CONSUMPTION PREFERENCES, TIME AND UNCERTAINTY: IMPACTS ON RETAIL PRICING TACTICS

by

MONIRE JALILI

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Student: Monire Jalili

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This dissertation has been accepted and approved in partial fulfillment of the requirements for the Doctor of Philosophy degree in the Department of Operations and Business Analytics by:

Michael Pangburn	Co-Chair
Nagesh Murthy	Co-Chair
Tolga Aydinliyim	Core Member
Bruce McGough	Institutional Representative

and

Scott L. Pratt	Dean of the Graduate School

Original approval signatures are on file with the University of Oregon Graduate School.

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DISSERTATION ABSTRACT

Monire Jalili

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Title: Consumption Preferences, Time and Uncertainty: Impacts on Retail Pricing Tactics

My dissertation is a collection of three essays with analytical models at the interface of marketing and operations with a focus on pricing. A unifying theme in this dissertation is the emphasis on understanding how consumer purchase behaviors impact the optimal pricing decisions of a firm. This dissertation includes co-authored material. In my *first* essay, I study the role of consumers' opposing perceptions of green quality on the optimal product line decisions, i.e., products, prices and quality by analyzing the firm's optimization problem and incorporating an endogenous demand model that emerges from the consumers' preferences while considering the cost implications of introducing a green product. My *second* essay is on optimal timing of price discounts. Delaying discounts, i.e., giving discounts on future spending based on current spending is a prevalent retail discounting practice. For a market of rational and forward-looking consumers who repeatedly visit and purchase with the firm, we analyze the relative efficacy of delayed credits vs. a natural alternative of immediate discounts. In my *third* essay, I explore a firm's optimal pricing strategy when it simultaneously rents and sells a product for which consumers have a priori valuation uncertainty.

This dissertation includes previously unpublished coauthored material.

CURRICULUM VITAE

NAME OF AUTHOR: Monire Jalili

GRADUATE AND UNDERGRADUATE SCHOOLS ATTENDED:

University of Oregon, Eugene
University of Tehran, Tehran
K. N. Toosi University of Technology, Tehran

DEGREES AWARDED:

Doctor of Philosophy, Operations and Business Analytics, 2017, University of Oregon
Master of Science, Industrial Engineering, 2009, University of Tehran
Bachelor of Science, Industrial Engineering, 2006, K. N. Toosi University of Technology

AREAS OF SPECIAL INTEREST:

OM/Marketing Interface
Retail Operations
Pricing

GRANTS, AWARDS AND HONORS:

Graduate Teaching Fellowship, University of Oregon, 2012 - 2017

Best Student Paper Award, Decision Science Institute, 2016

Roger and Robin Best Awards for Research, 2014, 2015 and 2016

Summer Research Award Receipt, 2012 to 2016

Summer Research Grant from Manufacturing 21, 2012 and 2013

Research and Travel Award Receipt from Doctoral Program, 2013 to 2017

Participant at POMS Doctoral Colloquium, Washington, DC, 2015

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CHAPTER I

INTRODUCTION

My dissertation is a collection of three essays with analytical models at the interface of marketing and operations, with a focus on pricing and retail operations. A unifying theme in this dissertation is the emphasis on understanding how consumer purchase behaviors impact the optimal pricing decisions of a firm. This dissertation includes co-authored material. In my *first* essay, a coauthored work with Dr. Tolga Aydinliyim and Dr. Nagesh Murthy, I study the role of consumers' opposing perceptions of green quality on the optimal product line decisions, i.e., products, prices, and quality. In this model, we study the firm's optimization problem by incorporating an endogenous demand model that emerges from the consumers' <u>preferences</u> while considering the cost implications of introducing a green product i.e., the material cost savings and the dis-economies of scope in production. My *second* essay, coauthored with Dr. Michael Pangburn, is on optimal <u>timing</u> of price discounts. Delaying discounts, i.e., giving discounts on future spending based on current spending is a prevalent retail discounting practice. For a market of rational and forward-looking consumers who repeatedly visit and purchase with the firm, we analyze the relative efficacy of delayed credits vs. a natural alternative of immediate discounts. In my *third* essay, coauthored with Dr. Michael Pangburn, I explore the role of consumer valuation <u>uncertainty</u> in determining how a firm should set its rental and selling prices, including when to offer a discount for converting trials to purchases. I hereby present a summary abstract of each work.

Essay 1: Key Factors for Green Product Line Design

Ever-increasing pressure to cut back the anti-environmental practices, from a growing group of consumers, have pushed the firms to rethink the type of products to manufacture or sell. Inclusion of a green variant - that has some recycled content - in the product line can contribute to virgin raw material saving and thus reduction of the firm's impact on the environment. The price and quality of the green variant not only affect the market demand, but also bear processing and material cost implications that notably influence the firm's profit and thus should not be overlooked. In this essay, we consider the price and quality optimization (i.e., product line design) problem of a monopolist selling at most two product variants, a base product and a green variant that comprises recycled/reused content, to a market of two distinct consumer types with heterogeneous valuations. We seek to answer couple of managerial questions. Firstly, should the firm target each customer type with a unique product, i.e., offer a green product with some recycled content for those who are willing to pay a higher price for green products (i.e, as we refer to them as naturalites), and a base product with no recycled content for those whose willingness-to-pay is lower for a green product (as we refer to them as conventionals); or should the firm offer only one product, and price appropriately to attract some demand from the segment with the opposing perception? Secondly, if the firm chooses to offer a green variant, just by itself, or, in addition to the base product, what should be the optimal percentage of recycled content used in the green variant? Using an endogenous demand model and non-linear programming theory, we characterize the economic conditions under which a monopolist can profitably serve both consumer types by maintaining a uniformly green product line, i.e., by selling only the green product variant.

When such equilibrium outcomes result, the firm's traditional profit maximization objective coincides with an environmentally-conscious outcome. We also assess the demand segmentation and firm profit consequences of underestimating the naturalites' (conventionals') marginal utility (dis-utility) for a green variant with more recycled/reused content, and show that such missteps may yield adverse implications for both firm profits and the environment.

