optimizing agent. As noted above, this focus is not shared by the majority of the literature in which emphasis is placed on the investment decision of outside agents (parents and government) in the human capital of the young (See Glomm and Ravikumar (1992) and Zhang (1995)) . Second, in the spirit of taking the Lucas Critique seriously and truly microfounding models of household behavior, I integrate insights from the behavioral economics literature into the model. Although there is a growing literature focusing on behavioral macroeconomics, there remains a considerable gap in the literature that we hope to fill. Namely, understanding the impact of present bias on investments in human capital.

Galor and Zeira (1993) investigated the role played by the societal distribution of wealth in determining human capital investment, growth and long run income inequality. They find that in the presence of imperfect credit markets, a society's initial distribution of wealth has large long run implications for human capital accumulation and growth. Glomm and Ravikumar (1992) used an overlapping generations model to explore the long run growth implications of public vs private funding for education via human capital accumulation. They focus on the existence and uniqueness of equilibria under a variety of assumptions regarding the law of motion for human capital accumulation. Both papers model investment in education as a choice made by parents and not young agents. Unlike these papers, I focus on the investment decision of young agents in order to understand how disutility from education crowds out human capital investments. This tension between current utility and future labor market returns is particularly interesting in the presence of present-biased agents who inherently overweight immediate returns over future payoffs.

A more recent literature focuses on the trade off between government outlays dedicated to funding social security and public education. Pecchenino and Pollard (2002) and Kaganovich and Zilcha (1999) analyze this trade off utilizing a 2 period OLG model in which all educational investment is made by parents and the government. Kaganovich and Zilcha find funding for social security is welfare and growth reducing relative to equivalent funding allocated to education. Pecchenino and Pollard corroborate this finding even in a model that does not assume crowding out in spending and allows for societal aging. My policy analysis leads to the same conclusions regarding the negative impact of social security for both RE and QH societies. Annabi et al. (2011) also find that higher education incentives can increase human capital and offset the effects of declining labor force growth. Glomm and Kaganovich (2008) analyze the trade off between funding social security and public education in a model with heterogeneous agents. They find that social security funding reduces income inequality and, under certain circumstances, does not decrease growth.

My work is perhaps most closely related to that of Krueger and Ludwig (2016) and Cadena and Keys (2015). Krueger and Ludwig compute the optimal degree of income tax progressivity and optimal education policies in a quantitative life-cycle framework. They find that generous tuition subsidies are optimal across a myriad of modeling assumptions. This result is mirrored in my work for both RE and QH discounters when optimal tax allocation is considered in section 3. Cadena and Keys examine the role of impatience in human capital formation. Using NLSY data, they find impatient individuals are more likely to drop out of college, earn less than their more patient peers, and express more regret when middle-aged. My model provides a clear, simple avenue through which these results can be contextualized; present-biased optimization leads to lower educational attainment, lower lifetime earnings, and a gap between desired and actual middle-aged human capital.

I also contribute to the the literature examining the theoretical and empirical effect of quasi-hyperbolic discounting in economics. Quasi-hyperbolic discounting has a long history of integration in the scientific literature. First introduced by Strotz (1956) and Phelps and Pollak (1968) in the middle of the 20th century, quasi-hyperbolic discounting gained significant traction in the economics literature with its reintroduction by Laibson (1997). Laibson found that QH discounters would overspend and under-save if given the opportunity. He found that commitment devices designed to constrain liquidity could help to offset the undesired aspects of QH optimization.

In the model outlined in the next section, optimizing agents are forced to commit to a level of human capital investment in the first period of their lives conditional on their expected savings and consumption profile in the second period of their lives. However, QH optimizers will choose to re-allocate savings to consumption when the second period arrives leaving themselves with much lower savings to consume out of when old and less human capital (and thus middle aged wages) than RE agents. Banks et al. (1998) find significant evidence that consumption falls when households retire and they conclude "the systematic arrival of unexpected adverse information" is the most likely culprit for this observation. However, our simple 3-period model in which agents face no aggregate or idiosyncratic uncertainty generates this same qualitative prediction. That is, when agents are present-biased, under-saving for retirement is a natural result of optimizing behavior. We therefore turn to the empirical literature in order to justify the use of present bias and quantify the degree to which individuals are present-biased.

There is a vast empirical literature outlining the impact of present-biasedness in a microeconomic setting. Dellavigna and Malmendier (2006) found that QH discounters lose hundreds of dollars per year in unused gym memberships and Milkman et al. (2009) find that quasi-hyperbolic discounting leads subscribers to a video rental service to request (and hold) movies they perceive to have intellectual merit (but do not enjoy) longer than mainstream movies that have little intellectual value but are more enjoyable.

Meier and Sprenger (2015) estimate the value of the present bias discount factor using a series of survey questions with a sample of poor, primarily minority women in Boston. They find the average degree of present bias to be between 0.69 and 0.82 depending upon their model specification. They also find a significant degree of present bias heterogeneity, with poorer women displaying a higher degree of present bias than wealthier women. Tanaka et al. (2010) perform a similar survey based experiment on 181 rural Vietnamese households. They find an average degree of present bias of 0.644. However, unlike Meier and Sprenger they do not find any correlation between household characteristics and the degree of present bias. Tanaka et al. do find that the exponential discount factor is higher for lower income households. A value of the present bias parameter that is in line with each of these studies will be used throughout the following sections. However, due to the conflicting evidence provided by these authors regarding present bias heterogeneity, the impact of heterogeneous present bias on schooling decisions will not be explored in this paper. See Chapter 2 of this prospectus proposal for an exposition of the role of present bias and preference heterogeneity in a general equilibrium framework.

In the macroeconomic literature, the impact of QH optimizers has been analyzed in a number of settings. Karp (2005) integrates QH discounting into a model of environmental preferences, finding QH discounting to be a realistic description of reality that alleviates some modeling difficulties inherent to using a constant discount rate to model long-lived environmental issues. İmrohoroğlu et al. (2003) integrate QH discounters into an overlapping generations model in order to examine the role of social security in a world populated by non-traditional optimizers. They include a number of frictions in their model that I do not consider, but their results are consistent with my findings; social security reduces output and consumption for both RE and QH agents and the effect of government intervention on welfare depends on the degree of time-inconsistency. Schwarz and Sheshinski (2007) further examine the impact of quasi-hyperbolic discounting on social security, finding the presence of QH agents breaks the equivalence between pay-as-you-go and fully-funded social security established in Sheshinksi and Weiss (1981).

The remainder of this paper is laid out in the following way: in the next section I outline some general features of the model and the corresponding equilibrium. I then delineate three different model specifications and the equilibrium differences that arise between QH and RE agents in these specifications where each model examines a different government policy regarding the disbursement of tax revenue. The final section concludes.

Modeling Overview

In each of the following models, I assume that there are an infinite number of periods populated by overlapping generations of three-period-lived agents. The subscript t denotes a period in time and a superscript t denotes the generation born at time t (e.g. the period $t+2$ consumption of an agent born in period t will be written as c^t_{t+2}). Each generation consists of a continuum of agents and the size of each generation is normalized to unity. There is a single good (c) that is produced, consumed, and saved at the rate r. We assume that agents live in a small, open economy so that r is exogenously determined and fixed. There is no population growth in the model economy, thus $r > n = 0$[1]. Credit access is limited only by the present discounted value of one's lifetime income.

Young agents are endowed with one unit of time and are faced with a trade-off between spending time working and receiving a wage w_t^t and attending non-mandatory schooling ($s_t^t \in [0,1]$) to increase their human capital when middle aged (h_{t+1}^t). I assume that agents receive disutility from their schooling investment. Without this assumption, agents in both RE and QH societies would chose the same level of schooling; that which maximizes their lifetime income[2]. Therefore, I proceed as in Cowan (2016) by imposing disutility from education. When young, agents can consume out of their exogenous endowment (x), using their wage earnings, or by financing their consumption through borrowing at the rate r. If they do not consume their entire income (the sum of wage earnings and the exogenous endowment), agents can store their savings (a_t^t) at the rate r. Middle age agents are endowed with one unit of time that they spend working for a wage of w_{t+1}^t, which is an increasing function of their human capital. When old, agents do not work, and consume whatever they have saved in the previous period as well as the interest accrued on their savings, $(1+r)a_{t+1}^t$.

The schooling variable has the following interpretation: $s = 0$ corresponds to the education of an individual who drops out after 10th grade and $s = 1$

[1]As the economy is dynamically efficient, $r > n$ leads to Social Security being welfare reducing.

[2]See Becker 1967 and Fuchs 1982 for further exposition on this topic.

corresponds to an individual who receives 4 years of education beyond a Bachelor's degree. Every one tenth increase in s maps to a 1 year increase in education beyond 10th grade. Although the true wage premium is certainly kinked at values of s corresponding to the completion of certain grade levels (i.e. high-school, Bachelor's degree, etc.), I abstract from this distinction in the simplified representation of this complex choice and instead focus on continuous, differentiable functions of human capital accumulation.

Members of generation t derive utility V_t^t from consumption in all three periods of life and experience disutility from schooling (s_t^t) when young. That is:

$$V_t = U(c_t^t, c_{t+1}^t, c_{t+2}^t, 1 - s_t^t)$$

$U(\cdot, \cdot, \cdot, \cdot)$ is strictly concave and increasing in all of its arguments. This utility function holds for all agents in all generations. A rational exponential (RE) discounter is distinguished from a quasi-hyperbolic (QH) discounter in the following way:

RE discounting of future utility:

$$V_t = U(c_t) + v(1 - s_t) + \sum_{i=1}^{2} \beta^i U(c_{t+i})$$

QH discounting of future utility:

$$V_t = U(c_t) + v(1 - s_t) + \delta \sum_{i=1}^{2} \beta^i U(c_{t+i})$$

where $\delta \in (0, 1)$[3]. A QH agent applies an additional discount factor δ to all future utility that an RE agent does not apply to future utility.

In the following section, the baseline model is calibrated using specific functional forms in order to explicate partial equilibrium distinctions that arise between the two types of agents. The role of the government is the only distinction between the following three models, therefore the discussion of the government is left for each subsection. An equilibrium is defined by prices, w and r, a collection of consumption and education decisions in which each generation solves their consumption and education profile according to the first order conditions implied by utility maximization, taking prices as given, and a government allocation of tax revenue that results in a balanced budget in each period.

Equilibrium Analysis: No Government Spending

The model in this section is distinguished by the assumed behavior of the government. In this section, all government revenue that is collected via taxes

[3] We have reversed the meaning of the β and δ parameters from Laibson's exposition of quasi-hyperbolic discounting so that the discount factor β retains its standard interpretation found throughout the macroeconomic literature.

on labor income is assumed to be immediately discarded so that the government budget is trivially balanced in each period.

Rational Solution

Consider the following optimization problem solved by an agent born in period "t".

$$\max_{\{c_i\}_{i=1}^{3},s} U_t = \ln(c_t^t) + \gamma \ln(1 - s_t^t) + \beta \ln(c_{t+1}^t) + \beta^2 \ln(c_{t+2}^t)$$

s.t.

$$c_t^t + a_t^t = x + (1-\tau)w_t^t(1 - s_t^t) \quad (2.1)$$

$$c_{t+1}^t + a_{t+1}^t = (1-\tau)w_{t+1}^t + (1+r)a_t^t \quad (2.2)$$

$$c_{t+2}^t = (1+r)a_{t+1}^t \quad (2.3)$$

Where $s_t^t \in [0,1]$ is the amount of schooling obtained by a young agent of generation t, $x \geq 0$ is the exogenous initial endowment received by young agents of each generation, w_t is the wage paid to young, unskilled workers, $\gamma > 0$ is the weight applied to disutility from schooling, $\tau \in [0, 1)$ is the exogenous tax rate, and $\beta \in (0, 1)$ is the discount factor applied to future utility. Human capital evolves according to $h_{t+1}^t = (1 + \phi s_t^t)$ where $\phi > 0$. The production technology owned by young and middle age agents is of the form $f(h_t^t) = W_u h_t^t$. W_u can be thought of as the unskilled wage and $W_u\phi$ can be thought of as the skill premium. An agent born in period t produces $W_u(1 - s_t^t)$ units of the consumption good when young and $W_u(1 + \phi s_t^t)$ units of the consumption good when middle-aged. Agents face no uncertainty regarding their life span or their lifetime earnings and asset holdings.

I assume $a_{t-i}^t \in \mathbb{R}\ \forall\ i \in \mathbb{Z}$. Thus, I can combine the three budget constraints listed above into one budget constraint via the agent's asset holdings. This makes it possible to express the agent's optimization problem as a Lagrangian:

$$\mathscr{L} = \ln(c_t^t) + \gamma \ln(1 - s_t^t) + \beta \ln(c_{t+1}^t) + \beta^2 \ln(c_{t+2}^t)$$

$$+ \lambda_t^t \left(x + (1-\tau)W_u(1 - s_t^t) + \frac{(1-\tau)W_u(1 + \phi s_t^t)}{1+r} - c_t^t - \frac{c_{t+1}^t}{1+r} - \frac{c_{t+2}^t}{(1+r)^2} \right)$$

Optimization yields the following first order conditions:

$$\mathscr{L}_1 = 0 \quad \Longrightarrow \quad \frac{1}{c_t^t} = \lambda_t^t \tag{2.4}$$

$$\mathscr{L}_2 = 0 \quad \Longrightarrow \quad \frac{\beta}{c_{t+1}^t} = \frac{\lambda_t^t}{1+r} \tag{2.5}$$

$$\mathscr{L}_3 = 0 \quad \Longrightarrow \quad \frac{\beta^2}{c_{t+2}^t} = \frac{\lambda_t^t}{(1+r)^2} \tag{2.6}$$

$$\mathscr{L}_4 = 0 \quad \Longrightarrow \quad -\frac{\gamma}{1 - s_t^t} - \lambda_t^t(1-\tau)W_u + \lambda_t^t \frac{(1-\tau)W_u\phi}{1+r} = 0 \tag{2.7}$$

$$\mathscr{L}_5 = 0 \quad \Longrightarrow \quad \text{Budget Constraint} \tag{2.8}$$

Combining (2.4)-(2.8) leads to the unique equilibrium for education and consumption in each period. As agents are identical, each generation will solve this problem in the same way. Therefore, I express rational equilibrium consumption with subscripts y, m and o for young, middle aged, and old and the superscript R^*. That is, the equilibrium consumption of an RE agent in the first, second, or third period of her life is denoted by $c_y^{R^*}$, $c_m^{R^*}$, and $c_o^{R^*}$ respectively. The equilibrium schooling decision of an RE agent is denoted by s^{R^*}.

$$c_y^{R^*} = \frac{1}{1+\beta+\beta^2+\gamma}\left[x + \frac{(1-\tau)W_u(1+\phi)}{1+r}\right] \tag{2.9}$$

$$c_m^{R^*} = \beta(1+r)c_y^{R^*} \tag{2.10}$$

$$c_o^{R^*} = \beta^2(1+r)^2 c_y^{R^*} \tag{2.11}$$

$$s^{R^*} = 1 - \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_y^{R^*} \tag{2.12}$$

Table 1 outlines the partial derivatives associated with the equilibrium consumption and education profile of an optimizing rational agent. It is clear that the optimal schooling

TABLE 1. Baseline Model Partial Derivatives

	γ	x	β	r	W_u	ϕ	τ
$c_y^{R^*}$	-	+	-	-	+	+	-
s^{R^*}	-	-	+	-	+	+	-
$c_m^{R^*}$	-	+	+	+	+	+	-
$c_o^{R^*}$	-	+	+	+	+	+	-

decision is decreasing in initial wealth, the interest rate, and the tax rate. Higher initial wealth decreases educational attainment as agents receive disutility from education. As there is no direct cost associated with college attendance, initial wealth acts as a means of avoiding a form of disutility (from education) while gaining higher utility from consumption (resulting from both higher initial wealth and increased earnings while young). A higher interest rate and higher taxes both reduce the relative return of investing in education relative to investing in capital. Thus, an increase in either of these variables, holding all else equal, reduces equilibrium schooling.[4].

Quasi-Hyperbolic Solution

I now set up the same model but rather than assuming rational behavior, the model is populated with identical quasi-hyperbolic discounters (QH agents). The corresponding model, with utility denoted by $\tilde{U}$, is:

$$Max\ \tilde{U}_t = \ln(\tilde{c}_t^t) + \gamma \ln(1 - \tilde{s}_t^t) + \delta\beta \ln(\tilde{c}_{t+1}^t) + \delta\beta^2 \ln(\tilde{c}_{t+2}^t)$$

s.t.

$$\tilde{c}_t^t + \frac{\tilde{c}_{t+1}^t}{1+r} + \frac{\tilde{c}_{t+2}^t}{(1+r)^2} = x + (1-\tau)W_u(1 - \tilde{s}_t^t) + \frac{(1-\tau)W_u(1+\phi\tilde{s}_t^t)}{1+r}$$

[4]This analysis holds true only when wages do not adjust following changes in the interest rate, as schooling is increasing in W_u. In future versions of this model, I intend to embed this problem in a general equilibrium setting in which prices respond to aggregate savings decisions.

Note, this is the same budget constraint faced by rational agents. Therefore, I proceed with optimization as shown above. The resulting expected consumption profile for a QH agent (delineated by a Q^* superscript for actual equilibrium behavior and a Q' superscript for expected equilibrium behavior) who optimizes for her lifetime is:

$$c_y^{Q^*} = \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right] \quad (2.13)$$

$$c_m^{Q'} = \delta\beta(1+r)c_y^{Q^*} \quad (2.14)$$

$$c_o^{Q'} = \delta\beta^2(1+r)^2c_y^{Q^*} \quad (2.15)$$

$$s^{Q^*} = 1-\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_y^{Q^*} \quad (2.16)$$

The above equilibrium for QH optimizers yields two immediate takeaways. First, QH agents consume more and invest less time to schooling when young than RE agents. Second, the planned middle aged and old age consumption profile for QH agents does *not* equal the actual consumption of middle aged and old QH agents. When optimizing agents are rational, the path of consumption solved for by a young agent, the collection ($c_y^{R^*}$, $c_m^{R^*}$, and $c_o^{R^*}$), is the same as the actual consumption of young, middle-aged and old agents. But now with QH optimizers, $c_m^{Q'}$ and $c_o^{Q'}$ are no longer the true consumption profiles for middle-aged and old agents. That is, young agents incorrectly optimize for their future selves. To see why this is, I consider the utility of a QH agent from the perspective of being young, middle-aged and old:

$$\tilde{U}_y = \ln(\tilde{c}_y^t) + \gamma \ln(1 - \tilde{s}^t) + \delta\beta \ln(\tilde{c}_m^t) + \delta\beta^2 \ln(\tilde{c}_o^t) \tag{2.17}$$

$$\tilde{U}_m = \beta^{-1} \ln(\tilde{c}_y^t) + \beta^{-1}\gamma \ln(1 - \tilde{s}^t) + \ln(\tilde{c}_m^t) + \delta\beta \ln(\tilde{c}_o^t) \tag{2.18}$$

$$\tilde{U}_o = \beta^{-2} \ln(\tilde{c}_y^t) + \beta^{-2}\gamma \ln(1 - \tilde{s}^t) + \beta^{-1} \ln(\tilde{c}_m^t) + \ln(\tilde{c}_o^t) \tag{2.19}$$

When agents are middle-aged, they use a different discount rate to look back at decisions made when young (β) than they did when they were young looking forward to being middle-aged ($\beta\delta$). Thus, middle-aged agents must re-optimize and solve for their new optimal consumption and savings plan subject to the remainder of their lifetime budget constraint, taking the decisions of their young selves as given.

Re-optimizing in (t+1)

$$Max\ \tilde{U}_{t+1} = \ln(\tilde{c}_{t+1}^t) + \delta\beta \ln(\tilde{c}_{t+2}^t)$$

s.t.

$$\tilde{c}_{t+1}^t + \tilde{a}_{t+1}^t = (1 - \tau)W_u(1 + \phi s^{Q^*}) + \tilde{a}_y^t(1 + r) \tag{2.20}$$

$$\tilde{c}_{t+2}^t = (1 + r)\tilde{a}_{t+1}^t \tag{2.21}$$

(2.21) can be thought of as the constrained analogue to (2.2) in which a middle-aged agent re-optimizes taking the choice of their younger self as given. With this interpretation in mind, $\tilde{a}_y^t$ is defined as the savings of a generation t agent made in the first period of her life. Solving the agent's optimization problem and substituting in the optimal first period consumption, education and savings yields the following equilibrium consumption profile for middle-aged and old agents:

$$c_m^{Q^*} = \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*} + (1-\tau)W_u(1+\phi s^{Q^*})\right] \tag{2.22}$$

$$c_o^{Q^*} = \beta\delta(1+r)c_m^{Q^*} \tag{2.23}$$

Where $a_y^{Q^*} = x + (1-\tau)W_u(1-s^{Q^*}) - c_y^{Q^*}$. For QH agents, realized equilibrium consumption does not equal planned equilibrium consumption when middle-aged (or old). That is $c_m^{Q'} \neq c_m^{Q^*}$ and $c_o^{Q'} \neq c_o^{Q^*}$. Unlike their RE counterparts, QH agents do not follow through with their planned lifetime consumption profile.

As our agent's utility is represented using a natural log function, I am able to analytically compare the equilibrium consumption and schooling profile of rational exponential discounting agents relative to quasi-hyperbolic discounting agents. It is important to note that these differences are not unique to a log specification. Numerical analysis confirms these results for a general CRRA utility function. Table 2 contains the analytical equilibrium differences in schooling and consumption in each period of life between rational and quasi-hyperbolic agents.

These differences hold for all values of τ between 0 and 1, all values of β and $\delta \in (0,1)$, all positive values of γ, and all non-negative values of x and W_u.

As shown in Table 2 below, for the parameter space outlined above, RE agents consume less when young, consume more when old, and dedicate more time to schooling when young than QH agents. Middle-aged rational agents will consume more than middle-aged QH agents in equilibrium so long as $1 + \gamma > \delta(\beta^3 + \beta^2)$. For any calibration of β and δ in line with estimates provided by previous literature, $\gamma > 0$ guarantees QH agents will consume less than RE agents in equilibrium when middle-aged. That is, as long as agents receive disutility from schooling, QH agents acquire less human capital than RE agents.