Essay 2: Delayed vs. Immediate Price Discounts

Price discounts, sometime called as rewards, are a ubiquitous practice to stimulate consumer demand. Of critical decisions with respect to rewards are the frequency, size and the timing of the discounts, which are highly correlated with each other. Recurring reward programs are a particular form of discount mechanism that have become prominent specially in retail and service sector. Generally, these programs fall into one of two varieties: delayed or immediate rewards. Delayed discounts are calculated based on today's spending yet redeemable only toward future purchases, and immediate discounts are applied instantaneously. In this paper, we contrast the immediate and delayed credit alternatives for a firm serving rational, forward-looking customers who repeatedly visit and purchase with the firm. We employ dynamic programming to construct the consumer surplus and the firm profits over an infinite horizon and subsequently optimize the firm's expected NPV of profit to determine the optimal size of immediate vs. delayed discounts. From the firm's perspective, paying out a discount later rather than sooner is naturally appealing, all other things being equal. However, the customers are rational and forward looking and rightfully account for the time-value of money and hence prefer immediate discounts, ceteris paribus. For the same reason, as we establish in our findings, the firm optimally scales up the size of a delayed discount (proportional to time value of money) to compensate the customer for the wait before redeeming their credit. To further the analysis and address the main question of long-term relative profitability of delayed vs. immediate discounts, we consider two main scenarios: 1) when the posted prices are steady over time and 2) when there is random occasional sales. We study the problem under each scenario for both a homogeneous and heterogeneous market. With steady prices, we find that the two tactics are equally profitable, regardless of the type of the market. Given price fluctuation and a homogeneous market, we find that delaying discounts increases the profits only if the representative customer's valuation falls within a specific middle range, otherwise both tactics yield equal profits and surplus. However, we continue to analysis to study a heterogeneous market (and non-customized discount percentage) and prove that delayed discounts segment the market more efficiently such that not only the achieved profits are higher, but also a larger portion of the marker shop frequently and this in turn leads to higher aggregate market surplus as well.

Essay 3: Try Before You Buy Pricing

Many products/services exhibit the nature of experience goods, for which the true quality and fit to a customer's taste and willingness to pay can only be learned via consumption. To mitigate customers' reluctance to purchase when facing uncertainty, firms follow various strategies such as different return policies, money-back guarantees, and trials. In this chapter, we analyze try-before-you-buy pricing tactic for a single firm that offers an experience good to consumers facing valuation uncertainty. By offering a low-cost trial (rental) option to its customers, the firm facilitates the resolution of valuation uncertainty — through learning via consumption — and therefore might want to charge a higher selling price to customers once they have realized a high valuation and are contemplating to upgrade to a purchase. However, a consumer's willingness to pay may decrease noticeably after the first consumption. This in turn might prompt the firm to lower the post-rental selling price to mitigate the risk of losing potential profitable sales to those who rented the product once. For some products, consumers may experience a significant drop in post-first-usage utility. For example, rewatching a thriller movie or rewearing a prom dress is much less enjoyable than the first time. However, products like ski gear and bikes don't suffer that much from this decrease in utility after the first usage. We analyze how to set rental and selling prices, and also consider the conditions under which a firm should refund — fully or partially — rental fees in order to optimally stimulate customer trials and subsequent purchase conversions. We conduct our analysis both for an ex-ante homogeneous and heterogeneous market. Among our findings, we establish that a small renal cost to selling cost ratio on the firm's side is necessary to practice try before you buy pricing and that the magnitude of rental utility relative to lifetime utility heavily affects the optimality of conversion discounts, especially in a heterogeneous market. We find that if the conditions are right, firms can sometimes benefit by charging a higher selling price after trial.

CHAPTER II

KEY FACTORS FOR GREEN PRODUCT LINE DESIGN

The excerpt to be included is co-authored material, a joint work with Dr. Tolga Aydinliyim and Dr. Nagesh Murthy.

Introduction

Over time, a continuously expanding consumer base and non-government organizations have pushed firms' operations and products to be less taxing on the environment. Despite the increasing pressure, firms have responded to such demands cautiously (especially when there is no binding legislation in place), as it is unclear whether the inclusion of products with less environmental impact in a firm's product line supports traditional profit measures. Take, for example, products with recycled content. On the cost side, provided unit collection costs are not excessive, using recycled materials may reduce variable input costs as recycled materials are typically procured at a lower cost compared to virgin materials. However, using recycled instead of virgin material may require a different production technology, modifying or upgrading existing equipment, or having to run existing equipment at a slower rate, thus implying an increase in production costs yielding diseconomies-in-scope. For example, Starbucks' white paper cups contain an industry-standard liner, which makes the hot beverage cups non-recyclable in most paper recycling systems, thus requiring Starbucks to subsidize recyclers' third-party's investment in necessary technologies. Even after such investments, if Starbucks procures fully recyclable cups from its suppliers, the unit cost for a cup would more than quadruple compared to the current design with only 10% recycled content, thus making this proposition economically hard to justify.[1] The net cost effect of including a green variant in a product line might be significant to justify the firm charging a premium for that product. As a case in point, consider Nike's "Reuse-a-shoe" and "Nike Grind" programs, where Nike re-purposes recycled materials by incorporating them in various products, such as Air Jordan XX3 and Nike Pegasus 25., which Nike sells at premium prices relative to other similar Nike shoes. On the demand side, some consumers are more environmentally conscious and, in some cases, are willing to pay more for green product variants. Recent surveys in Europe reveals that 30% of consumers in the United Kingdom say they plan to spend more, and 49% plan to spend the same amount on green products. In contrast, many other consumers, arguably the majority in the United States, perceive products with recycled (or reused) content inferior to those with no recycled content. This latter segment would not consider purchasing the green product unless that variant is available at a sufficiently high discount.