TABLE 2. Baseline 1 Equilibrium Differences for RE and QHD Agents

RE	**vs**	**QHD**	**Analytical Difference (RE - QHD)**
$c_y^{R^*}$	$<$	$c_y^{Q^*}$	$\frac{1}{1+\beta+\beta^2+\gamma}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right)-\frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}\right)$
s^{R^*}	$>$	s^{Q^*}	$\frac{\gamma(1-\delta)(\beta+\beta^2)[(1-\tau)W_u(1+\phi)+(1+r)x]}{(1-\tau)W_u[\phi-(1+r)](1+\beta+\beta^2+\gamma)(1+\delta\beta+\delta\beta^2+\gamma)}$
$c_m^{R^*}$	$>$	$c_m^{Q^*}$	$\frac{(1-\delta)\beta[(1-\tau)W_u(1+\phi)+(1+r)x][(\gamma+1)-\delta(\beta^3+\beta^2)]}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)(\beta^2+\beta+\gamma+1)}$
$c_o^{R^*}$	$>$	$c_o^{Q^*}$	$\frac{\beta^2(1-\delta)(1+r)[(1-\tau)W_u(1+\phi)+x(1+r)](\delta+\gamma+2\beta\delta+\delta\gamma+\beta^2\delta+\beta\delta\gamma+1)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)(\beta^2+\beta+\gamma+1)}$

Table 3 contains the analytical distinctions between expected (E) and actual (A) equilibrium consumption for quasi-hyperbolic discounting optimizers. As discussed above, agent re-optimization leads to an inequality between the planned consumption path and the realized equilibrium consumption path for optimizing QH agents . For all specifications of the parameters in the range outlined above, QH agents expect to consume more when old and less when middle-aged than they actually consume in equilibrium. This distinction creates a wedge between expected lifetime utility when young and realized lifetime utility. This wedge will be discussed further in the following section when we consider a slightly more complex model analytically.

TABLE 3. Baseline 1 Expected vs Realized Consumption for QHD Agents

QHD(E)	**vs**	**QHD(A)**	**Analytical Difference)**
$c_m^{Q'}$	$<$	$c_m^{Q^*}$	$\dfrac{(\delta-1)\beta^2\delta((1-\tau)W_u(1+\phi)+(1+r)x)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)}$
$c_o^{Q'}$	$>$	$c_o^{Q^*}$	$\dfrac{(1-\delta)(1+r)\beta^2\delta((1-\tau)W_u(1+\phi)+(1+r)x)}{(\beta\delta+1)(\delta\beta^2+\delta\beta+\gamma+1)}$

This section is concluded with a brief analysis of the relationship between equilibrium consumption and schooling for rational exponential discounters with a high discount rate (RH), quasi-hyperbolic discounters, and rational exponential discounters that discount future utility at the rate β. An RH agent is an individual that discounts utility from consumption i periods into the future at the rate $(\beta\delta)^i$. Recall, a QH agent and an RE agent discount consumption utility i periods into the future at the rate $\delta\beta^i$ and β^i, respectively.Populating the model with RH agents and optimizing leads to the following equilibrium orderings for schooling and consumption in each stage of life:

$$c_y^{RH^*} > c_y^{Q^*} > c_y^{R^*} \tag{2.24}$$

$$s^{RH^*} < s^{Q^*} < s^{R^*} \tag{2.25}$$

$$c_m^{RH^*} < c_m^{Q^*} < c_m^{R^*} \tag{2.26}$$

$$c_o^{RH^*} < c_o^{Q^*} < c_o^{R^*}\,^{5} \tag{2.27}$$

This analysis leads to the conclusion that populating a model with high discounting rational agents and typical RE agents (as outlined above) acts as a bound on the equilibrium behavior of QH agents. However, the desire to re-optimize when middle-aged remains a unique feature associated with QH agents. In the following section, I proceed by adding pay-as-you-go social security (PAYG) to the baseline model as a proposed first step for a policy maker who is unsure whether agents are rational or present-biased, and therefore aims to reduce the old age consumption gap that arises between QH and RE agents.

Equilibrium Analysis: Social Security

The set-up outlined in the previous section remains the same, but instead of assuming that all tax revenue is discarded, it is assumed that the government redistributes tax revenue via PAYG social security. The government balances

[5] When $\beta\delta \rightarrow 1$, $c_o^{Q^*} < c_o^{RH^*} < c_o^{R^*}$. However, discount rates of this magnitude are not empirically feasible and are therefore not considered.

its budget each period by distributing all taxes levied on young and middle-aged workers to old agents via a lump sum social security transfer.

Rational Solution

A rational agent is now faced with the following optimization problem:

$$Max\ U_t = \ln(c_t^t) + \gamma \ln(1 - s_t^t) + \beta \ln(c_{t+1}^t) + \beta^2 \ln(c_{t+2}^t)$$

s.t.

$$c_t^t + a_t^t = x + (1-\tau)W_u(1 - s_t^t) \quad (2.28)$$

$$c_{t+1}^t + a_{t+1}^t = (1-\tau)W_u(1 + \phi s_t^t) + (1+r)a_t^t \quad (2.29)$$

$$c_{t+2}^t = (1+r)a_{t+1}^t + b_{t+2}^t \quad (2.30)$$

The government collects labor income taxes from both young and middle aged workers and distributes tax revenue to old agents in the following way: $b_{t+2}^t = \tau W_u(1 - s_{t+2}^{t+2}) + \tau W_u(1 + \phi s_{t+1}^{t+1})$ is the transfer received in period $t+2$ by generation t agents from generation $t+1$ agents (middle aged when generation t agents are old) and generation $t+2$ agents (young agents when generation t agents are old). As in the previous section, I express an optimizing agent's problem as a Lagrangian by combining (2.28)-(2.30) into a single constraint. This leaves our agent with the following optimization problem:

$$\mathscr{L} = \ln(c_t^t) + \gamma \ln(1 - s_t^t) + \beta \ln(c_{t+1}^t) + \beta^2 \ln(c_{t+1}^t)$$

$$+ \lambda_t^t \left(x + (1-\tau)W_u(1-s_t^t) + \frac{(1-\tau)W_u(1+\phi s_t^t)}{1+r} + \frac{b_{t+2}^t}{(1+r)^2} - c_t^t - \frac{c_{t+1}^t}{1+r} - \frac{c_{t+2}^t}{(1+r)^2} \right)$$

Combining the first order conditions implied by the above optimization problem leads to the following consumption and education profile for agent t:

$$c_t^t = \frac{1}{1+\beta+\beta^2+\gamma}\left[x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b_{t+2}^t}{(1+r)^2}\right] \quad (2.31)$$

$$c_{t+1}^t = \beta(1+r)c_t^t \quad (2.32)$$

$$c_{t+2}^t = \beta^2(1+r)^2 c_t^t \quad (2.33)$$

$$s_t^t = 1 - \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}c_t^t \quad (2.34)$$

Unlike the baseline model outlined in the previous section, our equilibrium is not yet pinned down due to the inclusion of s_{t+1}^{t+1} and s_{t+2}^{t+2} in b_{t+2}^t. However, as all agents behave symmetrically in equilibrium, we know $s_{t+2}^{t+2} = s_{t+1}^{t+1} = s_t^t = s^{R^*}$. Thus, we replace the social security transfer received by old agents with its equilibrium value: $b^{R^*} = \tau W_u(1-s^{R^*}) + \tau W_u(1+\phi s^{R^*})$. We then substitute in for c_t^t in equation

(2.34). This leads to the following solution for equilibrium schooling:

$$s^{R^*} = \left(1 + \xi\left((\phi - 1)\frac{\tau W_u}{(1+r)^2}\right)\right)^{-1} \times \left(1 - \xi\left(x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{2\tau W_u}{(1+r)^2}\right)\right) \quad (2.35)$$

where $\xi = \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}\left(\frac{1}{1+\beta+\beta^2+\gamma}\right)$. Our equilibrium is given by s^{R^*} and:

$$b^{R^*} = \tau W_u(1 - s^{R^*}) + \tau W_u(1 + \phi s^{R^*}) \quad (2.36)$$

$$c_y^{R^*} = \frac{1}{1+\beta+\beta^2+\gamma}\left(x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b^{R^*}}{(1+r)^2}\right) \quad (2.37)$$

$$c_m^{R^*} = \beta(1+r)c_y^{R^*} \quad (2.38)$$

$$c_o^{R^*} = \beta^2(1+r)^2 c_y^{R^*} \quad (2.39)$$

Consumption in every period is an increasing function of the social security transfer b_{t+2}^t, however schooling is a decreasing function of the social security transfer b_{t+2}^t. Thus, in equilibrium social security plays a dual role in the determination of equilibrium consumption. By raising lifetime income for a given schooling decision relative to the model in previous section in which tax revenue is discarded, at first glance social security appears to increase equilibrium consumption. However, by disincentivizing investment in schooling, middle aged income is lower when taxes are levied and used to fund social security. We will discuss the net effect of social security on consumption and lifetime utility in a calibration exercise following our delineation of the QH solution below.

Quasi-Hyperbolic Solution

I now set up the same model but rather than assuming rational behavior, the model is populated with quasi-hyperbolic discounters. The corresponding model, with utility denoted by $\tilde{U}$, is:

$$Max\ \tilde{U}_t = \ln(\tilde{c}_t^t) + \gamma \ln(1 - \tilde{s}_t^t) + \delta\beta \ln(\tilde{c}_{t+1}^t) + \delta\beta^2 \ln(\tilde{c}_{t+2}^t)$$

s.t.

$$\tilde{c}_t^t + \frac{\tilde{c}_{t+1}^t}{1+r} + \frac{\tilde{c}_{t+2}^t}{(1+r)^2} = x + (1-\tau)W_u(1-\tilde{s}_t^t) + \frac{(1-\tau)W_u(1+\phi\tilde{s}_t^t)}{1+r} + \frac{\tilde{b}_{t+2}^t}{(1+r)^2}$$

The expected consumption profile for a QH agent is:

$$\tilde{c}_t^t = \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{\tilde{b}_{t+2}^t}{(1+r)^2}\right] \quad (2.40)$$

$$\tilde{c}_{t+1}^{t'} = \delta\beta(1+r)\tilde{c}_t^t \quad (2.41)$$

$$\tilde{c}_{t+2}^{t'} = \delta\beta^2(1+r)^2\tilde{c}_t^t \quad (2.42)$$

$$\tilde{s}^t = 1 - \frac{\gamma(1+r)}{1-\tau W_u[\phi-(1+r)]}\tilde{c}_t^t \quad (2.43)$$

We solve for s^{Q^*} by replacing $\tilde{b}_{t+2}^t$ with $b^{Q^*} = \tau W_u(1-s^{Q^*}) + \tau W_u(1+\phi s^{Q^*})$ in equation (2.40) and substituting (2.40) into (2.43):

$$s^{Q^*} = \left(1+\tilde{\xi}\left((\phi-1)\frac{\tau W_u}{(1+r)^2}\right)\right)^{-1} \times \left(1-\tilde{\xi}\left(x+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{2\tau W_u}{(1+r)^2}\right)\right) \quad (2.44)$$

Where $\tilde{\xi} = \frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]}\left(\frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\right)$. This leads to the following equilibrium social security transfer and expected consumption profile for QH optimizers:

$$b^{Q^*} = \tau W_u(1-s^{Q^*}) + \tau W_u(1+\phi s^{Q^*}) \quad (2.45)$$

$$c_y^{Q^*} = \frac{1}{1+\delta\beta+\delta\beta^2+\gamma}\left[x + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{\tilde{b}^{Q^*}}{(1+r)^2}\right] \quad (2.46)$$

$$c_m^{Q'} = \delta\beta(1+r)c_y^{Q^*} \quad (2.47)$$

$$c_o^{Q'} = \delta\beta^2(1+r)^2 c_y^{Q^*} \quad (2.48)$$

As in the previous section, QH agents will not follow through with their expected consumption profile when middle-aged. Thus, the re-optimization of QH agents must be accounted for.

Re-optimizing in t+1

$$Max\ \tilde{U}_{t+1} = \ln(\tilde{c}_{t+1}^t) + \delta\beta\ln(\tilde{c}_{t+2}^t)$$

s.t.

$$\tilde{c}_{t+1}^t + \tilde{a}_{t+1}^t = (1-\tau)w_{t+1}(1+\phi s^{Q^*}) + \tilde{a}_y^t(1+r) \quad (2.49)$$

$$\tilde{c}_{t+2}^t = (1+r)\tilde{a}_{t+1}^t + \tilde{b}_{t+2} \quad (2.50)$$

This yields the following equilibrium consumption profile for middle-aged and old agents:

$$c_m^{Q^*} = \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*} + (1-\tau)W_u(1+\phi s^{Q^*}) + \frac{b^{Q^*}}{1+r}\right] \quad (2.51)$$

$$c_o^{Q^*} = \beta\delta(1+r)c_m^{Q^*} \quad (2.52)$$

Where $a_y^{Q^*} = x + (1-\tau)W_u(1-s^{Q^*}) - c_y^{Q^*}$. The realized QH equilibrium is given by (2.44)-(2.46), (2.51), & (2.52). An analytical approach aimed at comparing this collection and the equilibrium consumption, education, and government transfers associated with RE optimization is no longer simple enough to illuminate the distinctions that arise between the two types of agents. Therefore, I proceed by calibrating the model in order to characterize the equilibrium distinctions that arise between RE and QH optimizers.

The return to education parameter, $\phi = 2.6$, was calculated by imposing a linear human capital production function, $h_{t+1} = (1+\phi s_t)$, and estimating the implied return to schooling using 2015 BLS data on median weekly earnings for high school dropouts ($s = 0$), high school graduates ($s = 0.2$), associate degree holders ($s = 0.4$), bachelor degree holders ($s = 0.6$), masters degree holders ($s = 0.8$) and professional degree holders ($s = 1$). As stated previously, I abstract from the kinked nature of this human capital production function and instead chose to average over the implied values of ϕ. Unskilled wage $W_u = \$235,000$ as this represent the income a full time low-skill worker (no high school diploma) would earn over 10 years according to the same 2015 BLS data used to calculate ϕ. I chose $\beta = 0.66$ which corresponds to an annual $\beta = 0.96$ raised to the 10^{th}, $r = 0.515$ so that $\beta(1+r) = 1$, and $\gamma = 0.5$. We set our agents' inheritance $x = 0$ in our baseline specification.

Table 4 outlines the differences in equilibrium behavior for RE and QH societies. Differences in expected utility for QH optimizers are also included at the bottom of Table 4. The preferred value of δ, the present bias parameter, is 0.7 corresponding to the findings of Laibson, Repetto, and Tobaman (2007). However, I choose to use values of $\delta = 0.8$ and $\delta = 0.6$ for our model 2 parameterization. I do so in order to explicate the impact of a high degree of present bias as well as a low degree of present bias that are conveniently centered around the preferred parameterization. Further, as outlined in the introduction, these values are in line with the lower and upper bound of δ consistent with the empirical literature on quasi-hyperbolic discounting. I will implement $\delta = 0.7$ in our analysis of education incentive pay. The calibrated results reported in Table 4 for $\delta = 0.8$ and $\delta = 0.6$ are complimented by Columns 2, 4, and 10 of Table 5 which outlines Model 2 equilibrium calibrations for the preferred specification of $\delta = 0.7$.

The measure of expected utility, the variables $E(U_y^{Q'})$ and $E(U_y^{Q^*})$, represent the utility associated with the expected consumption profile solved for by a young QH agent and the utility associated with the actual consumption profile that a QH agent will choose after re-optimizing when middle aged, respectively. These measures of utility are both calculated from the perspective of a young agent, so as to provide a measure of the utility loss associated with re-optimization performed by a QH agent.

I focus on changes in consumption and percent differences between consumption profiles for RE and QH agents. When $\tau = 0$, the results in table 4 are merely a calibrated version of the analytical equilibrium outlined previously. For each tax rate τ and QH discount rate δ considered, the analytical results outlined in Table 2 hold even with the inclusion of PAYG social security. I vary over several tax rates (and thus transfer amounts) in order to understand the differences in optimizing behavior that arise between RE and QH societies. I find that the gap between optimal rational and optimal QH behavior is decreasing in the discount factor δ. That is, the more present-biased a society is, the larger the gap between its equilibrium behavior and equilibrium behavior in a rational society[6].

For a given δ, increasing the role of social security (via increasing τ) leads to lower education for both RE and QH agents and a larger education gap between RE and QH societies. This comes from the inherent overweighting of current utility relative to future utility that distinguishes QH agents from RE agents. When RE and QH agents receive an increase to their old age income via a social security transfer, they immediately disinvest in education while young and reduce their planned savings when middle aged. However, QH agents do so at a much higher

[6] Although table 4 only highlights the use of two parameterizations of δ, numerical analysis corroborates these findings (the higher δ, the smaller the gap between RE and QH optimization) $\forall\ \delta \in (0, 1)$

TABLE 4. Differences for RE and QH agents: Social Security

δ	.8						.6					
τ	0.0		0.1		0.2		0.0		0.1		0.2	
		%Δ		%Δ		%Δ		%Δ		%Δ		%Δ
s^{R^*}	0.36	-16.3%	0.33	-18.5%	0.29	-21.6%	0.36	-36.0%	0.33	-40.6%	0.29	-47.4%
s^{Q^*}	0.30		0.27		0.23		0.23		0.19		0.15	
$c_y^{R^*}$	2.15	9.2%	2.04	9.0%	1.91	8.8%	2.15	20.3%	2.04	19.8%	1.91	19.2%
$c_y^{Q^*}$	2.35		2.22		2.08		2.59		2.44		2.28	
$c_m^{R^*}$	2.15	-5.1%	2.04	-5.2%	1.91	-5.5%	2.15	-14.1%	2.04	-14.5%	1.91	-14.9%
$c_m^{Q^*}$	2.04		01.93		1.81		1.85		1.74		1.63	
$c_o^{R^*}$	2.15	-24.0%	2.04	-24.2%	1.91	-24.4%	2.15	-48.5%	2.04	-48.7%	1.91	-50.0%
$c_o^{Q^*}$	1.63		1.54		1.45		1.11		1.04		0.98	
b^{R^*}	0.00	N/A	0.59	-3.8%	1.16	-4.0%	0.00	N/A	0.59	-8.4%	1.16	-8.9%
b^{Q^*}	0.00		0.57		1.11		0.00		0.54		1.05	
$E(U_y^{Q'})$	13.305	-.0483%	13.242	-.0485%	13.173	-.0488%	13.096	-0.250%	13.031	-0.251%	12.959	-0.253%
$E(U_y^{Q^*})$	13.298		13.236		13.167		13.063		12.998		12.926	

All values of consumption, c, and government transfers, b, stated above are in hundreds of thousands of units.

Note: In the table above, for all equilibrium values the %Δ is calculated as $\frac{x^{Q^*} - x^{R^*}}{x^{R^*}}$.

rate than RE agents, which leads to lower relative human capital and lifetime earnings.

Increasing social security decreases consumption in each period of life for RE and QH agents. This finding is in line with the majority of the literature regarding the welfare reducing effects of social security in overlapping generations models with human capital. When population growth is zero the economy is dynamically efficient and returns from other investments (education and savings) exceed the returns from social security. Thus, PAYG social security is a less effective means of saving than alternative investment options. Further, the utility gap between expected equilibrium consumption and actual equilibrium consumption of a young agent is increasing in the tax rate. This is outlined by the variables $E(U_y^{Q'})$ and $E(U_y^{Q^*})$ reported in the bottom two rows of Table 4. These variables are calculated by evaluating utility from consumption in the final two periods of life (from the perspective of a young agent) using actual equilibrium consumption ($c_m^{Q^*}$ and $c_o^{Q^*}$) to calculate $E(U_y^{Q^*})$ and planned equilibrium consumption ($c_m^{Q'}$ and $c_o^{Q'}$) to calculate $E(U_y^{Q'})$.

Both realized and planned consumption utility are decreasing in the tax rate for a given value of δ when social security is the only government outlay. From the perspective of a policy maker who is unsure whether agents in her society are rational or present-biased, social security proves to be an unequivocally poor policy intervention. It is both welfare reducing for RE and QH agents, it increases the old age consumption gap between these two theoretical societies, and it increases the utility gap between expected and realized consumption for a QH agent.

Thus far I have outlined distinctions that arise between RE and QH societies without discussing the likelihood that members of a society belong to one group or the other. The lifetime consumption profile of QH agents provides evidence that QH optimization may help to explain some features of observed consumption behavior that RE optimization cannot explain. The drop in old age consumption observed in time-series of U.S. data cannot be remedied with a model of RE optimization without the inclusion of several constraints, but it arises naturally in a simple 3-period model when optimizers are present-biased. Although this is not resounding evidence that societies behave in a systematically present-biased way, it is a feature of QH optimization that provides merit to the analysis of QH societies. In the next section, I proceed with an alternative policy intervention in which government outlays are split between funding an education incentives program and PAYG social security.

Equilibrium Analysis: Social Security & Education Incentives

In the following section, I augment the previous model by considering a second channel for the distribution of tax revenue. As social security unequivocally lowered household human capital and utility relative to a baseline in which the government does not levy taxes, I consider an alternative avenue for government spending.

Rational Solution

Agents are faced with the same optimization problem outlined in the previous section, but the budget constraint of young agents (previously 2.28) is now given by:

$$c_t^t + a_t^t = x + i_t^t s_t^t + (1-\tau)W_u(1-s_t^t) \tag{2.53}$$

where i_t^t is the incentive pay provided to students by the government. As before, the government taxes agents' labor income at the rate τ. Total tax revenue collected in period t remains the same. However, instead of directly transferring said revenue to old agents, the government splits its receipts between funding social security and education incentive pay. To maintain a balanced budget in every period, the government divides tax revenue in the following way:

$$i_t^t = \alpha[\tau W_u(1-s_t^t) + \tau W_u(1+\phi s_{t-1}^{t-1})] \tag{2.54}$$

$$b_t^{t-2} = (1-\alpha)[\tau W_u(1-s_t^t) + \tau W_u(1+\phi s_{t-1}^{t-1})] + \alpha(1-s_t^t)i_t^t \tag{2.55}$$

where $\alpha \in [0,1]$. The government collects sufficient taxes to pay every agent her education incentive pay for her entire schooling decision, even if all agents chooses $s_t^t = 1$. If agents chooses $s_t^t < 1$, then the excess revenue the government collects, $(1-s_t^t)i_t^t$, is added to the social security transferred to old agents. It is important to note that although I have referred to the education of the representative agent of generation t as s_t^t, this agent is actually an atomistic member of their generation. Thus, a young agent does not internalize the impact of their schooling decision on the gross education incentive pay, i_t.

Optimization leads to the following consumption and education profile for an agent born in period t:

$$\begin{aligned} c_t^t &= \frac{1}{1+\beta+\beta^2+\gamma}\left[x+i_t+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b_{t+2}}{(1+r)^2}\right] && (2.56)\\ c_{t+1}^t &= \beta(1+r)c_t^i && (2.57)\\ c_{t+2}^t &= \beta^2(1+r)^2c_t^i && (2.58)\\ s_t^t &= 1-\frac{\gamma(1+r)}{(1-\tau)W_u[\phi-(1+r)]+(1+r)i_t}c_t^t && (2.59) \end{aligned}$$

I must now account for the inclusion of s in both i_t and b_{t+2}. In equilibrium, i^{R^*} will be given by $\alpha[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})]$ and b^{R^*} will be given by $(1-\alpha)[\tau W_u(1-s^{R^*})+\tau W_u(1+\phi s^{R^*})]+(1-s^{R^*})i^{R^*}$. Thus, the equilibrium profile is not yet completely solved as s^{R^*} has not been pinned down. We substitute in for c_t^t in equation (2.59) and sub in i^{R^*} and b^{R^*} according to the above definition. This leads to the equilibrium solution for schooling:

$$s^{R^*}=1-\Xi\left(x+i^{R^*}+\frac{(1-\tau)W_u(1+\phi)}{1+r}+\frac{b^{R^*}}{(1+r)^2}\right) \quad (2.60)$$

where $\Xi=\dfrac{\gamma(1+r)}{((1-\tau)W_u[\phi-(1+r)]+(1+r)i^{R^*})(1+\beta+\beta^2+\gamma)}$.

Due to the non-linear nature of the above expression, I proceed numerically for the remaining equilibrium calibrations. Once s^{R^*} is obtained, the remaining equilibrium conditions are given by:

$$i^{R^*} = \alpha[\tau W_u(1 - s^{R^*}) + \tau W_u(1 + \phi s^{R^*})] \tag{2.61}$$

$$b^{R^*} = (1 - \alpha)[\tau W_u(1 - s^{R^*}) + \tau W_u(1 + \phi s^{R^*})] + (1 - s^{R^*})i^{R^*} \tag{2.62}$$

$$c_y^{R^*} = \frac{1}{1 + \beta + \beta^2 + \gamma}\left[x + i^{R^*} + \frac{(1 - \tau)W_u(1 + \phi)}{1 + r} + \frac{b^{R^*}}{(1 + r)^2}\right] \tag{2.63}$$

$$c_m^{R^*} = \beta(1 + r)c_y^{R^*} \tag{2.64}$$

$$c_o^{R^*} = \beta^2(1 + r)^2 c_y^{R^*} \tag{2.65}$$

Quasi-Hyperbolic Solution

Populating the model with QH agents leads to the same issue outlined above in regards to analytically deriving equilibrium schooling. Again, I proceed numerically to solve the following equation describing the nature of equilibrium schooling:

$$s^{Q^*} = 1 - \tilde{\Xi}\left(x + i^{Q^*} + \frac{(1 - \tau)W_u(1 + \phi)}{1 + r} + \frac{b^{Q^*}}{(1 + r)^2}\right) \tag{2.66}$$

Where $\tilde{\Xi} = \dfrac{\gamma(1 + r)}{((1 - \tau)W_u[\phi - (1 + r)] + (1 + r)i^{Q^*})(1 + \delta\beta + \delta\beta^2 + \gamma)}$.

This yields the following equilibrium government transfers and consumption profile for a QH agent:

$$i^{Q^*} = \alpha[\tau W_u(1 - s^{Q^*}) + \tau W_u(1 + \phi s^{Q^*})] \tag{2.67}$$

$$b^{Q^*} = (1-\alpha)[\tau W_u(1 - s^{Q^*}) + \tau W_u(1 + \phi s^{R^*})] + (1 - s^{Q^*})i^{Q^*} \tag{2.68}$$

$$c_y^{Q^*} = \frac{1}{1 + \delta\beta + \delta\beta^2 + \gamma}\left[x + i^{Q^*} + \frac{(1-\tau)W_u(1+\phi)}{1+r} + \frac{b^{Q^*}}{(1+r)^2}\right] \tag{2.69}$$

$$c_m^{Q^*} = \frac{1}{1+\beta\delta}\left[(1+r)a_y^{Q^*} + (1-\tau)W_u(1+\phi s^{Q^*}) + \frac{b^{Q^*}}{1+r}\right] \tag{2.70}$$

$$c_o^{Q^*} = \beta\delta(1+r)c_m^{Q^*} \tag{2.71}$$

where $a_y^{Q^*} = x + i^{Q^*}s^{Q^*} + (1-\tau)W_u(1-s^{Q^*}) - c_y^{Q^*}$.

Table 5 outlines the equilibrium consumption and education profiles for RE and QH agents. Rather than varying the degree of present-biasedness (as in Table 4), I instead focus on the tax rate and the allocation of taxes between funding education incentives and PAYG social security. I chose a value of $\delta = 0.7$ for our QH agents as this is the preferred value of δ [7]. When $\alpha = 0$, the calibrations in Table 5 represent the equilibrium consumption and schooling profile of RE and QH agents who only receive social security payments. Thus, columns 1,2 and 8 of table 5 are simply calibrated equilibrium profiles for model 2 when $\delta = 0.7$ and are readily comparable to the calibrations outlined in table 4.

[7] See page 31 for further exposition.

TABLE 5. Differences for RE and QH agents: Education Pay

τ	0.0		0.1						0.2					
α	n/a		0.0		0.5		1.0		0.0		0.5		1.0	
		%Δ		%Δ		%Δ		%Δ		%Δ		%Δ		
s^{R^*}	0.36	-25.7%	0.33	-29.0%	0.41	-21.5%	0.48	-17.0%	0.29	-33.9%	0.46	-18.2%	0.57	-12.5%
s^{Q^*}	0.27		0.23		0.32		0.40		0.19		0.38		0.50	
$c_y^{R^*}$	2.15	14.5%	2.04	14.2%	2.14	14.1%	2.24	14.2%	1.91	13.8%	2.13	13.9%	2.34	14.1%
$c_y^{Q^*}$	2.46		2.34		2.44		2.56		2.18		2.43		2.70	
$c_m^{R^*}$	2.15	-9.0%	2.04	-9.3%	2.14	-9.3%	2.24	-9.2%	1.91	-9.6%	2.13	-9.5%	2.34	-9.3%
$c_m^{Q^*}$	1.96		1.85		1.94		2.04		1.73		1.93		2.13	
$c_o^{R^*}$	2.15	-36.3%	2.04	-36.5%	2.14	-36.5%	2.24	-36.5%	1.91	-36.7%	2.13	-36.6%	2.35	-36.5%
$c_o^{Q^*}$	1.37		1.29		1.36		1.42		1.21		01.35		1.49	
b^{R^*}	0	N/A	0.59	-6.0	0.50	-0.0%	0.34	10.3%	1.16	-6.4%	0.99	0.3%	0.59	11.8%
b^{Q^*}	0		0.56		0.50		0.37		1.08		0.99		0.66	
i^{R^*}	0	N/A	0.00	N/A	0.31	-5.3%	0.65	-4.7%	0.00	N/A	0.64	-4.9%	1.37	-4.9%
i^{Q^*}	0		0.00		0.29		0.62		0.00		0.61		1.31	
$E(U_y^{Q'})$	13.210	.123%	13.147	-.123%	13.201	-.123%	13.253	-.122%	13.076	-.124%	13.196	-.123%	13.301	-.122%
$E(U_y^{Q^*})$	13.194		13.130		13.185		13.237		13.060		13.180		13.285	

All values of consumption, c, and government transfers, b and i, stated above are in hundreds of thousands of units.

Note: In the table above, for all equilibrium values the %Δ is calculated as $\dfrac{x^{Q^*} - x^{R^*}}{x^{R^*}}$.

By analyzing Table 5, it is clear that if the government is going to levy taxes, both consumption and schooling are higher for RE and QH agents in every period of life when $\alpha = 1$ relative to $\alpha < 1$. Unlike the model in in which all taxes fund PAYG social security, taxes no longer unequivocally lower consumption and education for RE or QH agents. Rather, higher taxes in conjunction with education incentive pay lead to higher equilibrium consumption and schooling relative to no taxation. When $\alpha = 1$, equilibrium schooling is over a year higher for both RE and QH agents for $\tau = 0.1$ and over two years higher for RE and QH agents when $\tau = 0.2$ relative to a regime with no taxation. Further, QH agents respond more drastically to higher education incentive pay than RE agents. This can be seen by comparing the schooling gap between RE and QH agents for a respective tax rate when $\alpha = 0$ and when $\alpha = 1$. Although endogenous growth and other positive spillovers from education aren't explored in this paper, Table 5 provides preliminary evidence that dedicating revenue from labor income taxation to education incentive pay leads to drastically higher schooling and consumption than an equivalent income tax rate with all proceeds being directed to PAYG social security.

As in Table 4, utility from expected consumption when middle aged and old is compared to utility from actual consumption when middle aged and old from the perspective of a young QH agent. When taxes are used for PAYG social security, a higher tax rate corresponds to lower utility from both expected and realized middle age and old age consumption and a higher utility gap between expected and realized consumption utility (see Table 4 rows 13 and 14). When the government splits tax revenue between PAYG social security and education incentive pay, the gap between expected and realized consumption utility is decreasing in α. That is, the higher the percentage of tax revenue dedicated to education incentive pay, the smaller the gap between utility associated with expected middle aged and old consumption and utility associated with actual middle aged and old consumption for a QH agent. Further,

when $\alpha = 1$, utility from expected consumption and realized consumption is higher than when $\alpha < 1$ and $\tau = 0$. Thus, not only does government funded education incentive pay increase consumption utility relative to a regime in which the government only issues social security, it increases consumption utility relative to an environment in which there is no taxation!

Numerical analysis confirms that equilibrium consumption utility is higher under linear education pay with $\alpha = 1$ for both QH and RE agents when viewed from the perspective of being young, middle aged, or old relative to no taxation. Thus, for a policy maker in either a QH or RE society, linear education pay is a simple policy measure that can be enacted in order to increase lifetime consumption and education of optimizing agents.[8]

[8]Note: we have assumed prices are exogenous. Changing this assumption could change the impact of equilibrium government intervention on both QH and RE agent's utility. We leave this analysis for future work.

Conclusion and Future Work

I build a unique model of human capital investment and analyze the optimizing behavior of both traditional exponential discounters and quasi-hyperbolic discounters. I find that agents in a quasi-hyperbolic society accumulate less human capital, consumes more when young and save less for retirement than agents in a rational, exponentially discounting society. The steep drop in retirement consumption generated by a simple three period OLG model with no idiosyncratic risk and perfect capital markets in the presence of QH agents provides further evidence that present-biased behavior matches certain empirical regularities [9]

I analyze several different arrangements for the disbursement of tax revenue. I find that both RE and QH societies are made worse off when taxes are levied and tax revenue is dedicated to funding PAYG social security relative to a regime in which no taxes are levied. Agents have lower utility, lower consumption in each period, and obtain less schooling relative to a tax free regime. Further, the gap between expected future utility from consumption and realized utility from consumption is increasing for QH agents in both the tax rate (τ) and the degree to which agents are present-biased (δ). However, taxes can be welfare improving if tax revenue is split between funding education incentive pay and social security. I show that agents in both RE and QH societies obtain more schooling, consume more in each period of life, and have higher lifetime utility when taxes are dedicated to

[9] See Angeletos et al. 2001 for further exposition on this topic.

education incentive pay. Further, education incentive pay reduces the welfare gap between expected and realized utility from consumption for QH agents.

In future iterations of this model I intend to add disutility from labor to the household's utility function. This additional complexity was omitted in this Chapter to preserve analytical tractability. The inclusion of disutility from labor will likely impact the magnitude of QH optimization on schooling investment as agents are now forced to trade-off between leisure, school, and work. However, the same mechanism leading present-biased agents to under-invest in education relative to RE agents remains the same. QH optimizers undervalue the future returns to education relative to RE discounters and will look to avoid within period disutility from both schooling and work when young. Future work will also include a normalized discount factor between RE and QH agents. Rather than assuming β is the same for all QH and RE agents and adding an additional discount factor δ for QH agents, I will normalize average discounting so that $\beta_{RE}=\delta\beta_{QH}$, where $\beta_{RE} < \beta_{QH}$. This approach is similar to the equilibrium selection of β_c outlined in Results section of Chapter 4.

CHAPTER III

PRESENT BIAS, PREFERENCE HETEROGENEITY, AND WEALTH INEQUALITY

Introduction

According to a national survey commissioned by Experian in 2016, 54% of Americans believe they will never fully pay off their debt, 46% of Americans have less savings today than they expected to have five years ago, and 71% of Americans aren't saving enough for retirement. Although some of these shortcomings can be explained by unexpected labor and capital earnings shocks or information frictions in the marketplace, a growing literature characterizing the widespread nature of present bias may provide an alternative viewpoint to help reconcile the observed gap between intentions and actions.

Present bias[1] refers to a preference anomaly wherein individuals discount trade-offs between all future periods and the current period at a higher rate than they discount tradeoffs between any two periods in the future. First outlined by Phelps and Pollak (1968) and Strotz (1956) and later reintroduced to the economics literature by Laibson (1997), present bias offers a convenient theoretical framework for explaining savings phenomena like those outlined in the Experian survey. Even in an environment in which individuals have perfect information regarding

[1]Also referred to as quasi-hyperbolic, quasi-geometric, or hyperbolic discounting.

future earnings, present bias creates a time inconsistency in preference that leads individuals to abandon their consumption savings plan in favor of re-optimizing (and over consuming relative to their previous plan) each period. I aim to elicit the avenues through which this consistent re-optimization impacts aggregate wealth and the timing of wealth accumulation over the life-cycle.

There is a robust empirical literature outlining the degree to which individuals are present-biased in both laboratory settings and over real world trade-offs that lends further credence to the examination of these types of non-standard preferences in a carefully calibrated general equilibrium framework[2]. I embed both present-biased and exponential discounters into a quantitative life-cycle model with uninsurable idiosyncratic risk in order to characterize the marginal contribution of each agent type towards generating wealth dispersion in the model economy. A consistent feature of this class of models is the gross under-prediction of wealth inequality in the model economy relative to U.S. data. Therefore, I am careful to characterize the contribution of each preference type to the overall dispersion of wealth and the evolution of wealth inequality over the life-cycle. I find the impact of present bias is largest when household planning horizons are short and for households with low wealth. As older households have both shorter planning horizons and lower wealth (on average) than working age households,

[2] See Della Vigna (2009) for a survey of the literature characterizing behavioral biases in economics.

special attention is paid to the role of preference heterogeneity and present bias on the accumulation of assets of older households.

Three key results stand out from my analysis. First, if present-biased agents are embedded in a partial equilibrium model, the impact of time inconsistency will be drastically overstated. In a model economy examined in general equilibrium (where prices adjust fully to the actions of agents) in which all agents are present-biased, present bias does very little to augment the aggregate wealth distribution relative to a model in which all agents behave rationally. This occurs as the inclusion of present-biased agents drives up the market clearing interest rate and changes the savings incentives of agents who would otherwise be tempted to over-consume and under-save. This result may be of particular interest as a number of previous studies examining the macroeconomic consequences of present-biased optimization embed agents in a partial equilibrium framework.

Second, although the aggregate wealth distribution is not particularly sensitive to the inclusion of present-biased agents[3], I find present bias increases wealth inequality as cohorts age and changes the timing of wealth accumulation in the model economy relative to a baseline model in which all agents behave rationally. Models that abstract from the richness of a life-cycle and instead place agents in an infinite horizon context will understate the role of present bias in the macroeconomy. This occurs as models with infinitely lived agents miss the

[3]When all agents are assumed to be present-biased.

interaction of shortened planning horizons and low average wealth inherent to older agents that amplifies the negative effects of present bias.

Third, the inclusion of discount rate and present bias heterogeneity improves the fit of the model economy to U.S. data. Although a number of authors have shown heterogeneous exponential discount factors lead to increased wealth dispersion in a life-cycle model, to the best of my knowledge this paper marks the first attempt to characterize the contribution of heterogeneous present bias to aggregate economic outcomes in a life-cycle framework. When some agents are present-biased and others are time-consistent, the time-consistent agents benefit from the aggregate mis-optimization of their present-biased peers. Thus, present bias heterogeneity leads to increased wealth dispersion and an increased concentration of wealth in the hands of the richest 1% relative to a model in which agents are only heterogeneous in the exponential discount factor. The remainder of the paper is structured in the following way: the next section reviews the literature, followed by sections describing my modeling environment, calibration approach, and results. This chapter ends with a concluding section.

Review of the Literature

Research on Wealth Inequality

There is a vast empirical literature outlining the extent of wealth and income inequality in the United States. The primary takeaways from this literature are:

wealth is much more highly concentrated than earnings[4], higher income households save a higher percentage of their wealth[5], and households with identical lifetime earnings retire with significantly different values of wealth[6]. Aiyagari (1994) and Huggett (1996) mark two of the first attempts to generate model economies for which the equilibrium wealth distribution matches the observed distribution of wealth in US data. Each author explores a model of idiosyncratic income risk in the presence of incomplete markets, with Aiyagari focusing on infinitely lived agents and Huggett focusing on finitely lived agents in a quantitative life-cycle framework. Each author finds that after endowing agents with a realistic degree of income risk and solving for optimal household, firm, and government decisions, the resulting equilibrium wealth distribution falls well short of recreating the concentration of wealth observed in the data. In particular, the richest 1%, 5% and 10% of households hold far too little wealth in equilibrium relative to the analogous wealth holdings of these percentiles in the data. This result stems from the model's inability to induce high savings for wealthy agents, who prefer to disinvest and prioritize consumption in model economies. Over the past 20 years, a literature aimed at remedying this shortcoming has emerged.

Heer (2001), DeNardi (2004) and DeNardi and Yang (2014) explore the role of intentional and unintentional bequests in cultivating wealth inequality

[4]See Piketty and Zucman (2014)

[5]See Dynan et al. (2004)

[6]See Hurst et al. (1998)

across generations. A heterogeneous agent overlapping generations model is augmented to accommodate lifetime uncertainty and the role of different bequest motives on equilibrium wealth accumulation is described. These authors find that modeling bequests, whether intentional or unintentional, does very little to augment the equilibrium distribution of wealth in a model economy. However, DeNardi and Yang find the introduction of non-linear bequest motives (only the richest households leave intentional bequests) improves the concentration of wealth in their model economy. Even in the presence of this assumption the proportion of wealth held by the richest agents still falls well short of that observed in the data.

Quadrini (2000), Cagetti and DeNardi (2006), and Benhabib, Bisin, and Zhu (2011) all consider models in which agents receive heterogeneous returns to capital income. Quadrini and Cagetti and DeNardi imbed entrepreneurs and workers in an overlapping generations framework and find the inclusion of entrepreneurs (who receive higher rates of return on their investments) considerably improves the fit of their models to U.S. data. Benhabib, Bisin, and Zhu find the inclusion of capital risk leads to an equilibrium wealth distribution that is much closer to the U.S. distribution of wealth than models with income risk alone. Although these authors are able to generate realistic wealth distributions in their model economies, they do so by making a significant departure from the benchmark models in the literature in which agents face a common interest rate and are only subject to risk in their labor endowments.

A separate strand of the literature more in line with the original works of Aiyagari and Huggett explores the role of preference heterogeneity in generating equilibrium wealth dispersion. Krusell and Smith (1998) find the inclusion of a small degree of heterogeneity in discount rates leads to a much more realistic wealth distribution in the model economy. However, this result hinges on a modeling environment in which agents are infinitely lived and receive frequent, transitory shocks to their labor earnings. If earnings are instead represented by a persistent process (as U.S. data indicates), then the Krusell Smith economy no longer generates a realistic wealth distribution. This results as agents no longer save high proportions of positive earnings shocks to protect against lower earnings in the future. Rather, an increase in earnings today leads to an increase in expected income tomorrow which induces agents to consume out of their new income at the expense of accumulating savings.

Hendricks (2007) augments a model closely resembling that of Huggett (1996) to accommodate heterogeneous discount factors. He exploits the fact that preference heterogeneity affects the distribution of wealth within a cohort as individuals age. Following this insight, the distribution of discount rates is targeted in equilibrium to match a set of age specific Gini coefficients. Hendricks finds a large degree of preference heterogeneity induces a modest increase in wealth inequality. In his model, the equilibrium wealth distribution is far less sensitive to preference heterogeneity than in the Krusell Smith model, exactly because agents face realistic earnings risk.

Present Bias and Preference Heterogeneity: Empirical Evidence

Following the re-introduction of present bias to economics by David Laibson (1997), a strong literature has emerged outlining the impact of present bias on consumer behavior. Meier and Sprenger (2009) find present-biased individuals are more likely to have credit card debt and have significantly higher credit card debt than their non present-biased peers. Recent work by Brown and Previtero (2017) indicates that present bias plays a large role in the determination of financial behaviors of households. They find that present-biased individuals are less likely to participate in a supplemental savings plan and, conditional on participation, they contribute less and are more likely to take a lump sum (vs an annuity) than individuals who display less procrastination.

Huffman et al. (2017) explore the role of discount rate heterogeneity and present bias among older households. Although they are unable to distinguish between present bias and heterogeneity in exponential discount rates, they find that individuals displaying less patience have lower retirement wealth and less planing for end of life care than more patient households. As in Brown and Previtero, Schreiber and Weber (2016) find individuals who answer time discounting questions inconsistently have a stronger tendency to choose lump sum payments over fair annuities. Further, they show the likelihood of an impatient household choosing a lump sum is increasing in age. My results offer a specific channel through which this result can be rationalized. In a general equilibrium framework, present-biased households behave near rationally when young and are more tempted by their biases as they reach retirement. The relationship between my work and the above literature is discussed further the conclusion of this Chapter.

Present Bias in Macroeconomics

İmrohoroğlu et al. (2003) consider the role of social security in an economy populated by overlapping generations of time-inconsistent optimizers. They find that unfunded social security lowers the capital stock, output and consumption for time-consistent and time-inconsistent individuals. However, time-inconsistent individuals may have higher welfare under a system providing unfunded social security, depending on the degree of their time inconsistency.

Angeletos et al. (2001) integrate hyperbolic discounting into a standard model of life-cycle behavior and find the inclusion of hyperbolic discounting can help rationalize observations in wealth holdings, debt accumulation and consumption paths in response to predictable income changes. However, their analysis is performed in a partial equilibrium setting in which prices are fixed and do not respond to aggregate behavior. I extend their model environment to a setting in which prices adjust to the savings decisions of households who are heterogeneous both in terms of the present-biased discount factor and the exponential discount factor.

My work most closely resembles that of Harris and Laibson (2001), Krusell et al. (2002) and Maliar and Maliar (2006). Harris and Laibson construct a partial equilibrium model in which present-biased agents are subject to transitory labor income shocks. Krusell et al. focus on a deterministic general equilibrium model with present-biased optimizers. However, agents are homogeneous in their degree of present bias and in their exponential discount factor. Mailar and Maliar introduce persistent labor earnings shocks and preference heterogeneity to a neoclassical growth model. However, they model infinitely lived agents and are unable to come remotely close to the wealth distribution of the US economy. I extend the work of Maliar and Maliar by embedding heterogeneous present-biased agents into a life-cycle model (as in Harris and Laibson and Angeletos et al.) in which agents are subject to persistent labor income shocks and preferences, earnings ability, and wealth are all partially inherited. Further, I consider heterogeneity in both the present bias discount factor (as in Maliar and Maliar) and in the exponential discount factor (as in Hendricks). This exercise is particularly valuable, as my results indicate present bias and discount rate heterogeneity have drastically different implications for the accumulation of wealth over the life-cycle. This result hinges on the interaction of preferences and age, and is therefore lost in infinite horizon settings.