In this paper, we consider the price and quality optimization (i.e., product line design) problem of a monopolist selling (at most) two product variants, a base product and a green variant that comprises recycled content, to a market of two distinct consumer types with heterogeneous valuations. Our setting features three distinguishing elements, which had hitherto not been considered together in the literature: (i) Consumer segments demonstrate opposing perceptions of the green variant. We refer to customers who associate dis-utility with a green variant with more recycled/reused content as "conventionals", whereas those who have a higher willingness-to-pay for the same product are referred to as "naturalites" (ii)

[1] In 2008, Starbucks announced that it does not have any plans to offer its beverages in fully recyclable cups until at least 2015. As of October 2016, Starbucks' cups are still not fully recyclable.

Including a green variant yields diseconomies-in-scope for the firm's production costs, which increases non-linearly as product variants become more vertically (and environmentally) differentiated. (iii) Using more recycled content for the green variant permits input material cost savings.

The aforementioned cost/demand dynamics give rise to a series of managerially relevant research questions. Firstly, given the varied (and opposing) consumer perceptions of the green product variants, what is the optimal product line for a monopolist, which also prices its product optimally? In other words, should the firm target each segment with a unique product, i.e., offer a green variant (with some recycled content) for naturalites, and a base product (with no recycled content) for the conventionals; or should the firm uniformly offer one product, and price it appropriately to attract some demand from the segment with the opposing perception? Secondly, if the firm chooses to offer a green variant, just by itself, or, in addition to the base product, what should be the optimal degree of vertical differentiation (as measured by the percentage of recycled content) between the product variants? Thirdly, how do the optimal quality and price decisions drive the firm's demand and profit, and consequently, how does the firm's optimal product line decision transition from one to another as key problem parameters, such as the relative proportion of the sizes of the naturalite and conventional segments, the marginal (dis)utility of each customer segment from additional recycled content, unit virgin and recycled material costs, and the degree of diseconomies-in-scope in production, change? Regarding the last point, we are particularly interested in characterizing the economic conditions under which a monopolist can profitably cover the entire market by maintaining a uniformly green product line, i.e., by selling only the green product variant. This is because, when such equilibrium outcomes result, the firm's traditional profit maximization objective coincides with an environmentally-conscious outcome. Finally, we assess the demand segmentation and firm profit consequences of underestimating the naturalites' (conventionals') marginal utility (dis-utility) for a green variant with more recycled/reused content, and show that such missteps may yield adverse implications for both firm profits and the environment.

Using an endogenous demand model that reflects consumers' self-selection regarding which (if any) of the product variants they choose to purchase, we characterize five distinct demand segmentation scenarios. The monopolist can induce one of these five demand scenarios by appropriately choosing prices for each product variant and/or how vertically differentiated these variants will be from one another (as measured by the green variant's recycled content). In two such scenarios, namely "Uniform Base" and "Uniform Green," all purchasing consumers (conventionals and/or naturalites) buy the same product variant, i.e., the base product or the green variant, respectively. Consequently, the monopolist sells only one product variant. In the other demand scenarios, which we refer to as "Targeted Marketing" (or "Targeting" in short) scenarios, as the firm includes both variants in its product line to target each segment with a unique product variant. In the "Perfect Targeting" scenario, the firm completely segments the market, i.e., all conventionals (naturalites) self-select to buy the base product (green variant). In the "Conventional (Naturalite) Targeting" scenario, all conventionals (naturalites) buy the base product (green variant), whereas only some naturalites (conventionals) self-select to buy the variant intended for their consumption, i.e., the green variant (base product).

We find that a uniformly green product line is optimal for a firm that cannot influence the green variant's product features (i.e., recycled content) when the consumer base comprises mostly of conventionals with mild dislike for the green variant, the naturalites' marginal willingness-to-pay for the green variant is low, the green variant has limited recycled content, and the cost differential for input material cost in virgin versus recycled forms is high. A uniformly green product line with a maximally green variant (i.e., one that contains the highest permissible amount of recycled content as per technological limits) sustains optimally only if the firm can influence the green variant's quality. In other words, a product line that optimizes the monopolist firm's traditional profit measure can induce all customers (of both types) to consume the green product variant with maximum recycled content yielding an environmentally-conscious outcome.

We also assess the potential profit loss for a firm that underestimates the naturalites' marginal utility or the conventionals' marginal dis-utility from a green variant with more recycled content. We find that underestimating naturalities' extra willingness-to-pay hurt firm profits the most when the virgin and recycled input material costs are comparable. In this case, even though a more environmentally-friendly outcome with a uniformly green product line results, firm profit suffers as the firm fails to segment the consumer base by underpricing the green variant and cannibalizing all demand for the base product. In contrast, when the conventionals' dislike for more recycled content in the green variant is underestimated, firm profits decline the most when input material cost in virgin form is high. In this case, the firm forgoes an opportunity to maintain a uniformly green product line by overpricing the green variant. As a result, each demand segment purchases only its intended product, i.e., the perfect targeting demand scenario, which implies an outcome wherein not only does the firm accrue profit loss, but also a less environmentally-friendly demand segmentation results yielding more virgin material consumption.

Literature Review

Our work lies at the intersection of sustainable operations management and marketing, with particular focus on price and quality optimization for product lines with green product variants. As such, following the classification of Guide and Van Wassenhove (2009), our paper contributes to the "Prices and Markets" phase of the research stream at this interface. In marketing, Mussa and Rosen (1978) and Moorthy (1984) are the two seminal works in product line design, which highlight that a monopolist may not be able to perfectly discriminate self-selecting consumers to product variants at distinct quality levels. In our context, this gives rise to the question whether a monopolist firm should offer both the base product and the green variant, or attract demand from both conventionals and naturalites for only one of the two products, and what firm cost and market demand conditions support each strategy.

The existing works in vertical differentiation and pricing research that follow the aforementioned seminal papers mostly assume that, when there are multiple consumer types, consumers' willingness-to-pay for a particular quality attribute is consistently increasing in quality for each type (i.e., single crossing property). However, in our context, conventionals' and naturalites' utility for the green variant change in opposing directions as the variant's recycled content increases. Our assumption that there are different (and possibly opposing) consumer perceptions for green product attributes finds empirical support in a number of studies such as Antil (1984), Roberts (1996), and Ubilava et al. (2010), which indicate that neither income level nor willingness-to-pay are closely tied to consumers' sensitivity to consumption with less waste.

Recent studies in operations management have considered demand models similar to ours. For example, Aydinliyim and Pangburn (2012) permit consumer utility from green consumptions to take negative values, Yenipazarli and Vakharia (2015) let two consumer segments (namely, green and brown) self-select one of two product variants, and Kim et al. (2013) consider two customer segments, each of which gives more importance to either a traditional quality attribute or a green product attribute. However, all three of these aforementioned studies consider fixed quality levels, and focus only on price optimization. In contrast, we study both price and quality optimization for a monopolist, and in the same spirit as Salant (1989) and Anderson and Dana (2009), we derive conditions relating production costs and the relative demand segments under which forgoing a full product line can be profitable. Chen (2001) also considers both price and quality optimization with two distinct consumer segments—ordinary and green consumers. However, in Chen's model, traditional and environmental quality levels add up to a fixed value (of one), and only green consumers value the environmental quality dimension. As such each consumer type's utility from distinct quality levels are consistently ranked, satisfying Moorthy's single crossing property.

In both marketing and sustainable operations literature, there are also studies that permit demand models that one can derive from utility functions for two attributes and violate Moorthy's single crossing property in a duopoly setting; see, for example, Vandenbosch and Weinberg (1995) and Chen and Liu (2014). These papers consider settings similar to those in Shaked and Sutton (1982) and Moorthy (1988), and extend their work by considering two product attributes instead of a single attribute. In contrast with our study, these studies permit each firm to sell only one product, and thus potential cannibalization effects within a firm's product lines are excluded from their analysis.

Past research in sustainable management literature has repeatedly argued that profit- and environment-related benefits are not necessarily antithetical; see, for example, Porter and van der Linde (1995), and Guide et al. (2003). Therefore, one of our objectives in this paper is to characterize situations wherein a monopolist firm can profitably sustain a uniformly green product line. Our focus on this particular matter relates to recent literature on closed loop supply chains (CLSC), which has mostly focused on pricing and inventory decisions for remanufactured products. Two almost uniformly applied assumptions in this research stream are that all consumers associate lower consumer valuations with remanufactured product variants, and that remanufactured products are cheaper to produce than new products; see, for example, Debo et al. (2005), Ferguson and Toktay (2006), Atasu, Guide and Van Wassenhove (2008), Atasu, Sarvary and Van Wassenhove (2008), and Abbey et al. (2015). Our models differ significantly from these aforementioned studies as the green product variant in our model may yield higher production costs due to diseconomies-in-scope, while inducing higher willingness-to-pay by naturalite consumers, which permits the possibility of a higher price than a base product's price at equilibrium.

Model

We consider a monopolist selling (at most) two product variants to a segmented consumer base with heterogeneous valuations. The *base* product does not contain any recycled content, whereas the *green* variant is produced by using a combination of virgin and recycled materials where the latter comprises fraction β of the input material. We refer to the consumer base who strictly prefers the green variant to the base product as *naturalites* and, those with opposing preferences for recycled content as *conventionals*; see Russo (2010) for more details on this terminology. We assume that all consumers' willingness-to-pay v for the base product is uniformly distributed over the unit interval. We further assume that the conventionals' valuation of the green variant is $(1-\alpha_c\beta)v$, where marginal dis-utility α_c reflects these consumers' utility loss percentage from using a green variant with only recycled content. Similarly, we denote the naturalites' marginal utility gain percentage from consuming a green variant with only recycled materials by α_n. Consequently, naturalities' valuation of a green variant with fraction β of recycled content is $(1+\alpha_n\beta)v$. We denote by $\bar{v}_c \equiv 1-\alpha_c\beta$ and $\bar{v}_n \equiv 1+\alpha_n\beta$ the largest of the conventional and the naturalite consumers' valuations, respectively, for the green variant. Without loss of generality, we normalize market size to 1, and assume that the conventionals comrpise fraction ω of the market.

On the cost side, including a green variant in its product line has material and production cost implications. Whereas the base product requires 100% virgin content at unit cost c_v, the unit material cost for the green variant with $100\beta\%$ recycled content is $\beta c_r + (1-\beta)c_v$, thus yielding material cost savings as long as the monopolist can acquire recycled material at a discount (i.e., $c_v > c_r$). On the other hand, consistent with the extant literature (e.g., Atasu, Guide and Van Wassenhove (2008)), we assume that maintaining a more vertically differentiated product line is more expensive. In other words, as the green variant's recycled content β increases, the monopolist experiences diseconomies-in-scope, which we model by the quadratic unit production cost $k(1+\beta)^2$. (Note that when the monopolist offers only the base product, i.e., $\beta = 0$, its unit production cost reduces to constant k.) Consequently, it costs $c_b \equiv c_v + k(1+\beta)^2$ and $c_g \equiv \beta c_r + (1-\beta)c_v + k(1+\beta)^2$ to deliver one unit of base and green product variants, respectively.

We will permit the monopolist to make both quality and price decisions. The quality decision comprises the recycled content fraction β to incorporate in the green product design, as choice of β yields a *vertical differentiation gap* between the naturalites' and the conventionals' valuation of the green variant. Considering two consumers with the same valuation v for the base product, the naturalite's valuation for the green variant will be $(\alpha_n + \alpha_c)\beta$ more than the conventional's valuation for the same. To reflect practice, we will assume that the quality decision comes first, and then price optimization, where the monopolist chooses p_b and p_g for the base and the green product variants, respectively, follows closer to when demand realizes.

Deriving the endogenous demand

We will employ an endogenous demand model to calculate the demand for each product variant given β, p_b, and p_g, where each consumer self-selects the option that gives him the highest net surplus whilst considering a purchase of one of the two variants or leaving without a purchase. We denote the resulting demand segments by D_{bc} (conventionals buying the base product), D_{gc} (conventionals buying the green variant), D_{bn} (naturalites buying the base product), and D_{gn} (naturalites buying the green variant), the relative magnitudes of which yield five different demand segmentation scenarios.