The Model

The modeling environment is a stylized version of the stochastic incomplete markets life-cycle model which is commonly used to examine the distribution of wealth. I build off of the work of Hugget (1996) and Hendricks (2007) while integrating insights from İmrohoroğlu et al. (2003) and Harris and Laibson (2001) with regard to modeling present bias. The economy is comprised of a continuum of agents of unit mass, a single representative firm, and a government. I restrict my attention to examining the economy in steady state with competitive markets.

Households

An agent is born at model age 1, works for the first R periods of life, and dies with certainty after N>R periods. The probability of an agent surviving to age t conditional on surviving to age $t-1$ is given by s_t. When an agent dies, they are replaced by a child of age 1 who inherits the after tax value of their parents' wealth and imperfectly inherits their parents preferences and labor endowments. The population grows at a constant rate n with stable demographic patterns so that age $t \leq$N agents make up a constant portion of the population.

Agents inelastically supply $l = h(t)e$ units of labor to the market each period from birth to retirement, where $h(t)$ is a deterministic age-earnings profile and e is a labor endowment shock. An agent's initial labor endowments (e_1) is inherited stochastically from their parents and earnings evolve over the life-cycle according to Markov transition matrix, P_e. A new agent inherits their parents age a_{IG} labor endowment with probability ρ_{IG}. With probability $1 - \rho_{IG}$ an agent draws an initial labor endowment from the distribution governing the economy wide distribution of initial labor shocks. An agent's draw of e_1 depends on their parent's labor endowment in period a_{IG} and not their parent's labor endowment in their terminal period[7]. Upon retirement, all agents receive a social security transfer, τ_R, and consume out of their savings and annual social security allocation.

Agents share identical preferences over consumption and leisure with the exception of their discount factors, β and δ. These heterogeneous preference parameters are drawn at birth from a finite set of discrete values J and remain constant over an agent's life. Preferences are stochastically inherited with probability ρ_j. With probability 1-ρ_j preferences are drawn randomly from the stable distribution over agent types. Upon drawing preference parameters, households maximize the expected discounted sum of lifetime utility in the following way:

$$U = \max \left\{ u(c_1) + \delta_j \; \mathrm{E}\left[\sum_{t=1}^{N} \beta_j^t \left(\prod_{i=1}^{t} s_i \right) u(c_{t+1}) \right] \right\} \tag{3.1}$$

where c_t represents age t consumption, s is the probability of surviving one additional year conditional on being alive today, δ_j is the present-biased discount

[7]This assumption implies parents stochastically pass on their human capital to their children during the their working lifetime, and not when old.

factor of a type j agent and β_j is the exponential discount factor of a type j agent[8]. All bequests are assumed to be unintentional, as previous research finds intentional bequests play a small role in generating realistic wealth dispersion in the absence of nonlinear bequest motives[9].

Firms

Output is produced using capital (K) and labor (L) by a representative firm with production given by $Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$. The representative firm seeks to maximize profit in a competitive market, $F(K, L) - q_K K - q_L L$, where q_K and q_L represent the rental rates for capital and labor, respectively. In each period, capital depreciates at a constant rate δ_k.

Government

The government levies taxes on labor income (τ_w) and bequests (τ_B) and provides state dependent lump sum transfers ($\tau(x)$). Transfers can be broken down into two categories; social security transfers to retired households (τ_R if $t > R$) as well as unconditional lump sum transfers ($\tilde{\tau}$) to all agents. The government does not tax capital income. Therefore, the relevant household prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$. Aggregate transfer payments are given by

[8] I have reversed the meaning of the β and δ parameters from Laibson's exposition of quasi-hyperbolic discounting so that the discount factor β retains its standard interpretation found throughout the macroeconomic literature.

[9] For further exposition on this topic, please see the literature review in which the results of DeNardi (2004) and DeNardi and Yang (2014) are discussed.

$T = \int_x \Lambda(x)\tau(x)dx$ where x denotes a potential state and $\Lambda(x)$ denotes the density of households over states. All tax revenue beyond that needed to fund aggregate transfers is assumed to be discarded so that the government balances its budget in each period.

Dynamic Programming Problem

An agent of type j has a state vector x given by $x = (k, e, t, j)$ where k is wealth, e is the current period's labor endowment shock, and t is agent age. The agent's optimization problem can be written as a dynamic program where the Bellman equation is given by:

$$V(x) = \max_{(c,\ k')}\ u(c) + \delta_j \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{3.2}$$

subject to

$$c + k' \leq (1+r)k + wl + \tilde{\tau} + \tau_R, \tag{3.3}$$

$$k' \geq \underline{k},\ c \geq 0,\ k' > 0 \text{ if } t = N,\ V(x) = \tilde{V}(x) = 0 \text{ if } t = N+1. \tag{3.4}$$

Common transfers $\tilde{\tau}$ and retirement transfers τ_R are independent of earnings history and depend only on agent age.[10] If δ_j=1, this is the well known value function of an exponential discounter. When $\delta_j < 1$, I must structure the beliefs of agents with regard to their future selves.

The two primary cases studied in the behavioral literature are naive and sophisticated present bias. A sophisticated agent will behave in a present-biased way today, but will seek commitment devices to keep their future self from violating their planned consumption, savings profile. I proceed by assuming that agents behave in a naive manor. This assumption is convenient for several reasons. First, there is not a strong consensus in the behavioral literature regarding the true nature of agent sophistication, but recent work from Laibson (2015) concludes "a demand for commitment is a special case rather than the general case". Second, my model does not offer agents any vehicle for commitment. Therefore, knowledge regarding one's own biases is not exploitable, as agents do not have savings vehicles that can effectively constrain their decisions even if they are wary of their future self's inability to follow through with a plan of action. A naive agent solves their dynamic programming problem under the belief that, although they have a history of present bias and behave in a present-biased way today, they will act in a time-consistent manner in the future. This assumption can be represented by defining

[10] I proceed with this assumption as this approach is shared by both Huggett (1996) and Hendricks (2007). Whenever possible, I aim to reduce the distinctions between my model and these previous models to limit the avenues through which my results may differ from their work.

the continuation payoff $\tilde{V}(x)$ as:

$$\tilde{V}(x) = \max_{(c,\ k')} \ u(c) + \beta_j s' \mathrm{E}\big[\tilde{V}(x'|x)\big] \tag{3.5}$$

subject to constraints (3.3) and (3.4) outlined above.

The continuation payoff of a present-biased agent, which informs an individuals' consumption-savings decisions over all future periods, is identical to the value function of an exponential discounter. The only distinction between an agent with $\delta_j < 1$ and an agent with $\delta_j = 1$ is the additional discounting of future utility made every period. It is exactly this additional discount factor that leads present-biased agents to behave inconsistently.

Equilibrium

A stationary, competitive equilibrium consists of aggregate quantities $(K,\ L,\ C,\ T,\ B)$, prices $(w,\ r,\ q_L,\ q_K)$, transfers $(\tau(x))$, current and continuation value functions ($V(x)$ and $\tilde{V}(x)$), policy functions ($c(x)$ and $k(x)$) and a distribution over agent types $(\Lambda(x))$ such that:

- The policy functions ($c(x)$ and $k(x)$) along with the value function $V(x)$ and the cont-
inuation value function $\tilde{V}(x)$ solve the agent's optimization problem.
- Firms maximize profits.
- The government balances its budget.

- The distribution of households over states, $\Lambda(x)$, is stationary.
- Prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$.
- Markets Clear:

  (i) $K = \int \Lambda(x)k(x)dx$

  (ii) $L = \int \Lambda(x)l(x)dx$

  (iii) $F(K, L) = C + \delta K$ where $C = \int_x \Lambda(x)c(x)dx$

Present Bias: The Euler Equation and Discounting the Future

When agents are not present-biased, $\delta = 1$, the discount factor is equal to β in every period. However, if individual's do behave in a present-biased manor, $\delta < 1$, the discount factor is no longer constant. Rather, the discount factor becomes an endogenous variable that depends on an agent's current state. I denote the effective discount factor as $\beta_{x'}$ to highlight the new dependence of discounting on the state next period, x'. Following from the optimization problem set up in the previous section, the present-biased Euler equation can be written as:

$$u'(c_t) \geq \beta_{x'}(1 + r)s_{t+1}E_t(u'(c_{t+1})) \tag{3.6}$$

The effective discount factor is equal to[11]:

$$\beta_{x'} \equiv \beta_{x'}(k_{t+1}, e_{t+1}, t + 1) = \beta \left[1 - \frac{1 - \delta}{1 + r} \frac{E_t[u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t + 1)]}{E_t[u'(c_{t+1})]} \right] \tag{3.7}$$

[11] For further exposition of this derivation, see Appendix B.

Where $c_k(k_{t+1}, e_{t+1}, t+1)$ is the derivative of the optimal consumption function with respect to capital selected in period t. When $\delta = 1$, $\beta_{x'} = \beta$ in each period. However, when $\delta < 1$, the effective discount factor is a function of the state in period $t + 1$[12]. This generates a direct relationship between the effective discount factor and an agent's wealth, k. As shown in Maliar and Maliar (2006), if the consumption function is strictly concave, then the effective discount factor of present-biased agents will be strictly increasing in wealth. Thus, if two agents have the same degree of present bias but different levels of wealth, the richer of the two agents will behave more patiently than the poorer agent.

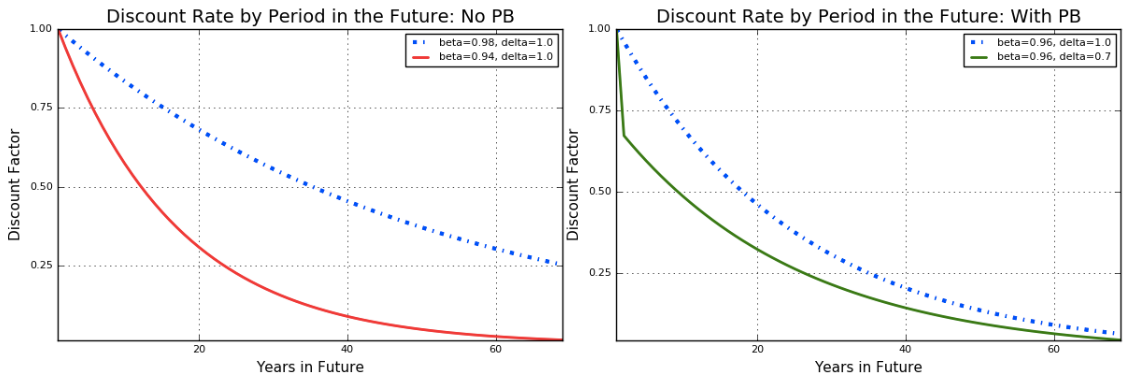
This result is not particularly surprising as diminishing marginal returns to consumption is a typical feature of an agent's utility function. A present-biased individual ends each day with a consumption savings plan laid out for their future self only to wake up the next morning and violate this plan. However, individuals who are very wealthy have less of an incentive to deviate from their planned consumption profile as their marginal utility gains are much smaller than those of a similarly biased individual with lower wealth (as wealthy agent's are at a flatter point on their utility curve with respect to consumption). It is this exact relationship between effective discounting and wealth that leads to increased dispersion in savings between rich and poor households. Thus, two present-biased

[12]For a full exposition regarding the effective discount factor under quasi-hyperbolic discounting, see Harris and Laibson (2001).

agents with identical lifetime earnings will display different savings behavior based on the amount of wealth they have accumulated.

FIGURE 1. Discounting of Future Utility

(a) homogeneous δ, heterogeneous β (b) heterogeneous δ, homogeneous β



The second avenue through which present bias impacts the accumulation of capital is the horizon over which decisions are being made. Consider Figure 1 which shows the effective discount rate applied to utility n periods in the future. Panel (a) shows the discount rate for exponential discounters ($\delta = 1$) with $\beta = 0.98$ and 0.94, respectively. The impact of heterogeneity in the exponential discount factor leads to small discrepancies in discounting over a short planning horizon but large discrepancies in discounting over trade-offs further into the future. This heterogeneity leads to the creation of savers and spenders in the model, as agents all face the same interest rate but have different preferences regarding trade-offs between now and any future period based on their discount factor. This intuition is confirmed in the results section, where I find the inclusion of heterogeneity in the exponential discount factor leads to a level shift in wealth inequality at each age in the model economy relative to a baseline model populated with homogeneous exponential discounters.

Panel (b) shows the discount rate for present-biased discounters with the same exponential discount factor ($\beta = 0.96$) and $\delta = 1.0$ and 0.6, respectively. Unlike heterogeneity in the exponential discount factor, which leads to large differences across household discounting as horizons move further into the future, present bias creates the largest disagreements regarding time discounting over short planning horizons. Further, as individuals apply their present-biased discount factor to every future period, this leads agents to continually re-evaluate their desired trade-off between current and future utility as they get closer to an event. This results in consistently higher discounting of a future event relative to an exponential discounter as the planning horizon between now and said event is shortened. It is this exact feature of present bias that leads agents in my model to decumulate wealth more quickly in old age when they are present-biased relative to exponential discounters.

To recap, if an individual is present-biased they behave in a more biased way the less wealth they have. Thus, in an economy comprised of present-biased individuals with identical preferences, there will still be a degree of effective preference heterogeneity if individuals have accumulated different amounts of wealth. This will serve to amplify wealth inequality, as it will generate savers and spenders relative to the prevailing market interest rate, albeit to a lesser extend than heterogeneous exponential discount factors. Further, when individuals are present-biased, the gap between their discounting of the future and the discounting of an exponential discounter is largest when planning horizons are short. Therefore, present bias will have it's greatest impact on agent behavior when: (a) wealth holdings are low and (b) planning horizons are short. This characterization of the impact of present bias on agent behavior offers support for the findings of Schreiber and Weber (2016). Younger agents are less tempted by their present bias than old agents, thus older households are more likely to behave irrationally and choose a lump sum over a fair annuity.

Calibrating Model Parameters

Demographics

Households are born at the age of 22 and live at most 69 periods (age 90). Households retire and begin receiving social security transfers at a model age of 43 (age 65). The probability of survival from one period to the next (1 year) is given by the mortality rates listed in the Social Security Life Tables [13].

Labor endowments

Agent's labor endowments consist of a deterministic age efficiency profile, $h(a)$, and a stochastic labor productivity shock, e. The age efficiency profile is

[13] A weighted average of Male and Female survival probabilities is used for model calibration. All values are taken from the 2015 Social Security Life Tables

modeled after Huggett 1996, using 1990 PUMS data. The transition matrix for labor endowment shocks, P_e, is the Markov transition matrix associated with the Markov approximation of the following autoregressive process:

$$\ln(e_t) = \rho \ln(e_{t-1}) + \epsilon_t \tag{3.8}$$

where e_{t-1} is the labor shock experienced in the previous period and $\epsilon_t \sim N(0, \sigma_\epsilon^2)\ \forall\ t$. I've selected values of ρ and σ_ϵ^2 that are consistent with the persistence of annual earnings and the variance of log earnings for individuals over their working lifetime. This provides a grid of potential labor shocks and a Markov transition matrix describing the probability of moving from shock e_i to any other shock e_j, $j \in [1, 7]$.

The initial productivity shock of young agents, e_1, is equal to their parent's shock at age a_{IG} with probability 0.41 and is drawn from a normal distribution governing initial labor endowments with probability 0.59[14]. Initial labor endowments are distributed $N \sim (\bar{e}_1, \sigma_1^2)$.

Although individuals in the model are not replaced by children until their death, I choose a value of $a_{IG} < a_D$. That is, agents inherit their parent's labor endowment in the middle of their parent's working life. This assumption is both reasonable, as human capital is likely transmitted between generations when parents are middle aged, and computationally convenient, as the observed

[14]I follow Hendricks (2007) in setting $\rho_{IG} = 0.41$ and setting $a_{IG} = 39$.

persistence of intergenerational earnings cannot be matched if endowments are transferred late in life.

Preferences

Preferences over consumption utility are given by $u(c) = c^{(1-\sigma)}/(1-\sigma)$. The curvature parameter of the household utility function is set to $\sigma = 1.5$. This is the value chosen in Hendricks (2007a), Huggett (1996), and DeNardi and Yang (2014), among others.

In terms of the calibration of present bias, Laibson, Repetto and Tobacman (2007) argue for a degree of present bias equal to 0.7, while Paserman (2008) finds an average degree of present bias of 0.65. However, Paserman finds the degree to which individuals are present-biased varies significantly with income. He finds low income households display a degree of present bias as low as 0.40 whereas high income households are significantly less time inconsistent (0.89). Meier and Sprenger (2015) estimate the value of the present bias discount factor using a series of survey questions and find average degree of present bias to be between 0.69 and 0.82 depending upon their model specification. They also find a significant degree of present bias heterogeneity, with poorer women displaying a higher degree of present bias than wealthier women. Tanaka et al. (2010) perform a similar survey based experiment on 181 rural Vietnamese households. They find an average degree of present bias of 0.644. However, unlike Meier and Sprenger or Paserman, they do not find any correlation between household characteristics and the degree of present bias. Due to the disparate findings outlined above in the empirical literature, I calibrate present bias as 0.7 in each experiment performed in this chapter.[15]

TABLE 6. Model Parameters

Demographics	
$a_D = 69$	Maximum lifetime (Physical age of 90)
$a_R = 43$	Retirement age (Physical age of 64)
P_s	Matches Mortality rates of couples found in Social Security Administration Period Life Tables 2015
Labor endowments	
$\eta_e = 7$	Size of labor endowment grid
$\rho = 0.96$	Persistence of labor endowments
$\sigma_e = 0.212$	Standard deviation of transitory shocks
$\rho_{IG} = 0.41$	Intergenerational persistence of labor endowments
$\sigma_{e_1} = 0.616$	Standard deviation of age 1 endowment shock
$a_{IG} = 19$	Age of intergenerational transmission (physical age 40) taken from Hendricks (2007)
Preferences	
$\sigma = 1.5$	Consistently used throughout the literature
$\rho_j = 0.5$	Intergenerational preference transmission is set to 0.5
Technology	
$\alpha = 0.36$	Capital income share
$\delta_k = 0.076$	Jointly set with A to normalize $q_L = 1$ when $r = 0.04$
$A = 0.89$	Jointly set with δ_k to normalize $q_L = 1$ when $r = 0.04$
Government	
$\tau_w = 0.40$	Tax rate on labor income, Trostel(1993), Hendricks (2007a)
$\tau_R = 0.4 * (AvgEarnings)$	Retirement transfer set to 40% of average household earnings

The parameters listed above are common to all model specifications. In the results section I distinguish experiments by the proportion of agents endowed with each β_j and δ_j.

Technology

The production function in the model economy is of the Cobb-Douglas form. $F(K, L) = AK^{\alpha}L^{(1-\alpha)}$. The capital share, α, is set equal to 0.36. A and δ are

[15]The results, while quantitatively sensitive to the selection of δ, do not change qualitatively when alternate parameterizations are considered.

chosen so that $q_L = 1$ when the interest rate, r, equals 0.04 and the capital to output ratio, $\frac{K}{Y}$, equals 3.1.

Government

Wages are taxed at a rate of 0.40, following Huggett (1996)[16]. Bequests are not taxed in the baseline case ($\tau_B = 0$) following a convention in much of the quantitative life-cycle literature. Preliminary analysis indicates my results are not sensitive to assumptions regarding bequest taxation.

Results

In this section, I draw distinctions between several experiments regarding the distribution of societal preferences over β and δ. As there is no consensus in the literature regarding the true distribution of household preferences, I remain agnostic with respect to which experiment corresponds to the "correct" model[17]. Instead, I analyze the wealth distribution corresponding to each distinct model economy and comment on the areas in which said distributions match wealth targets in SCF and PSID wealth data, which are commonly used as benchmarks in this literature. To impose discipline across modeling experiments, the aggregate capital to output ratio is set so that $\frac{K}{Y} = 3.1$ in each model economy and the

[16]Note, the results presented in the next section are robust to changes in the tax rate.

[17]See Frederick, Loewenstein and O'Donogue (2002) for a discussion of the empirical estimates of β dispersion. Their results indicate that low end estimates of β can be close to 0 and high end estimates can be greater than 1.

interest rate is set so that $r = 4\%$. Each experiment (and the subsequent distribution of wealth associated with the experiment) is disciplined by the selection of a common discount factor β_c applied to every household in the economy so that equilibrium r and $\frac{K}{Y}$ match the targets outlined above. In models without exponential discount factor heterogeneity, β_c is simply the exponential discount factor used by each household when making trade-offs between utility in any two future periods[18]. The term "Avg. β" refers to the average exponential discount factor across households in the model economy.

It is important to note that rather than iterating over β_c to match the targeted interest rate (and capital to output ratio), one could just as easily iterate over the interest rate for a given value of β_c to find an equilibrium. I avoid this approach for several reasons. First, changing the interest rate necessarily changes the capital to output ratio across modeling experiments and a fixed capital stock makes for more consistent graphical comparisons across models. Second, by constraining the capital to output ratio to be constant across models I am fixing an observable statistic in an economy (r) and varying an unobservable variable (β_c). Third, changing interest rates changes the price agents receive for renting their capital holdings and creates savers and spenders in a model economy with heterogeneous preferences. Changing the value of β_c does *the exact same thing* in the model economy.

[18]More detailed information regarding the solution algorithm can be found in Appendix B.

Although prices are fixed across experiments, an agent's response to prices is *not* fixed. Thus, a change in β_c can be interpreted as a change in the *relative price* of capital when discussing distinctions across models in the following section. When agents have heterogeneous preferences, a high value of β_c and a fixed interest rate of r=4% will generate individuals who view this interest rate as too high (relative to the value of β that would lead to consumption smoothing in their consumption Euler equation) and therefore become savers, and individuals who view this interest rate as too low and therefore become spenders. This mechanism for generating disparate savings decisions is equivalent to comparing model economies with a fixed value of β_c in which prices (r) respond to the distribution of households over types.

Outline of Experiments

The experiments outlined in the following section are characterized in Table 7 where the "Proportion of Households by Type" represents the stable distribution of households over each potential discount factor in the model economy. The experiment labeled "Base" corresponds to a baseline model in which all agents are exponential discounters ($\delta = 1.0$) and have the same $\beta = 0.963$ ($0.98 \times \beta_c = 0.98 \times 0.983$) in equilibrium. "BasePB" refers to a model in which all agents are present-biased ($\delta = 0.7$) and have an exponential discount factor equal to 0.999 in equilibrium."BaseHet" refers to a model in which all agents are exponential discounters and half of all agents have an equilibrium discount factor of 0.977

(0.98 × 0.997) and the other half have an equilibrium discount factor of 0.937 (0.94 × 0.997).