Next, we will describe these demand scenarios by highlighting their product line implications. As we will describe in more detail, the absolute price differential $\Delta_p^a \equiv p_g - p_b$ dictates the monopolist's product line choice. When the base product is so competitively priced that *all* purchasing consumers uniformly self-select to buy that variant, i.e., $D_{gc} = D_{gn} = 0$, the monopolist's product line comprises only of the base product. We refer to this scenario as the *uniform base* marketing case (UB), which realizes when $\Delta_p^a \geq \alpha_n\beta$ holds. In contrast, when $\Delta_p^a \leq -\alpha_c\beta$, the green variant is uniformly favored by *all* purchasing customers yielding $D_{bc} = D_{bn} = 0$. Consequently, the monopolist includes only the green variant in its product line, a scenario we refer to as the *uniform green* marketing scenario (UG).[2]

The remaining range of price and quality decisions ($-\alpha_c\beta < \Delta_p^a < \alpha_n\beta$) yield demand segmentation scenarios wherein the monopolist maintains a product line with both the base and the green variants. We refer to these scenarios as *targeted* marketing scenarios,[3] or *targeting* in short, as the monopolist targets each consumer segment with a unique product offering. For these targeted marketing scenarios, whether the monopolist can successfully direct each consumer type to its intended variant depends on the variants' relative price differential, which we denote by $\Delta_p^r \equiv \frac{p_g - p_b}{p_g}$. When $|\Delta_p^r|$ is small—more specifically, when $|\Delta_p^r| < \min\{\frac{\alpha_n\beta}{1+\alpha_n\beta}, \frac{\alpha_c\beta}{1-\alpha_c\beta}\}$, prices are not different enough to overcome consumers' opposing preferences of the product variants, and thus consumers' self-selection dynamics yield *perfect targeting* (PT), a marketing scenario wherein each consumer buys his preferred variant regardless of the (possible) discount offered for the other variant. On the other hand, when the prices are close enough in absolute sense but are different enough relatively, it is possible that one customer segment may purchase the product that is meant to target the other customer segment with opposing perceptions of the green variant. Specifically, when $\Delta_p^a < \alpha_n\beta$ and $\Delta_p^r > \frac{\alpha_n\beta}{1+\alpha_n\beta}$, some naturalites as well as *all* conventionals buy the base product—a marketing scenario we refer to as *conventional targeting* (CT), as that customer segment is successfully targeted by the base product. Similarly, when $\Delta_p^a > -\alpha_c\beta$ and $\Delta_p^r < \frac{-\alpha_c\beta}{1-\alpha_c\beta}$, some conventionals as well as all naturalites buy the green variant, which we refer to

[2]The UG demand scenario may realize in two different ways depending on the magnitude of demand segment D_{gc}, which we will discuss in more detail later in this section.

[3]See Ginsberg and Bloom (2004) for more details regarding this terminology.

as the *naturalite targeting* (NT) scenario. In the following lemma, we consolidate all of the aforementioned five demand segmentation scenarios that result from the monopolist's quality and price decisions. We relegate all proofs to the Appendix.

Lemma 1

The monopolist's quality and price decisions yield the following demand segments, and the corresponding product line and marketing scenarios as summarized in Table 1:

TABLE 1. Demand segments given prices and recycled content fraction.

s	Price range	D^s_{bc}	D^s_{gc}	D^s_{bn}	D^s_{gn}
UB	$p_b \leq p_g - \alpha_n\beta$	$w(1-p_b)$	0	$(1-w)(1-p_b)$	0
CT	$p_g - \alpha_n\beta < p_b < \frac{p_g}{1+\alpha_n\beta}$	$w(1-p_b)$	0	$(1-w)(\frac{p_g-p_b}{\alpha_n\beta} - p_b)$	$(1-w)(1-\frac{p_g-p_b}{\alpha_n\beta})$
PT	$\frac{p_g}{1+\alpha_n\beta} \leq p_b \leq \frac{p_g}{1-\alpha_c\beta}$	$w(1-p_b)$	0	0	$(1-w)(1-\frac{p_g}{1+\alpha_n\beta})$
NT	$\frac{p_g}{1-\alpha_c\beta} < p_b < p_g + \alpha_c\beta$	$w(1-\frac{p_b-p_g}{\alpha_c\beta})$	$w(\frac{p_b-p_g}{\alpha_c\beta} - \frac{p_g}{1-\alpha_c\beta})$	0	$(1-w)(1-\frac{p_g}{1+\alpha_n\beta})$
UG	$p_g + \alpha_c\beta \leq p_b$	0	$w(1-\frac{p_g}{1-\alpha_c\beta})^+$	0	$(1-w)(1-\frac{p_g}{1+\alpha_n\beta})$

Note: Scenario UG realizes as UG_{inc} or UG_{exc} depending on whether $w(1-\frac{p_g}{1-\alpha_c\beta})^+$ is positive or 0, respectively.

Note in Table 1 that scenario UG realizes in two different ways depending on what $D^{UG}_{gc} = w(1-\frac{p_g}{1-\alpha_c\beta})^+$ returns. Specifically, we refer to the case with $D^{UG}_{gc} = 0$ as the exclusive UG scenario, wherein the firm excludes the conventionals by setting $p_g \geq 1-\alpha_c\beta$, and denote it by UG_{exc}. In contrast, when $D^{UG}_{gc} > 0$, the firm serves both the conventionals and naturalites with the same green product variant; a scenario we refer to as inclusive UG and denote by UG_{inc}.