TABLE 7. Outline of Experiments

	Proportion of Households by Type						
	β			δ			
Experiment	(0.98	0.94	0.90)	(1.0	0.7)	β_c	Avg. β
Base	1.0	-	-	1.0	-	0.983	0.963
BasePB	1.0	-	-	-	1.0	1.017	0.999
BaseHet	0.5	0.5	-	1.0	-	0.997	0.957
	β			δ			
Experiment	(0.98	0.96	0.90)	(1.0	0.7)	β_c	Avg. β
Full	.44	.12	.44	1.0	-	1.003	0.945
FullPB	.44	.12	.44	-	1.0	1.027	0.968
*FullPBHet$_\lambda$	.44	.12	.44	λ	$1-\lambda$	-	-

All experiments labeled "Full" refer to a richer model in which agents are heterogeneous in their exponential discount factor β. Calibration for preference heterogeneity is modeled after Hendricks (2007) for a model with partial intergenerational transmission of preference and ability and full inheritance of wealth via accidental bequests. Hendricks backs out the distribution of households over preference types by matching age specific Gini coefficients in the model economy with their corresponding moments in PSID data. Although Hendricks allows for five potential discount factors, his approach for preference calibration results in meaningful weight being placed on just three potential β values. Therefore, I allow for just three values of β_j with weights close to those implied by Hendrick's calibration as opposed to selecting the degree of heterogeneity at random.

"Full" refers to a model in which agents are all exponential discounters ($\delta = 1.0$) with 44% of households endowed with $\beta = 0.98 \times 1.0032$, 12% of households endowed with $\beta = 0.96 \times 1.003$, and 44% of households endowed with $\beta = 0.90 \times 1.003$ where 1.003 is equilibrium β_c for the "Full" model. "FullPB" refers to a model with the same breakdown of exponential discount factors in the model economy and $\delta = 0.7$ for all agents. That is, every agent in the "FullPB" model economy is present-biased. Finally, the "FullPBHet" model refers to a set of calibrations in which households preferences are distributed over β as described in "Full", but a proportion λ of agents endowed with each β are not present-biased ($\delta = 1.0$) and $(1 - \lambda)$ are present-biased ($\delta = 0.7$). I consider values of $\lambda = 1$, 0.75, 0.5, 0.25, and 0 where $\lambda = 1$ corresponds to the "Full" model and $\lambda = 0$ corresponds to the "FullPB" model.

Results- Building Intuition

In this section I present results from the "Base" experiments. This exercise is useful as it provides a means of understanding the margins on which present bias and discount rate heterogeneity differentially impact the accumulation of wealth in model economies. Table 8 outlines the Gini coefficient and selected elements of the Lorenz Curve for both US data (SCF and PSID data) and for the "Base" model economies outlined above.

As shown in Huggett (1996), a model economy with homogeneous exponential discounters generates a Gini coefficient that is lower than that found in the data.

TABLE 8. Wealth Distribution in the U.S. and "Hugg" Model Economies

	Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)	0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)	0.80	34.7	23.1	11.3	12.7	17.2	1.0
Base	0.70	12.0	21.8	16.8	21.8	25.9	1.6
BasePB	0.72	11.8	22.4	17.4	22.3	25.3	0.7
BaseHet	0.74	13.1	24.0	18.3	20.2	23.6	0.8

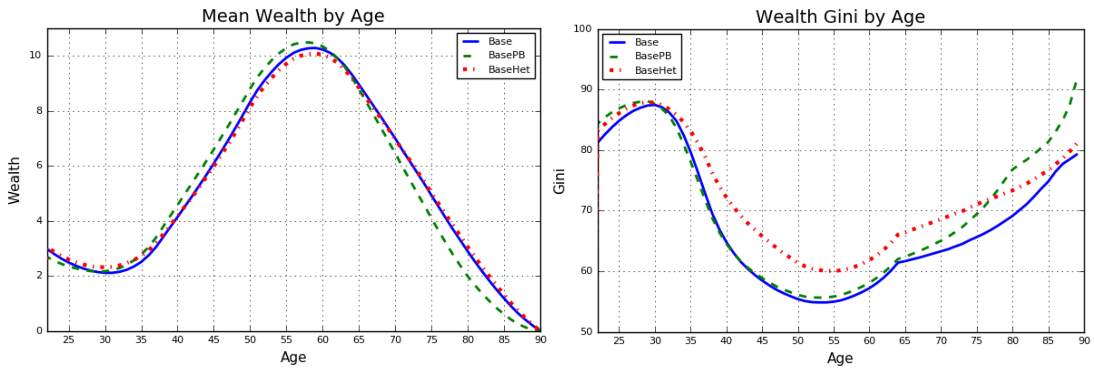
Further, the fraction of wealth held by the 99$^{\text{th}}$ percentile of households is just 12%, less than half of the wealth share of the top 1% of households reported in the PSID. Relative to the baseline model, adding present bias increases the Gini coefficient in the model economy to better match the data. However, this improved fit in the Gini coefficient is generated by a distribution of households in which the poorest 40% of households are too poor and the richest 1% of households are too poor relative to the data *and* the baseline model of time-consistent discounters. As shown in Krussell-Smith (1998) and Hendricks (2007a), the inclusion of heterogeneity in the exponential discount factor leads to a marked improvement in the Gini coefficient (0.74 compared to 0.70 in the baseline model) and the model's ability to match the wealth holdings of the top 1% of earners (13.1% compared with 12%).

To better understand the distinct role of present bias relative to that of preference heterogeneity in generating equilibrium wealth dispersion, consider the graphs in Figure 2 representing the average wealth accumulated in the model economy by age and the Gini coefficient in the model economy by age across different assumptions regarding economy wide preferences.

FIGURE 2. Evolution of Wealth over the Life-cycle

(a) Mean Wealth by Age

(b) Gini Coefficient by Age



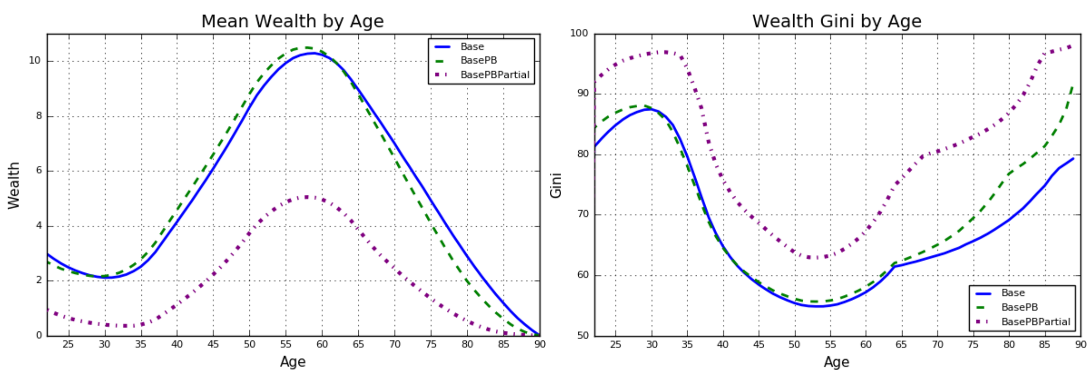
Average wealth in the "Base" model with homogeneous exponential discounters is nearly indistinguishable from the average wealth accumulated in the "BaseHet" model with heterogeneous exponential discounters. However, in the "BasePB" model economy, in which all agents are homogeneous in their exponential discount factor and present-biased, mean wealth peaks at a slightly earlier age and a slightly higher value than in the non present-biased economies. Further, wealth is depleted at a faster rate when all agents are present-biased following peak earnings, particularly after households retire (age 65). As each model is constrained to produce the same capital to output ratio and interest rate for the given distribution of households over types, total capital across model economies is also constrained to be identical. In spite of this fact, the inclusion of present-biased discounters has a significant impact on the timing of wealth accumulation.

Panel (b) sheds further light on the distinct role played by present bias relative to preference heterogeneity in generating wealth inequality over the life-cycle. Heterogeneity in discount rates leads to a level shift in the wealth Gini at every age over the life-cycle compared to a model with homogeneous discounters. Present bias, on the other hand, does very little to impact wealth inequality over the life-cycle *until households near retirement.* The Wealth Gini by Age in a present-biased economy is nearly identical to that in an economy populated by homogeneous exponential discounters until agents reach age 55. At this age, we see average wealth holdings rapidly decrease in the present-biased model economy and the Gini coefficient by age begins steadily increasing. At retirement, the "BaseHet" economy and the homogeneous exponential discounter economy see a slight uptick in wealth inequality, but wealth inequality in the present-biased economy increases at a much faster rate to a much higher level than either of the exponential economies. Present bias is the most costly to consumers when planning horizons are short and when agents have low wealth. We see wealth inequality drastically increase upon retirement as a wedge is driven between households with sufficient wealth to behave near rationally and households with low wealth who's biases are exacerbated by the short planning horizon over their remaining lifetime.

Early in the life-cycle, an economy comprised of present-biased optimizers looks very similar to one comprised of homogeneous discounters, as general equilibrium effects impose a higher β_c in the present-biased society than the "Base" society. Thus, a high relative value of savings overwhelms present-biased agents' desire to over consume while young, as they are extremely patient relative to the market interest rate in the long run compared with agents in the "Base" model who have a much lower β_c. Models that do not place present-biased agents in a general equilibrium framework will surely miss this fact. To this point, consider a version of the "BasePB" model that is *not* in general equilibrium, called "BasePB$_{partial}$". Every agent is endowed with $\beta_c = 0.963$ as in the "Base" economy and the interest rate is still set to $r = 4\%$.

FIGURE 3. Wealth Evolution: General vs Partial Equilibrium

(a) Mean Wealth by Age (b) Gini Coefficient by Age



The resulting economy has a Gini coefficient of 0.83 and the top 1% of households hold over 19% of all wealth. These are both significant improvements over the baseline model's ability to match inequality in the data, and may lead one to believe that homogeneous present bias plays a tremendous role in generating wealth inequality. However, this increase in wealth dispersion is generated by a much poorer society that has far greater wealth inequality at every age when compared with the the "Base" and "BasePB" societies. Figure 3 shows the mean wealth by age and Gini coefficient by age for the "Base" and "BasePB" models relative to the "BasePB$_{partial}$" economy. The above graphs shows a model analyzing the role of present bias outside of general equilibrium will drastically misrepresent the role played by present bias, particularly with regard to any measure that depends on aggregate wealth or the interest rate. By allowing the relative price of capital (the combination of β_c and r) to vary across modeling assumptions, model economies do not display increased wealth dispersion over an agent's working lifetime when all individuals are assumed to be present-biased.

In general equilibrium models, the largest deviation between present bias and non-present-biased societies comes late in life as households with low wealth are far more tempted by their shortened planning horizon than high wealth present-biased households. As there is already some degree of wealth inequality at retirement across all model economies due to heterogeneity in labor earnings, inequality is amplified in a present-biased society when the shortened planning horizon of old age is interacted with lower average wealth and increased wealth dispersion. Just as partial equilibrium models will overstate the importance of present bias in generating wealth inequality, infinite horizon models will understate the role of present bias *even in general equilibrium* as the largest impact of present bias on household outcomes occurs as households reach retirement. Thus, models missing the richness associated with life-cycle dynamics will understate the role of present bias.

Adding Present Bias to a Model with Preference Heterogeneity

Having established the role of present bias and preference heterogeneity in generating wealth inequality in the simple economy comprised of homogeneous exponential discounters, I now turn to a set of experiments in which agents display a large degree of exponential discount rate dispersion to better understand the margins on which present bias (and present bias heterogeneity) can improve the fit of my model to data. Results from the "Full" experiments (outlined in Table 7) are presented in Table 9. Recall, each "Full" experiment is calibrated so that 44% of the population has $\beta = 0.98$, 12% of the population has $\beta = 0.96$, and 44% of the population has $\beta = 0.90$, where each β is multiplied by the relevant β_c specific to each calibration[19]. In each experiment, I vary the percentage of the population with present-biased preferences, where a proportion λ of the agents in the model are not present-biased ($\delta = 1.0$) and $(1 - \lambda)$ of the agents are present-biased ($\delta = 0.7$). For each experiment, the proportion of individuals in the model economy endowed with each β is unaffected by the percentage of individuals that are present-biased. For example, when $\lambda = 0.5$, 44% of individuals have $\beta = 0.98$, 12% have $\beta = 0.96$, and

[19]This follows from the calibration of the distribution of preferences over household types in Hendricks (2007) for a model with accidental bequests.

44% have $\beta = 0.90$ but half of the individuals endowed with each β are endowed with $\delta = 1.0$ and half are endowed with $\delta = 0.7$.

The results from my calibration of the "Base" model as well as the wealth moments in PSID and SCF data are reported in Table 9 for ease of comparison. As shown in Hendricks (2007), the "Full" model in which there is a large degree of discount rate heterogeneity and no individuals are present-biased offers a significant improvement in fit relative to the baseline "Base" model. The Gini coefficient on wealth is 0.77 (compared with 0.70 in the "Base" model and 0.76-0.80 in the data) and the percentage of wealth held by 99^{th} percentile of households is 13.9 (compared to 12.0 in the "Base" calibration). Augmenting the baseline "Full" model so that every individual is present-biased ("FullPB") results in a slightly higher Gini coefficient of 0.79. As β_c in the "FullPB" model is larger than β_c in any of the other model calibrations considered, agents in this experiment have an incredibly high incentive to save. Thus, the percentage of wealth held by the top 1% of households is actually reduced by 0.2 relative to a model with no present bias. Again, we see the powerful role played by general equilibrium as the negative impact of agent's biases are somewhat mitigated by the increased relative prices of capital.

As I vary the proportion of individuals who are present-biased from 0 to 1, there is a steady increase in both the Gini coefficient **and** the percentage of wealth held by the 99^{th} percentile of earners. The model in which 75% of households are

TABLE 9. Wealth Distribution in the U.S. and "Full" Model Economies

	λ	β_c	Gini	99-100	95-99	90-95	80-90	40-80	0-40
PSID (2003)			0.76	25.3	21.8	14.0	16.3	21.8	0.9
SCF (1998)			0.80	34.7	23.1	11.3	12.7	17.2	1.0
Base	1	0.983	0.70	12.0	21.8	16.8	21.8	25.9	1.6
Full	1	1.003	0.77	13.8	26.3	19.6	20.6	19.7	0.1
FullPBHet$_{.75}$	3/4	1.008	0.78	14.0	27.2	19.6	20.7	18.5	0.0
FullPBHet$_{.5}$	1/2	1.013	0.79	14.2	27.9	19.6	20.8	17.5	0.0
FullPBHet$_{.25}$	1/4	1.020	0.80	14.4	27.9	19.9	20.7	17.1	0.0
FullPB	0	1.027	0.79	13.6	27.5	20.2	21.0	17.6	0.0

present-biased ("FullPB$_{.25}$") results in a wealth share for the top 1% of earners of 14.4, which is 20% higher than the equivalent wealth share in the "Base" calibration and 4.5% higher than the wealth share in the "Full" calibration in which no agents are present-biased. The Gini coefficient in this experiment is equal to 0.80, which matches the Gini reported in SCF data exactly. Thus, a model in which there is a high degree of discount rate heterogeneity and heterogeneity in whether households are present-biased generates an equilibrium wealth distribution that better matches both the Gini coefficient and the wealth share of the top 1% of households.

The rationale behind this finding is fairly straightforward. As shown in the previous section, if we fail to embed present-biased agents in a general equilibrium model in which prices respond to their actions, the resulting model economy will overstate the role of present bias in generating wealth inequality. When agents are heterogeneous with regard to their present bias, present-biased agents are essentially embedded in a partial equilibrium model, albeit one in which the price (or in this case, the value of β_c) is closer to their desired price than it would be if every agent in the economy was not present-biased. In the following section, I show the present-biased and non present-biased sub-economies when $\lambda = 0.75$, 0.50, and 0.25. As the percentage of agents in the model economy who are present-biased increases, the gap in average wealth between present-biased and non-present-biased individuals widens. This occurs as non present-biased individuals exploit an interest rate that is quite high relative to their discount factor exactly because the behavior of present-biased individuals has bid this value up in equilibrium. By examining mean wealth and Gini coefficients by age for both present-biased and non present-biased agents, I am able to show that the increased wealth dispersion reported in Table 9 is the result of *across* group inequality. *Within group* inequality is reduced as the percentage of individuals in the economy who are present-biased is increased.

The Present-Biased Sub-Economy

In this section I outline distinctions between two agent types in the "FullPBHet" model experiments: non present-biased (non PB) and present-biased (PB) individuals. Recall, each "FullPBHet" calibration includes a proportion λ of non PB agents and a proportion $(1-\lambda)$ of PB agents. In each of these calibrations, I define the economic behavior of non PB agents as the *exponential sub-economy* and the economic behavior of present-biased agents as the *present-biased sub-economy*. In Table 9, I report the equilibrium value of β_c for each experiment that

TABLE 10. Wealth Inequality for PB vs non PB Agents

	Gini	Top 1%	Top 5%	Top 20%
PSID (2003)	0.76	25.3	47.1	77.4
SCF (1998)	0.80	34.7	47.8	81.8
Full	0.77	13.8	40.1	80.3
FullPBHet ($\lambda = .75$)	0.78	14.0	41.2	81.5
-Non PB Agents	0.76	12.9	38.8	79.1
-PB Agents	0.83	17.8	48.1	88.7
FullPBHet ($\lambda = 0.50$)	0.79	14.2	42.1	82.5
-Non PB Agents	0.75	11.9	37.3	77.8
-PB Agents	0.82	16.6	46.0	86.4
FullPBHet ($\lambda = 0.25$)	0.80	14.4	42.3	82.9
-Non PB Agents	0.74	10.7	36.0	76.7
-PB Agents	0.81	15.3	43.6	84.6
FullPB	0.79	13.6	41.1	82.3

normalizes the interest rate and capital-to-output ratio across models. The "Full" model in which all agents are non PB results in a value of $\beta_c = 1.003$ and the "FullPB" model in which all agents are PB results in a value of $\beta_c = 1.027$. As the proportion of individuals endowed with $\delta = 0.7$ increases, the value of equilibrium β_c increases as well. Thus, when some agents are present-biased, non PB agents have access to an effective interest rate ($\beta_c \times r$) that is *higher* than what they face in a model comprised entirely of exponential discounters, and PB agents face an effective interest rate that is *lower* than what they face in a model economy comprised entirely of present-biased agents. It is this discrepancy that places both agents types in a partial equilibrium sub-economy in which prices have not fully adjusted to the individual preferences of each respective type.

Table 10 highlights the Gini coefficient and the wealth holdings of the top 1%, 5% and 20% of households in each "Full" model economy as well as in SCF and

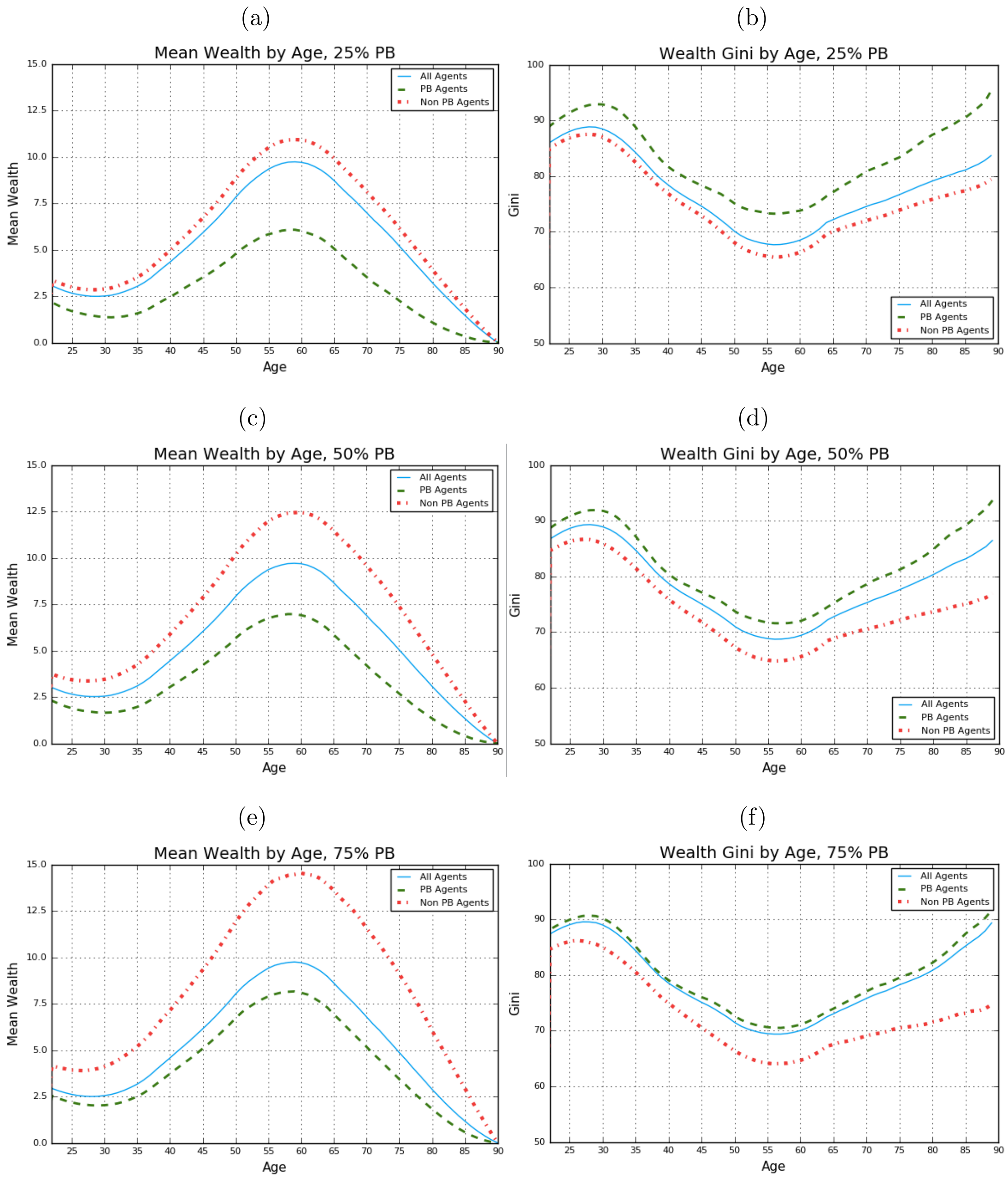
PSID data. For each "FullPBHet" calibration, these same statistics are evaluated for exponential and present-biased sub-economies. By evaluating the sub-economy associated with each agent type, I am able to provide insight into the avenue through which increased wealth dispersion is arising in the model economy. As shown in Table 9, as the percentage of agents who are present-biased increases, overall inequality increases. The Gini coefficient and wealth holdings of the top 1%, 5%, and 20% of households increase to levels much closer to that in the data in response to the increased proportion of agents displaying time inconsistent preferences. However, wealth inequality in each sub-economy is highest *for both PB and non PB agents* when the percentage of agents endowed with present-biased preferences is low!

When $\lambda = 0.75$, the present-biased sub-economy (25% of the total population) has a Gini coefficient on wealth of 0.83 and the wealth ownership of the top 1% of PB households is 17.8% of all wealth in the present-biased sub-economy. The exponential sub-economy has a Gini coefficient of 0.76 and wealth ownership of the top 1% of households is 12.9%. As λ increases, the Gini coefficient in each sub-economy decreases and the wealth holdings of the top 1%, 5% and 20% of households in each sub-economy decrease as well. However, in spite of the reduction in inequality within each sub-economy, overall inequality is increased as the proportion of present-biased agents increases.