When the Monopolist Cannot Influence Quality

In this section, we assume that the recycled content β in the green variant is fixed, and cannot be influenced by the monopolist firm. Note that as the demand expressions depend on prices inducing a particular demand scenario, the profit function we aim to optimize over the entire domain of prices changes. Therefore, we will use $\pi_s(p_b\,,p_g|\,\beta)$ (or, $\pi_s(\beta)$ in short) where subscript s takes values from set $\{UB, CT, PT, NT, UG\}$ to denote the firm's profit for each demand scenario s, and employ binary indicator variables $I_s(p_b\,,p_g)$ (or, I_s in short), which take a value of 1 if prices fall within the range prescribed for s in Table 1. As such, we can express the firm's profit $\pi(p_b\,,p_g|\,\beta)$ (or, $\pi(\beta)$ in short) as

$$\pi(\beta)=\sum_s I_s\pi_s=\sum_s I_s\Big((p_b-c_b)(D^s_{bc}+D^s_{bn})+(p_g-c_g)(D^s_{gc}+D^s_{gn})\Big). \tag{2.1}$$

The firm's optimization requires finding optimal prices p_b^* and p_g^* for the base product and the green variant, respectively, which we can formally state with the following mixed binary-integer non-linear program:

$$\begin{aligned} \max_{p_b\,,p_g} \quad & \pi(\beta) & & \\ \text{subject to} \quad & p_b\le 1 & & p_g\le \bar{v}_n \\ & \textstyle\sum_s I_s=1 & & I_s=\{0,1\}\,,\ \forall s \\ & D^s_{bc}\,,D^s_{gc}\,,D^s_{bn}\,,D^s_{gn} & & \forall s \text{ are as defined in Table 1} \end{aligned} \tag{2.2}$$

We will first investigate the structural properties of $\pi_s(\beta)$ (for all s) and $\pi(\beta)$ to see whether standard non-linear optimization approaches can be utilized to identify globally optimal prices, which the next lemma clarifies:

Lemma 2

For each demand scenario s highlighted in Table 1, function $\pi_s(p_b\,,p_g|\,\beta)$ is jointly concave in p_b and p_g. However, function $\pi(p_b\,,p_g|\,\beta)$ is not jointly concave over its entire support $(p_b\,,p_g)\in[0,\,1]\times[0\,,\bar{v}_n]$.

Lemma 2 highlights that the globally optimal prices in each demand scenario s must satisfy the Karush-Kuhn-Tucker (KKT) conditions for each $\pi_s(\beta)$ and the constraints that ensure scenario s realizes, yet further analysis is necessary to determine the firm's optimal product line strategy and the corresponding optimal prices p_b^* and p_g^*. Consequently, the globally optimal prices may exist within one of the five price range intervals that imply each demand scenario, or they may coincide with one of the four bounds of two subsequent price ranges. Next, we will identify the parameter spaces for all s (in terms of critical problem parameters β, ω, c_v, c_r, α_c, α_n, k), and their subsets when necessary, over which the firm optimally implements each demand scenario and the corresponding product line and prices. The next proposition formally characterizes our result:

Proposition 1

Define the following constant parameter combinations:

$$z \equiv \; 1/(\tfrac{\omega}{\bar{v}_c} + \tfrac{1-\omega}{\bar{v}_n})$$

$$y \equiv \; \alpha_c \beta (1 - z\tfrac{\omega}{\bar{v}_c})$$

$$x \equiv \; \bar{v}_c\Big((1-c_b)^2 + z(1-w)(\tfrac{\bar{v}_n}{\bar{v}_c} + \tfrac{\bar{v}_c}{\bar{v}_n} - 2)\Big)$$

Also, define the following parameter sets:

$$\Gamma^*_{UB} \equiv \{c_b \leq \min\{c_g - \alpha_n\beta, 1\}\}$$

$$\Gamma^*_{CT} \equiv \{c_g - \alpha_n\beta < c_b < \min\{\tfrac{c_g}{\bar{v}_n}, 1\}\}$$

$$\Gamma^1_{PT} \equiv \{\tfrac{c_g}{\bar{v}_n} < c_b < \min\{\tfrac{c_g}{\bar{v}_c} + (\tfrac{\bar{v}_n}{\bar{v}_c} - 1), 1\}\}$$

$$\Gamma^2_{PT} \equiv \Gamma^*_{NT} \bigcup \Gamma^*_{UG^1_{inc}}$$

$$\Gamma^*_{NT} \equiv \{\tfrac{c_g}{\bar{v}_c} + \sqrt{y}(\tfrac{\bar{v}_n}{\bar{v}_c} - 1) \leq c_b \leq c_g + \alpha_c\beta\}$$

$$\Gamma^*_{UG^1_{inc}} \equiv \{c_b < 1\} \bigcap \{c_g \leq \min\{\bar{v}_c - \sqrt{x}, c_b - \alpha_c\beta\}\}$$

$$\Gamma^*_{UG^2_{inc}} \equiv \{c_b \geq 1\} \bigcap \{c_g \leq \bar{v}_c - \sqrt{\bar{v}_c\,(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))}\}$$

$$\Gamma^*_{UG_{exc}} \equiv \{\{c_b > 1\} \bigcap \{\bar{v}_c - \sqrt{\bar{v}_c\,(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))} < c_g < \bar{v}_n\}\}$$

Then, the monopolist's optimal price decisions p_b^* and p_g^* for each demand scenario s, and the corresponding optimal profit $\pi_s^*(\beta)$ and optimality region $\Gamma_s^*(\beta)$ are summarized as in Table 2:

TABLE 2. Optimal prices, profit and regions

Scenario s	p_b^*	p_g^*	$\pi_s^*(\beta)$	$\Gamma_s^*(\beta)$
UB	$\frac{1+c_b}{2}$	$\frac{\bar{v}_n+c_b+\alpha_n\beta}{2}$	$\frac{(1-c_b)^2}{4}$	Γ^*_{UB}
CT	$\frac{1+c_b}{2}$	$\frac{\bar{v}_n+c_g}{2}$	$\frac{(1-c_b)^2}{4} + (1-w)\frac{(c_b-c_g+\alpha_n\beta)^2}{4\alpha_n\beta}$	Γ^*_{CT}
PT	$\frac{1+c_b}{2}$	$\frac{\bar{v}_n+c_g}{2}$	$w\frac{(1-c_b)^2}{4} + (1-w)\frac{(\bar{v}_n-c_g)^2}{4\bar{v}_n}$	$\Gamma^1_{PT} \setminus \Gamma^2_{PT}$
NT	$\frac{\alpha_c\beta+c_b+z}{2}$	$\frac{c_g+z}{2}$	$w\frac{(c_g-c_b+\alpha_c\beta)^2}{4\alpha_c\beta} + \frac{(z-c_g)^2}{4z}$	Γ^*_{NT}
UG_{inc}	$\frac{2\alpha_c\beta+c_g+z}{2}$	$\frac{c_g+z}{2}$	$\frac{(z-c_g)^2}{4z}$	$\Gamma^*_{UG^1_{inc}} \bigcup \Gamma^*_{UG^2_{inc}}$
UG_{exc}	$\frac{2\alpha_c\beta+c_g+\bar{v}_n}{2}$	$\frac{c_g+\bar{v}_n}{2}$	$\frac{(1-\omega)(\bar{v}_n-c_g)^2}{4\bar{v}_n}$	$\Gamma^*_{UG_{exc}}$