Again, we return to the importance of general equilibrium effects when agents are endowed with time inconsistent preferences. As more present-biased individuals are added to the model economy, the common discount factor β_c is bid up by the impatient behavior of these individuals. As demonstrated by the distinction between the "FullPB" and "Full" model economies, if all agents are present-biased wealth inequality is *reduced* relative to a world in which all agents are exponential discounters. Thus, the higher the proportion of present-biased agents in the economy, the greater the savings incentive for both present-biased and non present-biased agents. As all agents face an increased savings motive when more time inconsistent discounters are in the economy, within type inequality is reduced because both present-biased and non present-biased agents accumulate more wealth in the face of relatively inexpensive capital. Despite this fact, inequality in the economy as a whole increases because wealth dispersion *across* agent types increases with the percentage of present-biased agents in the model economy.

Figure 4 provides further insight into the evolution of wealth inequality in each "FullPBHet" calibration. The dashed line represents the mean wealth by age of present-biased agents and the wealth Gini by age across present-biased agents for $\lambda = 0.75$, 0.50, and 0.25. The dash-dot line represents these same statistics for the non present-biased agents in each experiment, and the solid line represents the mean wealth by age and wealth Gini by age for all agents in the model economy. Comparing panels (a), (c), and (e), it is apparent that as the proportion of individuals in the model economy endowed with present bias increases, the gap in mean wealth holdings between PB and non PB agents increases. Although a

FIGURE 4. Evolution of Wealth over the Life-Cycle- Mixed Economy



higher β_c leads to an increase in the mean wealth holding by age for both the present-biased and exponential sub-economies, as $(1-\lambda)$ increases, agents in the exponential sub-economy take advantage of the increased effective interest rate they face when coexisting in an economy with a large number of present-biased individuals and amass much higher average wealth than their present-biased peers. Thus, as the percentage of individuals in the economy endowed with present-biased preferences increases, inequality between time-inconsistent and time-consistent agents increases.

As cross group inequality increases in response to a higher proportion of present-biased agents in the model economy, within group inequality is somewhat reduced in both the exponential and present-biased sub-economies. Consider the Gini coefficient on wealth by age displayed in panels (b), (d), and (f) of Figure 4. As more present-biased agents are added to the model economy, the Gini coefficient at every age is lowered (very slightly) for both present-biased and exponential agents. As shown in Table 10, this decrease in within group inequality results from an increased savings incentive for all agents as an increase in $(1-\lambda)$ drives up β_c. When all agents face an increased savings incentive, they accumulate more wealth earlier in their lifetime and inequality is driven down by an economy comprised of net savers. Further, the gap in wealth Gini by age between the present-biased and exponential sub economies is largely unaffected by the percentage of individuals endowed with PB preferences. That is, the distance between the dashed line

(PB agents) and the dashdot line (non PB agents) is nearly identical for each calibration. This gap is smallest when agents are young and begins widening at a much faster rate when agents near retirement and the shortened planning horizon of older households interacts with the natural inclination to under-save inherent to present-biased optimizers.

A final note of interest from Figure 4 pertains to the exact shape of the wealth Gini by age for all agents in the model economy (the solid line in all three experiments). In panels (b), (d), and (f) this line reaches its minimum between age 55 and 60, but the rate at which inequality evolves following its trough depends on the percentage of agents in the model economy who are present-biased. In panel (b), when only 25% of agents are present-biased, the wealth Gini by age increases at a much slower rate following age 60 to a lower overall level than it does in panel (d) (50% of agents PB) or panel (f) (75% of agents PB). Thus, my model offers a convenient means of backing out the percentage of households who display present-biased preferences via the examination of the evolution of wealth inequality as agents age. Exponential discounting will result in a low degree of wealth dispersion following age 60, while present bias implies a rapid increase in wealth dispersion after age 60. By measuring the actual dispersion of wealth amongst older households, one could infer the percentage of households endowed with each preference type.

This result has the potential to resolve an issue discussed in Hendricks (2007). Recall Hendricks' insight that heterogeneous exponential discounting leads to dispersion in wealth as households age. Thus, he calibrated the distribution of households over exponential discount factors to minimize the distance between model Gini coefficients on wealth by age and moments in U.S. data. Although it may appear that this approach could be used to replicate the wealth distribution perfectly, Hendricks notes "since preference heterogeneity increases inequality among young and old households, it is not possible to match inequality among the old without overstating inequality among the young". As shown throughout this section, present bias offers an avenue through which old age wealth inequality can be increased without overstating inequality among younger households. A future endeavor aimed at calibrating the distribution of households over exponential and present-biased discount factors may offer a resolution to this issue proposed by Hendricks. A calibration approach of this type could, at the very least, shed light on the percentage of households endowed with present-biased preferences.

Conclusion

I embed present-biased agents into an overlapping generations, quantitative life-cycle general equilibrium framework in which agents face uninsurable idiosyncratic income shocks. If all agents are assumed to be present-biased, the distribution of wealth is largely unaffected relative to a baseline economy in which all agents are exponential discounters. However, stark differences arise between a present-biased society and a society of homogeneous, exponential discounters when the accumulation of wealth over the life-cycle is analyzed. A present-biased society is characterized by an earlier peak in mean wealth by age and by increased dispersion in wealth as household reach retirement. I then consider a model economy populated by agents who are heterogeneous across both their exponential and present-biased discount factor. I find the inclusion of some present-biased households improves the fit of the model economy to the data. The increase in wealth dispersion resulting from an increase in the percentage of households that are present-biased arises as time-consistent discounters amass higher wealth relative to their present-biased peers due to the general equilibrium impact of present-biased optimization on the effective interest rate.

The work outlined above highlights several avenues through which present bias may help to rationalize gaps in the literature pertaining to the savings decisions of retired households. As noted in Schreiber and Weber (2016) and Huffman et al. (2017), some puzzling behaviors associated with retired households appear to be correlated with present-biased preferences. Schreiber and Weber highlight the fact that older households characterized as impatient are more likely to choose a lump sum over a fair annuity relative to patient older households and younger (age 30-50) impatient households. This result fits nicely with the general equilibrium implications of present bias; older households are more tempted by their behavioral biases than younger households due to their shortened planning horizon and lower average wealth. Huffman et al. find there is a large degree of heterogeneity in time discounting of older households, but due to data restrictions they are unable to distinguish between heterogeneity in the exponential discount factor and heterogeneity in the present-biased discount factor. Thus, they conclude that as less patient households have significantly lower net wealth in retirement, this is "probably indicating that the least patient save less and therefore arrive at old age with fewer assets." My results imply that this may not be the entire story. If agents are present-biased, then wealth dispersion in retirement will be a function of both pre-retirement savings decisions **and** post-retirement savings decisions. Thus, further work eliciting the proportion of individuals displaying present-biased preferences could help to distinguish the role played by high discounting due to a low draw of β versus a draw of $\delta < 1$.

Although I have primarily focused on the interaction of present bias and old age, it is important to note that the key interaction of shortened planning horizons and low wealth that leads older populations to suffer most from their biases is not an age dependent phenomenon. In the modeling environment utilized throughout this paper, the only finite horizon for an agent is the entire life-cycle. However, as post-secondary educational investment occurs early in life, a young agent is effectively looking at a short finite horizon over which they must decide how much schooling to attain. Previous work by Nighswander (2017) embeds present-biased agents in a simple three period framework in which time in the first period of life is split between supplying low skill labor and acquiring human capital via costly education. I find that present-biased individuals acquire less education leading to lower lifetime earnings and lower retirement consumption than time-consistent discounters. I intend to extend this simple model of educational choice to a life-cycle framework in which agents must decide how much education to acquire while young before entering the labor market. This endeavor may help to amplify lifetime inequality in the model economy to levels closer to US data.

CHAPTER IV

COLLEGE INVESTMENT & HETEROGENEOUS PATIENCE

Introduction

In the United States, the average college graduate earns over 80% more than the average individual with only a high school degree[1]. Further, the returns to a college degree have been consistently increasing since the early 1970s, which has led to a large increase in the percentage of high school graduates enrolling in four-year colleges. In spite of this increase in college attendance, there has been a very modest increase in the percentage of individuals who actually complete a bachelor's degree. In 1970, 23% of high school graduates had obtained a bachelor's degree by age 23 (51% had attempted some college) and by 1999 67% of 23 year olds had attempted some college, but only 24% had earned a college degree (Turner 2004). As the market returns to college enrollment are primarily captured upon the completion of one's degree (Carnevale et al. 2011) and tuition costs have increased substantially over the last 30 years, the fact that over 40% of individuals who enroll full time in four-year colleges fail to obtain a bachelor's degree within 6 years is truly puzzling (Velez 2014).

In this chapter, I utilize the skills and insights gained in the previous two chapters to outline a model economy in which dropouts are induced via a novel

[1] See Carnevale et al. 2001.

mechanism: present-biased optimization. Previous attempts to rationalize college completion and dropout rates have focused primarily on ability uncertainty. The typical explanation is that students begin college with very little information about their schooling ability and, through the process of credit accumulation and noisy signals received while attending college, students update their beliefs regarding their ability to complete a degree. However, a growing empirical literature indicates that behavioral biases, and present bias in particular, may shed light on previously unexplained aspects of college enrollment and completion. As theoretical work on this topic is extremely limited, I provide support for this empirical literature by outlining the pathway through which present bias leads to college dropouts in a model with no ability uncertainty.

As outlined in Chapters 2 and 3, a present-biased agent will continually re-evaluate their planned consumption, savings, and time allocations in favor of increased within period utility. Further, the degree to which present bias leads agents to re-evaluate their plans is decreasing in their wealth. This leads present-biased individuals with low initial assets to behave in a more biased fashion than agents with identical preferences and higher initial wealth. This occurs as high wealth individuals have high consumption and receive smaller gains in utility from deviating from their previous plan than low wealth individuals who receive large utility gains from small changes in their current consumption[2].

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[2]See Appendix A from Chapter 3 for more detail.

In the context of the education model outlined in this chapter, for certain ranges of starting capital present-biased individuals will begin attending college and accumulating credits at a slow rate that would not result in the completion of a degree because they expect to accumulate credits more rapidly in the future. However, unlike an impatient exponential discounter (low β) who might accumulate credits too slowly to complete a degree in the allotted time frame and therefore choose not to attend college, a present-biased agent will expect their future self to behave in a more constrained way and increase the rate at which credits are accumulated. In Section 5, I find when credit constraints bind and agents are unable to borrow against their future earnings, the poorest present-biased agents who enroll in college dropout and enter the labor market without completing their college degree.

Although there is a growing empirical literature outlining the role that behavioral biases (and present bias in particular) play in generating observed college enrollment and completion, to the best of my knowledge this paper marks the first attempt to generate college dropouts via present bias in a model of optimizing consumer behavior. The next section reviews the literature on college completion and human capital investment in life-cycle economies. I then set up a model of optimizing consumer behavior in a life-cycle economy and outline the calibration approach for this model. Next, I present results from a simplified version of the model followed by a brief concluding section.

Literature Review

Although human capital investment and present bias have not been jointly examined in a quantitative life-cycle model, each of these topics has been explored independently in this modeling environment. As previous chapters have focused on present bias in overlapping generations model economies, this literature review will be focused on papers exploring investment in post-secondary education[3]. Lochner and Monge (2011) explore the relationship between endogenous borrowing constraints and schooling investment. They find when borrowing constraints are endogenous, federal policies aimed at increasing schooling have both a first order impact on college enrollment and a secondary impact on enrollment via the expansion of private credit which increases credit access and schooling among constrained households. Kruger and Ludwig (2016) examine the optimal degree of tax progressivity and educational subsidies in a life-cycle economy. They find the optimal mix includes large education subsidies and moderate tax progressivity to avoid crowding out the returns to human capital investment. While neither of these papers includes an avenue for college completion risk, each provides guidance for modeling human capital investment when earnings are stochastic and related to one's education.

[3]See Chapter 3 for a discussion of present bias in quantitative life-cycle models, including Imrohoroglu et al (2003), Maliar and Maliar (2006), Angeletos et al. (2001), Laibson et al. (1998) and others.

Arcidiacono et al. (2016) estimates a dynamic structural model of investment in higher education to outline the role of imperfect information regarding one's own ability on schooling. They find that the elimination of informational frictions would increase college completion rates by 9 percentage points, due almost entirely to a reduction in college dropouts. Krivorotov (2016) uses a model of ability learning in which agents choose to enroll in school after observing a noisy signal of their ability. They then perform Bayesian updating to infer their true ability from the grades they earn while enrolled in college.

Krivorotov finds an increase in the college wage premium in the model economy that matches the increase observed in US data over the last 30 years induces individuals with lower signals of their ability to enroll in college on the off chance that they are more capable than they initially appear. Not only are lower ability agents more likely to enroll in college when the wage premium increases, these agents are more likely to stay enrolled in school after receiving poor grades. These agents persist in school longer than they should "on the off-chance that those low grades were due to bad luck" and not indicative of their innate ability. Krivorotov's model accommodates the fact that a rising college wage premium leads to both longer time to degree and a longer time before dropping out, explaining roughly half of the increase in time to degree from the early 1970s to the late 1990s in the United States.

Hendricks and Leukhina (2018) analyze the impact of graduating from college on lifetime earnings utilizing transcript data to carefully model agents' estimation of their ability. In their model, all individuals attempt the same number of credits in each period and some combination of luck and ability generates a completed subset of attempted credits. They use ability uncertainty to match college completion and dropouts in the High School & Beyond data set and conclude 54% of the difference in lifetime earnings between college graduates and non-graduates is the result of ability differences.

A more recent empirical literature indicates that ability uncertainty may not be the only channel through which college dropouts are generated. Cadena and Keys (2015) use NLSY data to show that individuals labeled as impatient (which is used as a proxy for present bias) are more likely to drop out of college with one year of full time credits or less remaining than those who are not impatient. This result holds even when a number of demographic factors are taken into account, including measures of student ability and family wealth. Further, these impatient individuals earn less and express more regret upon reaching middle age than patient individuals. DePaola and Scoppa (2015) use data sampling a large number of Italian undergraduates and show individuals who procrastinate more perform worse in academic environments, even when controlling for cognitive abilities, background characteristics, and family income. In subsequent work utilizing the same data, DePaola and Gioia (2017) find a negative relationship between time preferences and academic performance. They find that impatient students are more likely to drop out of college and take longer than they had expected to acquire a college degree.

I utilize insights from this empirical literature in conjunction with the modeling environment outlined above to highlight the role played by present-biased optimization in the determination of college completion in a life-cycle model economy.

The Model

I aim to characterize the role played by preference heterogeneity and present-biased optimization on college enrollment, completion, and dropouts in quantitative lifecycle model. As such, I build on the stochastic incomplete markets life-cycle literature established by Bewley (1986), Huggett (1996), Aiyagari (1994) and others in which overlapping generations of optimizing agents are subject to uninsurable, idiosyncratic earnings risks. However, unlike the model utilized in my third chapter in which all agents face the same earnings process, I allow for different earnings processes calibrated to match the distribution of lifetime earnings for workers of different skill types. My calibration of the model outlined in this section follows closely from the approach established in Karahan and Ozkan (2012) and utilized in Kruger and Ludwig (2016) for establishing unique earnings processes for different skill levels. However, unlike Kruger and Ludwig who model a deterministic education decision in which agents invest in their human capital before optimizing in the model economy, agents in my model must continually acquire credits to graduate from college while splitting time between low skilled work, leisure, and education in the first stage of life.

The College Investment Decision

Each agent i's lifetime can be broken down into 3 distinct phases. Phase one is the college education phase, lasting from period 1 to period C, where $C \in [1, \bar{C}]$. The college phase can end in three distinct ways. (1) Agents obtain sufficient credits to graduate and enter the labor market as college graduates in the next period. (2) Agents hit the maximum allotted time to complete a college degree ($\bar{C}$) with insufficient credits to graduate and enter the labor market as an unskilled worker in the next period. (3) Agents with insufficient credits to graduate decide to acquire 0 credits in any period prior to $\bar{C}$ and enter the labor market as an unskilled worker. Any agent that ends their education phase with a positive number of credits but too few credits to graduate is labeled a college dropout, whether they end college via (2) or (3).

Phase 2 is the working lifetime, lasting from time $\tau = (C + 1)$ to R, where R is the exogenously imposed retirement age. If agents elect not to attend college at all, then $C = 0$ and an agent's working lifetime begins at model age 1. During phase 2, agents select their hours worked (n), consumption (c), and savings (k') in each period. Total income in a given period is a function of a deterministic age earnings profile, a stochastic component calibrated to approximate earnings risk corresponding to each worker type, and a skill premium for college graduates.

Phase 3 of life represents an agent's retired years, lasting from $R + 1$ to N where N is the exogenously imposed time of death for all agents. During this phase, agents no longer work and consume out of their savings and social security transfers.

In phase 1, an agent will decide to enroll in college if: (1) she can acquire sufficient credits to graduate within 6 years of initial enrollment given her draw of ability, time preferences, and initial financial resource, (2) the wage premium associated with her college investment decision is sufficient to balance out the disutility from education, the foregone unskilled wages she would receive if all non-leisure time was spent working, and the explicit tuition costs of her college credits. While enrolled in college, an agent has 1 unit of time in each period to dedicate to acquiring college credits, earning a wage in the unskilled labor market, and enjoying leisure time. In order to graduate, an agent must earn sufficient credit hours in a 6 year window to surpass the graduation threshold χ_g. Agents acquire credits by investing time in schooling, s^a, in each period. The notation s^a is used to make explicit the relationship between time in school and ability. The higher an agent's initial draw of a, the less time it will take them to earn a credit hour χ.

As disutility comes from time spent in school, higher ability agents will have a lower per credit disutility and will leave themselves more time for leisure or work in the unskilled labor market during college. Agents pay for their credits according to the market wide tuition cost d, where each college credit costs d regardless of the time needed to acquire that credit. As the majority of financial returns to education are captured only upon the completion of a college degree[4], all workers who have not acquired sufficient credits to earn a college degree will enter the market as unskilled workers (type u), and all workers who have acquired sufficient credits will enter the market as college graduates (type g).

The model of human capital accumulation presented in this chapter differs from the model outlined in Chapter 2 in several ways. The first is that credit accumulation is assumed to be continuous in Chapter 2 and discrete in Chapter 4. The second is the returns to college depends on an indivisible education investment in Chapter 2 whereas in Chapter 4, all time spent in school increased middle-aged wages. The third is that human capital accumulation occurs in a single period in Chapter 2 and accumulating sufficient human capital to receive a wage premium requires a multi-period investment in Chapter 4. As shown in Chapter 2, present-biased agents will invest less time in education than exponential discounters when their education choice is made in a single period. In the model outlined in Chapter 4, I am now interested in exploring how the re-optimization inherent to present-biased discounters leads agents to pursue different human capital investments than they had initially intended. As I am focused on college dropouts and completion

[4]See Carnevale et al. 2011.

rates in Chapter 4, a model of discrete credit accumulation is appropriate given the current structure of post-secondary education in the United States.

The Household's Problem

Agents are born into the model at age 18 (time 1) at which point they receive a draw of unskilled labor productivity ($e_{u,1}$), initial wealth (k_1), initial college credit hours ($\chi_0 = 0$), and a type (j) that governs an individual's time invariant ability (a_j) and discount factors (β_j, δ_j). Following this initial draw, agents are tasked with deciding between taking their unskilled labor to the market and forgoing a college education or enrolling in college and splitting their non-leisure time between working in the unskilled labor market and obtaining college credits.

If an agent elects to work during college or skips college to work in the unskilled labor market permanently, their labor productivity evolves according to the Markov transition matrix P_u, governing the evolution of unskilled labor productivity from the initial draw $e_{u,1}$. If an agent attends college they will receive a new initial draw of labor productivity upon entering the working stage of life in period τ ($e_{\omega,\tau}$) where $\omega \in \{u, g\}$. If credits earned are above χ_g, then an agent draws $e_{g,\tau}$ which evolves according to the Markov transition matrix P_g for college graduates. If credits earned are below χ_g, then an agents draws a new unskilled labor shock $e_{u,\tau}$ which evolves according to P_u for high school graduates.

In a typical life-cycle model, all agents receive labor endowment shocks from the same Markov transition probability matrix governing labor endowment parameter draws. However, as unemployment risk and earnings volatility differ across skilled and unskilled workers, I accommodate this fact by allowing for a different idiosyncratic earnings process for each worker type.

The total compensation per unit of time worked ($y_{t,\omega}$) for an individual of age t and skill type $\omega \in \{u, g\}$ is given by:

$$y_{t,\omega} = w_t \times h(t) \times e_\omega \times p_\omega \tag{4.1}$$

where u corresponds to an individual who's highest degree is a high school degree and g corresponds to an individual who's highest degree is a college degree. w_t is the market wage in period t, $h(t)$ is a deterministic age earnings profile shared by all workers of age t, e_ω is a labor endowment shock drawn from the corresponding distribution of labor market outcomes for individuals with education level ω, and p_ω represents the college wage premium, where $p_g > 1$ and $p_u = 1$.

The initial productivity draw $e_{u,1}$ plays a dual role in the college investment decision as a higher productivity draw both increases an agent's ability to fund consumption during college and increases the opportunity cost of time spent in school and not working. Below, I outline first the optimization problem of a household that has exited the college education phase of their life followed by the optimization problem solved by a household deciding whether to obtain an additional year of schooling.

Not enrolled in college:

Recall the first period of an agent's working life is denoted by $\tau = C + 1$. Once an individual stops attending college (either having completed some or all of a college degree or having decided to forgo college entirely), their optimization problem becomes:

$$U_{WL} = \max_{c,n,k'} u(c_\tau, n_\tau) + \delta_j E\left\{ \sum_{i=1}^{N} \beta_j^i u(c_{\tau+i}, n_{\tau+i}) \right\} \tag{4.2}$$

where each period the budget constraint is given by:

$$c_t + k_{t+1} = (1+r)k_t + n_t y_{t,\omega} + \tau_R(\omega) \tag{4.3}$$

$$k' \geq \underline{k},\ n_t + l_t = 1 \tag{4.4}$$

U_{WL} is discounted expected utility over one's working life, δ_j and β_j are the present-biased and exponential discount factors of a type j agent, respectively. $\tau_R(\omega)$ is a social security payment made to type ω agents once they retire at age 65, and l_t is leisure in period t. The restriction $k' \geq \underline{k}$ introduces a borrowing constraint so that agents are unable to perfectly smooth their consumption in the face of idiosyncratic earnings shocks.

An agent of type j has a state vector x which is given by $x = (k, e, t, j, \omega)$ where k is wealth, e is the current period's labor shock, t is the agent's age, j

governs the agent's draw of β, δ, and a, and $\omega \in \{u, g\}$ determines whether a worker is unskilled or college educated. Conditional on this state x, we can define the agent's optimization problem over their working lifetime by defining their working life value function (V_W) in the following way:

$$V_W(x) = \max_{c,k',n} \; u(c,n) + \delta\beta E[\tilde{V}_W(x'|x)] \tag{4.5}$$

subject to:

$$c + k' = (1+r)k + ny(x) + \tau_R(x), \;\; n + l = 1 \tag{4.6}$$

$$k' \geq \underline{k}, c \geq 0, n \in [0,1] \;\&\; k' \geq 0 \text{ if } t = \bar{N}, \; V_W(x) = \tilde{V}_W(x) = 0 \text{ if } t = \bar{N} + 1 \tag{4.7}$$

As outlined above, compensation per unit of time worked depends on t, e, and ω and retirement transfers depend on t and ω, thus both y and τ_R are expressed as functions of the state variable x. If an agent draws a value of $\delta_j = 1$, then they are not present-biased and $V_W = \tilde{V}_W$. However, if an agent draws $\delta_j < 1$, I assume that agents are naive regarding their present bias and thus the appropriate continuation value function for their working life Bellman equation is given by[5]:

$$\tilde{V}_W(x) = \max_{c,k',n} \; u(c,n) + \beta E[\tilde{V}_W(x'|x)]$$

[5] For a discussion of naive vs sophisticated present bias, please see Chapter 3.

That is, the continuation value function of a present-biased agent is simply the value function of an exponential discounter, as a present-biased agent expects to display higher patience in the future than they are willing to exercise today.

While enrolled in college:

In the first period an agent is enrolled in college, they solve the following optimization problem:

$$U = \max_{c,k',n,s^a} u(c_1, s_1^a, n_1) + \delta_j E\Bigg\{\Bigg[\sum_{i=1}^{C} \beta_j^i u(c_{1+i}, s_{1+1}^a, n_{1+i}) + \beta^{C+1} U_{WL}\Bigg]\Bigg\} \tag{4.8}$$

and in each period the budget constraint is given by:

$$c_t + k_{t+1} = (1+r)k_t - d_t\chi_t + n_t y_{t,u} \tag{4.9}$$

$$n_t + s_t^a + l_t = 1, \; k \geq \underline{k} + d_t\chi_t \tag{4.10}$$

Where d_t is the direct education cost paid per credit hour acquired and $y_{t,u}$ is the compensation of an unskilled worker. Following the approach of Kruger and Ludwig (2016) and Hendricks and Leukhina (2016), each agent's borrowing constraint is relaxed during the education phase of their life so that agents can borrow beyond the fixed borrowing limit $\underline{k}$ in order to finance their higher education. However, borrowing while enrolled in college cannot exceed the cost of tuition and therefore is not a means of consumption smoothing for credit constrained agents.

Student ability augments the amount of s needed to accumulate credit hours. Thus, when referring to credit accumulation below we use the notation $\chi(s^a)$ to denote that credits accumulated is a function of ability augmented time invested in schooling. In each period an agent is enrolled in college, they begin with some amount of credits χ and they end with $\chi' = \chi + \chi(s^a)$. That is, accumulated credits at the end of the period are the sum of initial credits and new credits earned via schooling investment s^a. Each period, agents can attempt 12, 24, or 36 credits (per year) and the graduation threshold is set to $\chi_g = 125$.

Again, we turn to representing our agent's optimization problem as a value function where the state is given by $x = (k, e, t, j, \chi)$ where χ is the acquired college credits an agent has earned by the beginning of the current period. In the specification of x during an agent's working life, $\omega \in \{u, g\}$ was in the agent's state instead of χ. This substitution was made as ω is uniquely determined by χ, and credits earned are fixed once an agent begins their working life. We can represent an optimizing college agent's Bellman equation (V_C) in the following way:

$$V_C(x) = \max_{c,k',n,s^a} u(c, n, s^a) + \delta\beta E[\tilde{V}_{C/W}(x'|x)] \tag{4.11}$$

subject to:

$$c + k' = (1 + r)k + y(x)n - d\chi(s^a), \; n + s + l = 1 \tag{4.12}$$

$$k' \geq \underline{k} + d\chi(s^a), c \geq 0, n, s \in [0, 1] \; \& \; \tilde{V}_{C/W}(x) = \tilde{V}_W(x) \text{ if } t = \bar{C} \tag{4.13}$$

where $\tilde{V}_{C/W}$ is the expected value function of working or continuing school in the next period. This value function is given by $\tilde{V}_{C/W}(x'|x) = E\max[\tilde{V}_C(x'|x), \tilde{V}_W(x'|x)]$. As discussed above while defining the working life value function, if $\delta_j = 1$, then $\tilde{V}_{C/W}(x'|x) = V_{C/W}(x'|x) = E\max[V_C(x'|x), V_W(x'|x)]$. As the only uncertainty in the model economy comes from the employment shock parameter e, exponential discounters will only drop out of college if the knife edge case occurs in which they were just on the cusp of being able to afford college attendance and they receive an unexpected negative shock to labor earnings that pushes them below a feasible consumption stream while also accumulating college credits.

If $\delta < 1$, then the expected continuation value functions are given by:

$$\tilde{V}_W(x) = \max_{c,k',n} u(c,n) + \beta E[\tilde{V}_W(x'|x)] \quad (4.14)$$

$$\tilde{V}_C(c) = \max_{c,k',n,s^a} u(c,n,s^a) + \beta E[\tilde{V}_{C/W}(x'|x)] \quad (4.15)$$

As noted in the introduction, a present-biased individual has an additional pathway through which they may begin investing in a college degree only to drop out and enter the labor market as an unskilled worker: their mis-estimation of their future value function $\tilde{V}_{C/W}$. A present-biased agent calculates their optimal schooling investment today based on maximizing their expected utility in the next period from either entering the workforce or continuing to accumulate credits in school. However, as the exposition of $\tilde{V}_W(x)$ and $\tilde{V}_C(c)$ make clear, a present-biased agent makes their schooling investment today with the expectation that they will behave in an exponential fashion next period. Meaning the optimal decision of a present-biased agent today is based on a continuation value function calculated using values of future consumption, schooling, and labor choices that the agent will not select when they arrive at that future period.

Firms

Output is produced using capital (K) and Labor (L) where total labor supply is a function of both college educated workers (L_g) and workers without a college degree (L_u). Thus, the labor supply in period t is given by $L_t = (L_{t,u}^{\rho} + L_{t,g}^{\rho})^{\frac{1}{\rho}}$ and production is given by:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} = AK_t^{\alpha}\left[(L_{t,N}^{\rho} + L_{t,C}^{\rho})^{\frac{1}{\rho}}\right]^{1-\alpha}$$

I assume $\rho = 1$ so that worker types are perfect substitutes in the production process:

$$Y_t = AK_t^{\alpha}[L_{t,N} + L_{t,C}]^{1-\alpha} \tag{4.16}$$

Factor prices (determined in a competitive market by the marginal product of capital and labor, respectively) are given by:

$$q_K = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{4.17}$$

$$q_L = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{4.18}$$

Note, both high and low skilled workers are compensated according to the same marginal product q_L. The differences in compensation per unit of time worked across agent types will be determined by their effective units of labor, jointly determined by skill specific earnings shocks ($e_{t,\omega}$) and the skill premium (p_ω).

Equilibrium

A stationary competitive equilibrium consists of aggregate quantities ($K,\ L_u,\ L_g, C, T$), prices ($w,\ r,\ q_L,\ q_K$), value functions $\{V_C, V_W\}$, continuation value functions $\{\tilde{V}_C, \tilde{V}_W\}$, policy functions $\{c(x),\ n(x),\ k(x),\ s^a(x)\}$, and a distribution over agent types ($\Lambda(x)$) such that:

- The policy functions $c(x)$, $n(x)$, $s^a(x)$, and $k(x)$ along with the value functions $V_C(x)$,

  $V_W(x)$ and the continuation value functions $\tilde{V}_C(x)$ and $\tilde{V}_W(x)$ solve the agent's optimi-

  zation problem.
- Firms maximize profits.

- The distribution of households over states, $\Lambda(x)$, is stationary.
- Prices are given by $w = (1 - \tau_w)q_L$ and $r = q_K - \delta$.
- Markets Clear:

  (i) $K = \int \Lambda(x)k(x)dx$

  (ii) $L = L_u + L_g = \int \Lambda(x)l_u(x)dx + \int \Lambda(x)l_g(x)dx$

  (iii) $F(K, L) = C + \delta K$ where $C = \int_x \Lambda(x)c(x)dx$

Calibrating Model Parameters

Demographics

Households are born at the age of 18 (model age 1) at which point they receive their initial draw of preferences, ability, wealth, and unskilled labor earnings. Agents may attend school for at most $\bar{C} = 6$ years, after which point they must enter the working stage of life. All agents work in every period until retirement at age 65 (model age 47) and live at most 90 years (model age 73).

Labor Endowments

An agent's labor endowment consists of a deterministic age efficiency profile, $h(t)$, a stochastic labor productivity shock corresponding to their skill level, e_ω, and a skill premium p_ω. The age efficiency profile is modeled using 1990 PUMS data and is meant to recreate the hump-shaped lifetime earnings profile in the model economy that we observe in US data. The transition matrix for labor endowment shocks, P_ω, is the Markov transition matrix associated with the discretized Markov approximation of the following autoregressive process for each skill type:

$$\ln(e_{t,\omega}) = \rho_\omega \ln(e_{t-1,\omega}) + \epsilon_{t,\omega} \tag{4.19}$$

where $e_{t-1,\omega}$ is the labor shock experienced in the previous period by a worker of type $\omega \in \{u, g\}$and $\epsilon_{t,\omega} \sim N(0, \sigma^2_{\epsilon,\omega}) \ \forall \ t$.

As both the persistence and variance of earnings shocks differ across college educated (type g) and unskilled (type u) workers, ρ_ω and $\sigma^2_{\epsilon,\omega}$ are calibrated to match earnings volatility for workers of each skill type. Following the calibration outlined in Krueger and Ludwig (2016) using PSID data, the selected parameters are $\rho_g = 0.969$, $\rho_u = 0.928$, $\sigma^2_{\epsilon,g} = 0.0100$ and $\sigma^2_{\epsilon,u} = 0.0192$. The main takeaway from this calibration is that earnings of college graduates display higher persistence and lower variance than the earnings of unskilled workers, so a college education affords workers both higher lifetime pay through the college wage premium (p_g) and less uncertainty regarding lifetime earnings.

Preferences

Preferences during the education phase of life are given by $u(c, s^a, n) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma\frac{(1-(n+s^a))^{1-\phi}}{1-\phi}$. During one's working lifetime, households no longer dedicate time to education ($s^a = 0 \ \forall$ agents) and utility is given by $u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} + \gamma\frac{(1-n)^{1-\phi}}{1-\phi}$. The curvature parameter governing utility from consumption is set to $\sigma = 1.5$[6] and the parameters γ and ϕ governing the disutility from non-leisure endeavors are jointly calibrated with the schooling ability parameter s^a to match college enrollment, completion, and time spent working during college as reported in the CPS *2015 Digest of Education Statistics*.

A Simple Model

Using a tractable three period version of the problem described above, I establish that dropouts occur when agents are present-biased. Earnings are no longer assumed to be stochastic or age dependent and agents split time between schooling and working in periods 1 and 2. Factor prices and the wage premium (w, r, and p_g) are exogenously determined, $\bar{C} = 2$, and agents are assumed to die at the end of their working lifetime, so that $a_D = a_R = 3$. Earnings for an agent born in period t are given by:

$$y_t = w(1 - s_t) \tag{4.20}$$

$$y_{t+1} = w(1 - s_{t+1}) \tag{4.21}$$

$$y_{t+2} = wf(s_t + s_{t+1}) \tag{4.22}$$

[6]This is the value chosen in Hendricks (2007a), Huggett (1996), and DeNardi and Yang (2014), among others.

where individuals can invest $s \in \{0, s_L, s_H\}$ units of time in school in periods 1 and 2. The function $f(s_t + s_{t+1}) = 1$ if $s_t + s_{t+1} < s_g$ and p_g if $s_t + s_{t+1} \geq s_g$. I calibrate $s_L + s_H = s_g$, meaning in order to accumulate sufficient schooling to be a college graduate ($s_t + s_{t+1} \geq s_g$) an individual must choose the high schooling investment (s_H) in either the first or second period in which they are enrolled in college.

Proposition 1: Agents who enroll in school will always select s_L in period 1 and expect to select s_H in period 2 [7].

Exercise 1: Perfect Capital Markets

Utility over consumption is given by $u(c_t) = \ln(c_t)$ and disutility from schooling is given by $\gamma * (1 - s_t)^{1-\phi}/(1 - \phi)$. Unlike the preferences outlined in the calibration section, I now drop the disutility from labor portion of the agent's optimization problem as agents are assumed to inelastically supply all time to the labor market that is not spent acquiring education. Upon receiving an initial draw of capital ($k_t \geq 0$) agents solve the following optimization problem:

$$U_t = \max_{c,s,k'} \ln(c_t) + \frac{\gamma(1 - s_t)^{1-\phi}}{1 - \phi} + \beta\delta\left[\ln(c_{t+1}) + \frac{\gamma(1 - s_{t+1})^{1-\phi}}{1 - \phi}\right] + \beta^2\delta \ln(c_{t+2}) \quad (4.23)$$

[7]Proof of Proposition 1 in Appendix C.

s.t.

$$c_t + k_{t+1} = k_t + y_t \tag{4.24}$$

$$c_{t+1} + k_{t+2} = (1+r)k_{t+1} + y_{t+1} \tag{4.25}$$

$$c_{t+2} = (1+r)k_{t+2} + y_{t+2} \tag{4.26}$$

When capital markets are perfect, agents can borrow freely against expected future earnings in order to smooth consumption over their lifetime. Thus, for a given selection of s_t and s_{t+1}, optimal consumption in period t ($c_t^\star$) and expected consumption in period $t+1$ and $t+2$ ($c_{t+1}^{\star E}$ and $c_{t+2}^{\star E}$) are given by:

$$c_t^\star = \frac{k_t + y_t + \frac{y_{t+1}}{1+r} + \frac{y_{t+2}}{(1+r)^2}}{1 + \delta\beta + \delta\beta^2} \tag{4.27}$$

$$c_{t+1}^{\star E} = \beta\delta(1+r)c_t^\star \tag{4.28}$$

$$c_{t+2}^{\star E} = \beta^2\delta(1+r)^2 c_t^\star \tag{4.29}$$

As schooling has both a direct utility cost via disutility from education and an indirect utility cost through lowered income in periods t and t+1, an agent will only select $s_t > 0$ if they expect to acquire sufficient credits to graduate and earn the wage premium p_g in period $t+2$. Thus, agents must choose between $s_t = s_{t+1} = 0$ and $s_t + s_{t+1}^E = s_L + s_H$ where s_{t+1}^E is expected schooling in period $t+1$. An agent will enroll in college if their expected utility from enrolling exceeds their expected utility from not enrolling[8].

We can now compare expected utility from enrolling in college ($s_t = s_L$ and $s^E_{t+1} = s_H$) to expected utility from not enrolling by substituting in for y_t, y_{t+1}, and y_{t+2} in equations (4.27)-(4.29). c^N_t is the optimal period 1 consumption for an agent who does not enroll in college and c^S_t is the optimal period 1 consumption for an agent who does enroll in college. If an agent elects to forgo a college education, their income is equal to w in each period and their consumption is equal to:

$$c^N_t = \frac{k_t + w + \frac{w}{1+r} + \frac{w}{(1+r)^2}}{1 + \delta\beta + \delta\beta^2} \tag{4.30}$$

$$c^{NE}_{t+1} = \beta\delta(1+r)c^N_t \tag{4.31}$$

$$c^{NE}_{t+2} = \beta^2\delta(1+r)^2 c^N_t \tag{4.32}$$

For agents who do enroll in college, their income in period t is given by $(1 - s_L)w$ and their expected income in periods $t + 1$ and $t + 2$ (conditional on following through with their education investment and selecting $s_{t+1} = s_H$) are given by

[8]Although their is no uncertainty in the model, if agents are present-biased then U_t is calculated using values of c^E_{t+1}, s^E_{t+1} and c^E_{t+2} that an agent might not chose when they arrive in periods $t + 1$ and $t + 2$. If an agent draws $\delta = 1$, then they are able to perfectly forecast their future consumption and schooling decisions and U_t is known with certainty.

$w(1-s_H)$ and $p_g w$ respectively:

$$c_t^S = \frac{k_t + (1-s_L)w + \frac{(1-s_H)w}{1+r} + \frac{p_g w}{(1+r)^2}}{1+\delta\beta+\delta\beta^2} \quad (4.33)$$

$$c_{t+1}^{SE} = \beta\delta(1+r)c_t^S \quad (4.34)$$

$$c_{t+2}^{SE} = \beta^2\delta(1+r)^2 c_t^S \quad (4.35)$$

Thus, agents will enroll in college if, for a given draw of k_t:

$$U_t^E(c_t^N, c_{t+1}^{NE}, c_{t+2}^{NE}, s_t = 0, s_{t+1} = 0) < U_t^E(c_t^S, c_{t+1}^{SE}, c_{t+2}^{SE}, s_t = s_L, s_{t+1}^E = s_H), \quad (4.36)$$

where U_t^E is expected utility associated with time t optimal c_t and s_t and expected future schooling and consumption.

If $\delta = 1$, then optimal $c_{t+1}^\star = c_{t+1}^{\star E}$ and optimal $c_{t+2}^\star = c_{t+2}^{\star E}$. Thus, no agent will enroll in college in period t and then drop out in period $t+1$ if they are not present-biased, as non present-biased agents perfectly predict their future utility from consumption and schooling. If $\delta < 1$, then agents will re-optimize in period $t+1$ taking the value of $k_{t+1}^\star = k_t + (1-s_t)w - c_t^\star$ as given. In period $t+1$, present-biased agents solve the following optimization problem:

$$U_{t+1} = \max_{c,s,k'} \ln(c_{t+1}) + \frac{\gamma(1-s_{t+1})^{1-\phi}}{1-\phi} + \beta\delta\ln(c_{t+2}) \quad (4.37)$$

s.t.

$$c_{t+1} + k_{t+2} = (1+r)k^{\star}_{t+1} + y_{t+1} \tag{4.38}$$

$$c_{t+2} = (1+r)k_{t+2} + y_{t+2} \tag{4.39}$$

A college drop out is an individual that selects $s_t = s_L$ but chooses $s_{t+1} = 0$ instead of following through with $s^E_{t+1} = s_H$. When deciding whether to drop out, agents solve the above utility maximization problem outlined by equations (4.37)-(4.39) where $k^{\star}_{t+1} = k^S_{t+1} = k_t + (1 - s_L)w - c^S_t$.

Optimal consumption in periods $t+1$ and $t+2$ for an individual choosing no school in period $t+1$ is given by:

$$c^N_{t+1} = \frac{(1+r)[k_t + (1-s_L)w - c^S_t] + w + \frac{w}{1+r}}{1+\delta\beta} \tag{4.40}$$

$$c^N_{t+2} = (1+r)\beta\delta c^N_{t+1} \tag{4.41}$$

and optimal consumption for an agent who continues their education in period $t+1$ is given by:

$$c^S_{t+1} = \frac{(1+r)[k_t + (1-s_L)w - c^S_t] + w(1-s_H) + \frac{p_g w}{1+r}}{1+\delta\beta} \tag{4.42}$$

$$c^S_{t+2} = (1+r)\beta\delta c^N_{t+1} \tag{4.43}$$

An individual who enrolled in college in period t (see (4.36)) will choose to drop out of college in period $t+1$ if:

$$U_{t+1}(c^N_{t+1}, c^N_{t+2}, s_{t+1} = 0) > U_{t+1}(c^S_{t+1}, c^S_{t+2}, s_{t+1} = s_H) \tag{4.44}$$

When capital markets are perfect, agents who enroll in school in period t are already consuming out of their expected future income from completing their degree. Thus, dropping out of college will reduce consumption in periods $t+1$ and $t+2$[9]. Although dropping out will lower consumption utility in periods $t+1$ and $t+2$, agents may still be induced to drop out of college if the utility gains from not spending s_H units of time in school are sufficient to balance out their reduction in consumption.

Exercise 1: Results

Consider the following calibration: The schooling parameters are given by $\{s_L,\ s_H\} = \{.1,\ .6\}$, preference parameters are given by $\{\beta, \delta, \phi, \gamma\} = \{0.96, .6, 2.5, .15\}$, and prices and the wage premium are given by $\{w, r, p_g\} = \{1, .01, 3.5\}$. After selecting the schooling parameters, preferences, and prices in the model economy, I search over values of initial capital for which equation (4.36) is satisfied and agents enroll in school during period t and equation (4.44) is satisfied

[9]See proof in Appendix D.

and agents choose not to complete their education. That is:

$$U_t^E(c_t^N, c_{t+1}^{NE}, c_{t+2}^{NE}, s_t = 0, s_{t+1} = 0) < U_t^E(c_t^S, c_{t+1}^{SE}, c_{t+2}^{SE}, s_t = s_L, s_{t+1}^E = s_H)$$

and:

$$U_{t+1}(c_{t+1}^N, c_{t+2}^N, s_{t+1} = 0) > U_{t+1}(c_{t+1}^S, c_{t+2}^S, s_{t+1} = s_H)$$

For the parameterization outlined above, I find agents will choose $s_t = s_L$ and then select $s_{t+1} = 0$ for initial capital in the range of $k_t \in [15.77, 16.14]$. This constitutes values of initial capital between 3.34 and 3.41 times greater than the discounted lifetime earnings of a college graduate[10]. Thus, even in a model with no uncertainty regarding ability, earnings, or the credit accumulation process, if agents are present-biased then some individuals who enroll in college will elect not to complete their degree. This occurs even though college enrollment has both a utility cost from time spent in school during period t and an uncompleted degree leads to a reduction in lifetime income as agents earn only part time unskilled wages in period t and do not earn the skill premium in period $t + 2$.

When capital markets are perfect, present-biased individuals who enroll in school and borrow against their future earnings ($k_{t+1} < 0$) will never drop out of college, as they have already consumed against their future income stream. If these

[10]Discounted lifetime earnings from the perspective of an agent in period t, and are equal to $y_t + y_{t+1}/(1 + r) + y_{t+2}/(1 + r)^2$.

individuals decide to drop out in period $t+1$, then their consumption must decrease considerably in both period $t+1$ and period $t+2$ relative to their previous plan, as they now have to repay the debt they used to finance age t consumption out of an unskilled worker's salary. Depending on the value of their initial capital, they may be unable to repay this debt without completing their degree or it may simply cost too much in terms of consumption utility in period $t+1$ and $t+2$.

However, US data indicates that students from poor households are far more likely to drop out of college than students from wealthier households. Only 32% of the poorest 25% of college enrollees complete their degree within 6 years of initial enrollment compared with a 68% completion rate for the richest 25% of students (Shankie 2014). As the only agents to drop out of college in a model of unconstrained borrowing are individuals with considerably higher starting wealth than expected lifetime income, I now turn to a model in which agents face a binding borrowing constraint to see if present bias can account for dropouts among lower wealth households.