Proposition 1 offers a full characterization of the cost (c_v, c_r, k), product (β), and market demand (α_c, α_n, ω) conditions under which a specific product line and pricing strategy optimally sustains for a monopolist firm. Yet, the resulting optimality region closed-form expressions Γ_s^* for each demand scenario are difficult to interpret except for UB and CT.

Using relations we established earlier between virgin and recycled material costs (c_v and c_r) and unit costs for each product variant (c_b and c_g), we can conclude that unit cost differential $c_b - c_g$ equals $\beta(c_v - c_r)$. As such, the optimality region Γ_{UB}^* can simply be expressed as $\{c_v - c_r < -\alpha_n\}$, which yields the following corollary:[4]

Corollary 1

Unless virgin materials can be procured at a discount relative to recycled materials, i.e., $c_v < c_r$, the monopolist firm always maintains a product line including the green variant.

In most practical settings, firms can acquire and utilize recycled input material at a lower cost than they can procure input material in virgin form, and thus a monopolist selling to a heterogeneous market should never maintain a product line with only the base product. Similar algebra permits Γ_{CT}^* to be expressed as $\{-\alpha_n < c_v - c_r < -\alpha_n c_g\}$, and thus we have the following corollary:

Corollary 2

Unless virgin materials can be procured at a discount relative to recycled materials, i.e., $c_v < c_r$, the naturalites always purchase the green variant.

Corollary 2 highlights that, the input material being cheaper in virgin form than it is in recycled form is a necessary condition for a monopolist to profitably induce

[4]Note that $\min\{c_g - \alpha_n\beta, 1\} = c_g - \alpha_n\beta$ as the principle of optimality dictates $c_g < p_g^* < \bar{v}_n = 1 + \alpha_n\beta$.

base product purchases by some naturalites. To appreciate this Corollary, note that when the input material cost differential $c_v - c_r$ is positive, $\frac{\bar{v}_n v}{c_g}$ exceeds $\frac{v}{c_b}$, and thus a monopolist can always extract more surplus from a naturalite buying a green variant than the surplus when the same consumer purchases a base product. Consequently, $c_v > c_r$ is a sufficient condition for one of PT, NT, or UG to sustain as the monopolist's optimal strategy.

As we highlighted via Proposition 1, it is analytically tractable to attain a full characterization of what exact conditions induce each optimal strategy, yet, the interpretation of the optimality conditions yielding Γ_s^* for $\{PT, NT, UG\}$ is not straightforward as the boundaries of these surfaces are highly non-linear in problem parameters. In what follows, we derive a combination of necessary and sufficient conditions, which imply thresholds on critical parameters β, ω, and α_n for a "greener" product line (i.e., NT and UG) to sustain optimally.

Proposition 2

Define the following constant parameter combinations:

$$\begin{aligned}
\omega^{NT} &\equiv 1 - \frac{\left(1+\alpha_n\beta\right)\left(\frac{c_v - c_r - \alpha_c(c_v + k(1+\beta)^2)}{\alpha_n+\alpha_c}\right)^2}{\beta\left(\alpha_c + (\alpha_n - \alpha_c)\left(\frac{c_v - c_r - \alpha_c(c_v + k(1+\beta)^2)}{\alpha_n+\alpha_c}\right)^2\right)} \\
\alpha_n^{NT} &\equiv \frac{\sqrt{\alpha_c\beta(1-\alpha_c)(1-\alpha_c-2k(1+\beta)^2)}}{\beta} - \alpha_c \\
\beta^{NT} &\equiv \frac{\left(\sqrt{\alpha_c(1-w)(\alpha_n+\alpha_c)^2 + 12\alpha_c k\left((1-\alpha_c)c_v - \alpha_c k - c_r\right)} - (\alpha_n+\alpha_c)\sqrt{\alpha_c(1-w)}\right)^2}{36\alpha_c^2 k^2}
\end{aligned}$$

If $\alpha_c(c_v + k) < c_v - c_r < \alpha_c$ and $k < \frac{c_v(1-\alpha_c) - c_r - \alpha_c\sqrt{\alpha_c\beta(1-\omega)}}{\alpha_c(1+\beta)^2}$, then the following statements hold true:

(i) Strategy NT is optimal if and only if $\omega > \omega^{NT}$.

(ii) Strategy NT is optimal if $\alpha_n < \alpha_n^{NT}$.

(iii) Strategy NT is optimal if $\beta < \beta^{NT}$. Furthermore, β^{NT} increases with a decrease in α_n and/or in α_c.

Proposition 2 highlights that product line strategy NT is only profitable for the monopolist firm when the unit input material cost savings $c_v - c_r$ for using recycled material are high, the diseconomies-of-scope effects are not strong, i.e., low k, and the conventionals' dis-utility for using the green should not be significant, i.e., low α_c. If these conditions are not satisfied, the monopolist is better-off by appropriately pricing both the base product and the green variant to perfectly segment the market, i.e., scenario PT wherein each consumer type purchases the variant that is intended to be sold to that consumer type.