Exercise 2: No Borrowing

Consider the same optimization problem outlined in equations (4.23)-(4.26) with the additional constraint that $k_{t+i} \geq 0 \; \forall \; i \in [1,2]$. When a no-borrowing constraint is imposed, optimal consumption at time t is calculated for three distinct cases. Case 1: $k_{t+1} > 0$ and $k_{t+2} > 0$, Case 2: $k_{t+1} > 0$ and $k_{t+2} = 0$, and Case 3:

$k_{t+1} = k_{t+2} = 0$.[11] Optimal consumption for an agent electing not to attend school is outlined below for each distinct case.

Case 1: $k_{t+1} > 0$ and $k_{t+2} > 0$ (identical to the unconstrained values in equations (4.30)-(4.32))

$$c_t^N = \frac{k_t + w + \frac{w}{1+r} + \frac{w}{(1+r)^2}}{1 + \delta\beta + \delta\beta^2} \tag{4.45}$$

$$c_{t+1}^{NE} = \beta\delta(1+r)c_t^N \tag{4.46}$$

$$c_{t+2}^{NE} = \beta^2\delta(1+r)^2 c_t^N \tag{4.47}$$

Case 2: $k_{t+1} > 0$ and $k_{t+2} = 0$

$$c_t^N = \frac{k_t + w + \frac{w}{1+r}}{1 + \delta\beta} \tag{4.48}$$

$$c_{t+1}^{NE} = \beta\delta(1+r)c_t^N \tag{4.49}$$

$$c_{t+2}^{NE} = w \tag{4.50}$$

Note, for Case 2 agents are smoothing their consumption across periods t and $t+1$ as in the unconstrained solution, however in period $t+2$ agents simply consume their income.

[11]We do not need to consider the case in which $k_{t+1} = 0$ and $k_{t+2} > 0$ as income is never expected to be higher in period $t+1$ than in period t so an optimizing agent would never exhaust their savings in period t only to save in period $t+1$.

Case 3: $k_{t+1} = k_{t+2} = 0$

$$c_t^N = k_t + w \tag{4.51}$$

$$c_{t+1}^{NE} = w \tag{4.52}$$

$$c_{t+2}^{NE} = w \tag{4.53}$$

Where the solution for Case 3 highlights the fact that when borrowing constraints bind in every period, optimizing agents simply consume all of their available resources in each period.

These same three cases are relevant for determining optimal consumption when agents select $s_t = s_L$ with the expectation that $s_{t+1} = s_H$. Consumption for Case 1 is identical to the values outlined in equations (4.33)-(4.45) for the unconstrained problem. Consumption for Case 2 is given by:

$$c_t^S = \frac{k_t + (1 - s_L)w + \frac{(1-s_H)w}{1+r}}{1 + \delta\beta} \tag{4.54}$$

$$c_{t+1}^{SE} = \beta\delta(1 + r)c_t^S \tag{4.55}$$

$$c_{t+2}^{SE} = p_g w \tag{4.56}$$

Optimal consumption for Case 3 is given by:

$$c_t^S = k_t + (1 - s_L)w \quad (4.57)$$

$$c_{t+1}^{SE} = (1 - s_H)w \quad (4.58)$$

$$c_{t+2}^{SE} = p_g w \quad (4.59)$$

As in the unconstrained case, agents will choose to enroll in school for a given draw of k_t if:

$$U_t^E(c_t^N, c_{t+1}^{NE}, c_{t+2}^{NE}, s_t = 0, s_{t+1} = 0) < U_t^E(c_t^S, c_{t+1}^{SE}, c_{t+2}^{SE}, s_t = s_L, s_{t+1}^E = s_H) \quad (4.60)$$

That is, if the utility from their optimally selected c_t^S and expected c_{t+1}^{SE} and c_{t+2}^{SE} along with $s_t = s_L$ and expected $s_{t+1} = s_H$ is greater than their utility from consuming c_t^N, c_{t+1}^{NE}, and c_{t+2}^{NE} with $s_t = s_{t+1} = 0$.

As in Exercise 1 when capital markets are assumed to be perfect, if an agent draws $\delta = 1$, then their actual consumption in periods $t+1$ and $t+2$ is equal to their expected future consumption in periods $t+1$ and $t+2$ and schooling in period $t+1$ equals expected schooling in period $t+1$. If $\delta < 1$, then agents must re-solve their optimal consumption and schooling decision in period $t+1$ taking optimal savings from period t, $k_{t+1}^S = k_t + (1 - s_t)w - c_t^S$, as given.

Agents solve the utility maximization problem outlined in equations (4.37)-(4.39) with the additional constraint that $k_{t+2} \geq 0$. This leaves us with two distinct cases for agents who elect to drop out ($s_{t+1} = 0$) and 2 distinct cases for agents who decide to complete their education ($s_{t+1} = s_H$). Case 1: $k_{t+2} > 0$ and Case 2: $k_{t+2} = 0$[12]. For agents who elect to drop out of school, consumption in Case 1 is given by:

$$c_{t+1}^N = \frac{(1+r)[k_t + (1-s_L)w - c_t^S] + w + \frac{w}{1+r}}{1+\delta\beta} \quad (4.61)$$

$$c_{t+2}^N = (1+r)\beta\delta c_{t+1}^N \quad (4.62)$$

and consumption in Case 2 is given by:

$$c_{t+1}^N = (1+r)[k_t + (1-s_L)w - c_t^S] + w \quad (4.63)$$

$$c_{t+2}^N = w \quad (4.64)$$

For agents who elect to continue school in period $t+1$, consumption in Case 1 is given by:

$$c_{t+1}^S = \frac{(1+r)[k_t + (1-s_L)w - c_t^S] + (1-s_L)w + \frac{p_g w}{1+r}}{1+\delta\beta} \quad (4.65)$$

$$c_{t+2}^S = (1+r)\beta\delta c_{t+1}^S \quad (4.66)$$

[12]Although agents never expect to select $k_{t+1} = 0$ and $k_{t+2} > 0$, present bias may lead some agents to pursue this previously unexpected savings plan.

and consumption in Case 2 is given by:

$$c^S_{t+1} = (1+r)[k_t + (1-s_L)w - c^S_t] + (1-s_L)w \tag{4.67}$$

$$c^S_{t+2} = w \tag{4.68}$$

As before, an individual will elect to drop out of college in period $t+1$ if:

$$U_{t+1}(c^N_{t+1}, c^N_{t+2}, s_{t+1} = 0) > U_{t+1}(c^S_{t+1}, c^S_{t+2}, s_{t+1} = s_H) \tag{4.69}$$

Exercise 2: Results

When credit markets are perfect and agents can borrow and lend freely at the market interest rate, present-biased agents will be induced to drop out for relatively high values of initial capital. This range of capital is sufficiently high that it is only agents who always select positive capital holding (i.e. non-borrowers) who drop out of college. When credit markets are imperfect and agents are constrained to select non-negative savings in every period, the capital range for agents who drop out is lowered considerably and it is primarily credit constrained present-biased individuals who drop out of college.

As in Exercise 1, the schooling parameters are given by $\{s_L,\ s_H\} = \{.1,\ .6\}$, preference parameters are given by $\{\beta, \delta, \phi, \gamma\} = \{0.96, .6, 2.5, .15\}$, and prices and the wage premium are given by $\{w, r, p_g\} = \{1, .01, 3.5\}$. Once schooling parameters, preferences, and prices are set I search over values of initial capital for which equation (4.36) is satisfied and agents enroll in school during period t and equation (4.44) is satisfied and agents choose not to complete their education.

When capital markets are not perfect and agents are unable to borrow against their future earnings, wealthy present-biased agents with $k_t \in [15.77, 16.14]$ will drop out, as in the unconstrained case. However, now agents with $k_t \in [.53, 1.76]$ will also enroll in school during period t only to drop out in period $t + 1$. This range of initial capital corresponds to initial assets between 11.2% and 37.2% of the discounted lifetime earnings of a college graduate. Not only is this range significantly lower than the range of capital that induced dropouts in the unconstrained model (agents with k_t roughly 3.4 times greater than discounted lifetime earnings were the only dropouts), it is also a much wider range of initial asset holdings. Thus, dropouts are less of a knife edge case (as they were in the unconstrained model) and more a general feature of the optimization of present-biased households with low initial wealth.

When credit constraints bind, agents with lower initial wealth are induced to drop out because deviating from their expected period $t + 1$ schooling decision no longer comes with explicitly lower consumption in period $t+1$, as was the case when capital markets were perfect (see Appendix D). If agents start with sufficiently low capital that they cannot perfectly smooth consumption across all three periods, then they cannot fully consume out of their expected college wage premium that they intend to receive in period $t + 2$. Thus, dropping out of college in period $t + 1$ now gives agents both an increase in within period utility due to the disutility of education they avoid and also an increase in period $t + 1$ consumption, as agents who drop out receive a higher wage in period $t + 1$. Present-biased agents now have two avenues through which they are tempted to drop out, leading to the wide range of initial capital that induces present-biased agents to dropout when borrowing constraints bind.

Conclusion

In this Chapter, I outline a quantitative life-cycle model economy in which agents are tasked with choosing between investing time in school in order to complete a college degree and working for an unskilled wage. I propose a novel mechanism that leads some agents to begin investing in a college education only to abandon their investment and enter the labor pool as an unskilled worker: present-biased optimization.

I show that for a simple three period version of the more complex quantitative life-cycle model discussed in the modeling section, there is a range of initial capital for which present-biased agents will begin school only to drop out in the following period. When capital markets are perfect, this range of capital is very narrow and constitutes high initial wealth relative to discounted lifetime earnings. When a no borrowing constraint is imposed, a much larger range of initial capital will induce present-biased agents to begin a college degree that they will never finish. This result is generated in a model economy in which there is no individual or aggregate uncertainty in the earnings process or in an individual's ability to complete their college degree, a necessary feature in previous models attempting to rationalize college dropouts as the result of optimizing consumer behavior.

After characterizing the role played by discount rate heterogeneity and present bias in college completion and dropouts, in future projects I aim to extend this model in order to outline the role of initial conditions in generating inequality over the life-cycle. In a typical quantitative life-cycle model in which income is assumed to be exogenously determined, the distribution of wealth in the model economy displays significantly less inequality than what we observe in US data (as outlined in Chapter 3). A primary culprit for this observation is the model economy's inability to account for the high savings propensities of earnings-rich households.

Although not the focus of this fourth chapter, my model offers a new channel through which earnings-rich households will continue accumulating wealth; the same discount factor that leads agents to invest in obtaining a college education also guides their consumption-savings decisions throughout their working lifetime. As all earnings and wealth differences come from differences in either initial conditions (initial wealth, ability, and time preferences) or shocks experienced over the life-cycle, this model provides an excellent foundation for outlining the relative importance of these different elements in generating inequality over the life-cycle.

CHAPTER V

CONCLUSION

In my dissertation, I show that present-biased optimization can lead to underinvestment in education, decreased wealth in retirement, and college dropouts. Chapter 2 focuses on highlighting the role of present bias in a simple three period model in order to clearly elicit the pathway through which present bias leads agents to re-optimize and abandon their planned consumption and savings profile in favor of within period utility. When prices are exogenous and all agents are either present-biased or exponential discounters, I show that present-biased agents will make schooling decisions that they regret later in life (backward looking present-biased agents would select a higher time investment in schooling when young). Government policies aimed at increasing educational attainment through education incentive pay increase education and lifetime consumption utility for both present-biased and exponential societies. These increases are relative to an environment in which the government only funds social security *and* relative to a society in which no taxes are levied.

In Chapter 3, I show that the impact of present-biased optimization on consumption and savings depends on several factors. The first is whether agents are embedded in a general or partial equilibrium framework. The second is whether agents' lifetimes are assumed to be finite or infinite. The third is that the role of present bias in generating inequality over the life-cycle depends on the proportion of agents assumed to behave in a biased fashion. The more agents who are present-biased, the more prices respond to their biased behavior and the more time-consistent discounters benefit from the time-inconsistent optimization of their present-biased peers.

In Chapter 4, I build a model in which agents with a realistic life-cycle (as in Chapter 3) are forced to make an education/ low skill labor trade-off (as in Chapter 2). I outline the full specification of the modeling environment for this chapter and proceed to show that present-biased optimization is sufficient to generate college dropouts in a simplified version of the model economy. Further, I find credit constraints are essential for generating dropouts among low wealth households (individuals who are most likely to drop out according to data) in the simple model economy.

The results presented in my dissertation have led to several policy-relevant research questions that I intend to pursue using a quantitative life-cycle model similar to the model outlined in Chapter 2. Building on my finding that present-biased optimization has its largest impact on consumption and savings late in the life-cycle, I intend to explore the role of present-biased optimization on the timing of retirement and social security uptake. I believe that present bias can help to rationalize the high number of retiring individuals who elect to draw social security prior to age 65, which reduces their monthly benefits throughout retirement and is currently difficult to rationalize in a model economy populated with exponential discounters. Other extensions will focus on the savings response of present-biased households relative to exponential households in response to announced social security reform. As such reform is very likely given the current state of social security in the United States, a detailed understanding of how different agent types respond to potential reforms could provide useful insights to policy makers.

APPENDIX

SUPPLEMENTAL EQUATIONS

A: Outline of Equilibrium Solution Algorithm:

1. Propose a candidate interest rate, r, and common discount factor, β_c.For a given β_c:

2. Solve the household problem (find $c(x)$ and $k(x)$) given prices r and w that solve the firm optimization problem for the capital stock implied by the interest rate r.

3. Using the optimized capital decision rule, $k(x)$, compute individual savings decisions for the stable distribution of households.

4. Compute aggregate capital, K_1, the capital stock in the model economy given preferences β_c and the interest rate r. Compute the implied capital to output ratio, K_1/Y_1.

5. If K_1/Y_1 is sufficiently close to the target capital output ratio of 3.10, stop. If not, propose a new β_c and repeat steps (a)-(d) until convergence is achieved.

B: Present-Biased Euler Equation:

If the Bellman Equation outlined by equations (3.2)-(3.4) has an interior solution, such a solution satisfies the present-biased Euler Equation:

$$u'(c_t) \geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1 - \delta)c_k(k_{t+1}, e_{t+1}, t + 1)]\} \tag{A.1}$$

where c_k is the derivative of the optimal consumption function w.r.t assets.

Equation (A.1) can be re-written in the following way:

$$\begin{aligned}
u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})[1 + r - (1 - \delta)c_k(k_{t+1}, e_{t+1}, t + 1)]\} \\
\Leftrightarrow u'(c_t) &\geq \beta s_{t+1} E_t\{u'(c_{t+1})\}(1 + r) - \beta s_{t+1} E_t\{u'(c_{t+1})(1 - \delta)c_k(k_{t+1}, e_{t+1}, t + 1)\} \\
\Leftrightarrow u'(c_t) &\geq \beta(1 + r)s_{t+1} E_t\{u'(c_{t+1})\}\left[1 - \frac{1 - \delta}{1 + r} E_t\{c_k(k_{t+1}, e_{t+1}, t + 1)\}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta(1 + r)s_{t+1} E_t\{u'(c_{t+1})\}\left[1 - \frac{1 - \delta}{1 + r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t + 1)\}}{E_t\{u'(c_{t+1}\}}\right] \\
\Leftrightarrow u'(c_t) &\geq \beta\left[1 - \frac{1 - \delta}{1 + r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t + 1)\}}{E_t\{u'(c_{t+1}\}}\right](1 + r)s_{t+1} E_t\{u'(c_{t+1})\}
\end{aligned}$$

let $\beta_{x'} = \beta\left[1 - \frac{1 - \delta}{1 + r} \frac{E_t\{u'(c_{t+1})c_k(k_{t+1}, e_{t+1}, t + 1)\}}{E_t\{u'(c_{t+1})\}}\right]$, then the present-biased Euler Equation can be written as:

$$u'(c_t) \geq \beta_{x'}(1 + r)s_{t+1} E_t\{u'(c_{t+1})\}$$

which is exactly equation (3.6) in the text.

C: Proof of Proposition 1:

If an agent decides to attend college, in order to complete their degree they must obtain $s_L + s_H$ units of schooling ($s_L < s_H$) between their first period enrolled in school (t) and their second period enrolled in school ($t+1$).

Claim: Agents who enroll in school will always select s_L in period t and expect to select s_H in period $t+2$.

An agent will select s_L in period t and s_H in period $t + 1$ if the utility from this choice is greater than the utility from from selecting s_H in t and s_L in $t + 1$. As utility is additively separable in consumption and schooling, I begin by showing agents always prefer to obtain the low level of schooling in period t based on their disutility from education. I then show that agents always prefer the low schooling investment in period t based on utility from consumption. As agents prefer selecting s_L in period t and s_H in period $t + 1$ based on both components of utility, individual's will always select s_L then s_H if they enroll in school.

Recall that disutility from schooling in period t is given by $\gamma(1 - s_t)^{1-\phi}/(1 - \phi) + \beta\delta\gamma(1 - s_{t+1})^{1-\phi}/(1 - \phi)$[1]. Thus:

[1]This equation corresponds to the schooling portion of utility, outlined in equation (4.23).

$$\text{Schooling utility from } (s_L, s_H) > \text{Schooling utility from } (s_H, s_L) \quad \text{(A.2)}$$

$$\frac{\gamma(1-s_L)^{1-\phi}}{1-\phi} + \beta\delta\frac{\gamma(1-s_H)^{1-\phi}}{1-\phi} > \frac{\gamma(1-s_H)^{1-\phi}}{1-\phi} + \beta\delta\frac{\gamma(1-s_L)^{1-\phi}}{1-\phi} \quad \text{(A.3)}$$

$$(1-s_L)^{1-\phi} + \beta\delta(1-s_H)^{1-\phi} > (1-s_H)^{1-\phi} + \beta\delta(1-s_L)^{1-\phi} \quad \text{(A.4)}$$

$$(1-s_L)^{1-\phi}(1-\beta\delta) > (1-s_H)^{1-\phi}(1-\beta\delta) \quad \text{(A.5)}$$

$$1-s_L > 1-s_H \quad \text{(A.6)}$$

As $s_L < s_H$ by assumption, (A.6) is satisfied and therefore based on the disutility from education, agents always prefer postponing their high schooling investment to period $t+1$.

The second consideration for the timing of selecting s_L and s_H comes from consumption utility. Optimal consumption in period t is given by equation (4.27) and optimal expected consumption in periods $t+1$ and $t+2$ are given by (4.28) and (4.29), respectively. As $c^{\star E}_{t+1}$ and $c^{\star E}_{t+2}$ are both increasing in $c^{\star}_t$ and utility is strictly increasing in consumption, it is sufficient to show that optimal consumption in period t for an agent who enrolls in school is maximized by selecting $s_t = s_L$ and $s_{t+1} = s_H$. When $s_t = s_L$ and $s_{t+1} = s_H$, $y_t = (1-s_L)w$ and $y_{t+1} = (1-s_H)w$ and when $s_t = s_H$ and $s_{t+1} = s_L$, $y_t = (1-s_H)w$ and $y_{t+1} = (1-s_L)w$. Substituting for

y_t and y_{t+1} in equation (4.27) yields:

$$\begin{aligned}
c_t^*(s_t = s_L, s_{t+1} = s_H) &> c_t^*(s_t = s_H, s_{t+1} = s_L) && \text{(A.7)}\\
\frac{k_t + (1-s_L)w + \frac{(1-s_H)w}{1+r} + \frac{y_{t+2}}{(1+r)^2}}{1+\delta\beta+\delta\beta^2} &> \frac{k_t + (1-s_H)w + \frac{(1-s_L)w}{1+r} + \frac{y_{t+2}}{(1+r)^2}}{1+\delta\beta+\delta\beta^2} && \text{(A.8)}\\
(1-s_L)w + \frac{(1-s_H)w}{1+r} &> (1-s_H)w + \frac{(1-s_L)w}{1+r} && \text{(A.9)}\\
(1-s_L)(1-1/(1+r)) &> (1-s_H)(1-1/(1+r)) && \text{(A.10)}\\
1-s_L &> 1-s_H && \text{(A.11)}
\end{aligned}$$

As $s_L < s_H$ by assumption, (A.11) is satisfied. Therefore, based on utility from consumption agents always prefer postponing their high schooling investment to period $t+1$ as this provides them with higher consumption in all periods. As agents always prefer $s_t = s_L$ and $s_{t+1} = s_H$ from both a schooling disutility and consumption utility perspective, proposition 1 holds.

D: Consumption for Dropouts:

Claim: When credit markets are perfect and an individual has selected $s_t = s_L$, then consumption in periods $t+1$ and $t+2$ must decrease if that agent drops out of college (chooses $s_{t+1} = 0$). Note, consumption in period $t+1$ for a dropout is given by (4.40) and consumption for an individual who continues school and graduates in period $t+1$ is given by (4.42).

Proof: Suppose consumption increases when an individual drops out of college in period 2. That is, suppose $c_{t+1}^N > c_{t+1}^S$:

$$c_{t+1}^N = \frac{(1+r)k_{t+1}^S + w + \frac{w}{1+r}}{1+\delta\beta} > \frac{(1+r)k_{t+1}^S + (1-s_H)w + \frac{p_g w}{1+r}}{1+\delta\beta} = c_{t+1}^S \quad \text{(A.12)}$$

$$w + \frac{w}{1+r} > (1-s_H)w + \frac{p_g w}{1+r} \quad \text{(A.13)}$$

$$s_H + \frac{1}{1+r} > \frac{p_g}{1+r} \quad \text{(A.14)}$$

$$(1+r)s_H + 1 > p_g \quad \text{(A.15)}$$

In order for consumption to increase when an agent drops out of college, equation (A.15) must hold. However, in order for an individual to become a drop out, they must first invest $s_t = s_L$ units of time in school during period t. As schooling has a utility cost, it must be the case that an individual will only choose to enroll in school if their consumption is increasing in the schooling decision. That is, $c_t^S > c_t^N$ or else optimizing agents would never accept the disutility associated with schooling. Substituting in for c_t^S and c_t^N using equations (4.33) and (4.30), respectively, and canceling common terms leaves:

$$
\begin{aligned}
c_t^S &> c_t^N && \text{(A.16)}\\
(1-s_L)w + \frac{(1-s_H)w}{1+r} + \frac{p_g w}{(1+r)^2} &> w + \frac{w}{1+r} + \frac{w}{(1+r)^2} && \text{(A.17)}\\
(1-s_L) + \frac{(1-s_H)}{1+r} + \frac{p_g}{(1+r)^2} &> 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} && \text{(A.18)}\\
\frac{p_g}{(1+r)^2} &> s_L + \frac{s_H}{1+r} + \frac{1}{(1+r)^2} && \text{(A.19)}\\
p_g &> (1+r)^2 s_L + (1+r)s_H + 1 && \text{(A.20)}
\end{aligned}
$$

As $(1+r)^2 s_L > 0$, it is impossible for equation (A.15) and (A.20) to be simultaneously satisfied. Thus, consumption must be strictly lower in periods $t+1$ and $t+2$ for an agent who drops out of college relative to the value of consumption they would select if they selected $s_{t+1} = s_H$ and graduated college.

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