What is surprising that Proposition 2 highlights is that a greener product line (NT when compared to PT) does not sustain when there are many naturalites in the customer base, i.e., low ω, who are willing to pay a significant premium for the green variant, i.e., high α_n. This is because, in such scenarios, it is better for the monopolist to increase the price of the green variant and emphasize higher margins it may attain from these high-paying customers, especially when there are plenty of such customers. In that case, instead of trying to convince some conventionals to purchase the green variant (which yields net unit cost savings due to high $c_v - c_r$ and low k), it makes more sense for the buyer to extract more surplus from conventionals by making all of them purchase the appropriately priced base product. Also unexpected, and highlighted by Proposition 2, is that it becomes less likely for a monopolist to optimally sustain strategy NT when the green variant contains high recycled content. As β increases, the product variants become more vertically differentiated due to consumers' opposing perceptions of recycled content, thus giving the monopolist an opportunity to segment the market using two distinct product variants, while, at the same time, exploiting the increased unit cost savings of the green variant. Finally, Proposition 2 highlights that β^{NT} increases as α_c and/or α_n decreases. In other words, optimally sustaining strategy NT is more likely when the consumer' preference gap between the two variants, i.e., $\beta(\alpha_n + \alpha_c)$, is smaller, so that the green variant is not only preferred by the naturalites, but is acceptable to the conventionals if its price is right.

<u>Proposition 3</u>

Define the following constant parameter combinations:

$$
\begin{aligned}
S \equiv\ & (1 - c_v - k(1+\beta)^2 + \beta(c_v - c_r - \alpha_c))^2 - (1-\alpha_c\beta)(1 - c_v - k(1+\beta)^2)^2 \\
\omega_{\alpha_1} \equiv\ & 1 - \tfrac{S}{\alpha_c\beta(S+(1-\alpha_c\beta)\alpha_c\beta)} \\
\omega^{UG_1} \equiv\ & 1 - \tfrac{S(1+\alpha_n\beta)}{\beta^2(\alpha_n+\alpha_c)^2(1-\alpha_c\beta)+S\beta(\alpha_n+\alpha_c)} \\
\omega^{UG_2} \equiv\ & -\tfrac{(1-\alpha_c\beta)\left(\beta(c_r-c_v-\alpha_n)+c_v+k(1+\beta)^2-1\right)\left(\beta(2\alpha_c+c_r-c_v+\alpha_n)+c_v+k(1+\beta)^2-1\right)}{\beta(\alpha_c+\alpha_n)(-2(1-\alpha_c\beta)(\beta c_r+c_v(1-\beta)+k(1+\beta)^2)+(\beta c_r+(1-\beta)c_v+k(1+\beta)^2)^2+(1-\alpha_c\beta)(1+\alpha_n\beta))} \\
\alpha_n^{UG_1} \equiv\ & \tfrac{\sqrt{S}\sqrt{Sw^2+4(1-w)(1-\alpha_c\beta)^2}+Sw}{2\beta(1-w)(1-\alpha_c\beta)} - \alpha_c \\
\alpha_n^{UG_2} \equiv\ & \tfrac{1-\beta(2\alpha_c+c_r)-(1-\beta)c_v-k(1+\beta)^2}{\beta} \\
\beta^{UG_2} \equiv\ & \tfrac{c_v-c_r-2k-2\alpha_c-\alpha_n+\sqrt{(\alpha_n+c_r-c_v+2(\alpha_c+k))^2-4k(c_v+k-1)}}{2k}
\end{aligned}
$$

If $c_v - c_r > \alpha_c$ and $k < \min\{1-c_v, \frac{1}{4}(1-c_v+\sqrt{(1-\alpha_c) - \frac{(1-\omega)(\alpha_n+\alpha_c)^2}{1-\alpha_c+\omega(\alpha_n+\alpha_c)}})\}$, then the following statements hold true:

(a) If $c_v < 1 - k(1+\beta)^2$, then:

  (i) Strategy UG_{inc} is optimal if and only if $\omega > \omega^{UG_1}$.

  (ii) If $\omega > \omega_{\alpha_1}$, then strategy UG_{inc} is optimal if $\alpha_n < \alpha_n^{UG_1}$

  (iii) There exists $\beta^{UG_1} \in [0,1]$ such that strategy UG_{inc} is optimal if $\beta < \beta^{UG_1}$.

(b) If $c_v > 1 - k(1+\beta)^2$, then:

  (i) Strategy UG_{inc} is optimal if and only if $\omega > \omega^{UG_2}$.

(ii) Strategy UG_{inc} is optimal if $\alpha_n < \alpha_n^{UG_2}$

(iii) Strategy UG_{inc} is optimal if $\beta < \beta^{UG_2}$.

(iv) Strategy UG_{exc} is optimal if and only if $\omega < \omega^{UG_2}$.

Proposition 3 highlights that conditions similar to those we stated for strategy NT to be optimal are required for the monopolist to optimally maintain a "green-only" product line, i.e., strategy UG: The unit material cost differential $c_v - c_r$ must be high, the diseconomies-in-scope parameter k must be low, the number of naturilites in the market must be limited, (i.e., high ω), and the consumers' preference gap for the two variants $\beta(\alpha_n + \alpha_c)$ must remain low to make the green variant acceptable to the entire consumer base. The only distinction is that, for the monopolist to optimally exclude the base product from its product line, an even stricter set of threshold conditions must hold. For example, the unit material cost differential $c_v - c_r$ should exceed α_c for strategy UG to sustain optimally. In contrast, $\alpha_c(c_v + k) < c_v - c_r < \alpha_c$ suffices for strategy NT to be optimal. We illustrate these dynamics more clearly in Figure 1. In this figure, we observe he transitions for the monopolist firm's optimal product line strategy as either ω, α_n, or β changes. In each panel, c_v and c_r vary between 0 and 1 within the vertical and horizontal axis, respectively; $k = 0.15$ and $\alpha_c = 0.3$. (a) From left-to-right, the conventional consumer segment percentage ω increases. $(20\%, 60\%, 80\%, 95\%)$. (b) From left-to-right, the naturalites' marginal utility from recycled content α_n increases. $(5\%, 25\%, 50\%, 95\%)$. (c) From left-to-right, the green variant's recycled content fraction β increases. $(5\%, 25\%, 45\%, 75\%)$

Also of note in each panel of Figure 1 is what we characterize in Proposition 3(b), where the monopolist firm can sustain a profitable business only by focusing on a solely green product line. In such case, the virgin material cost

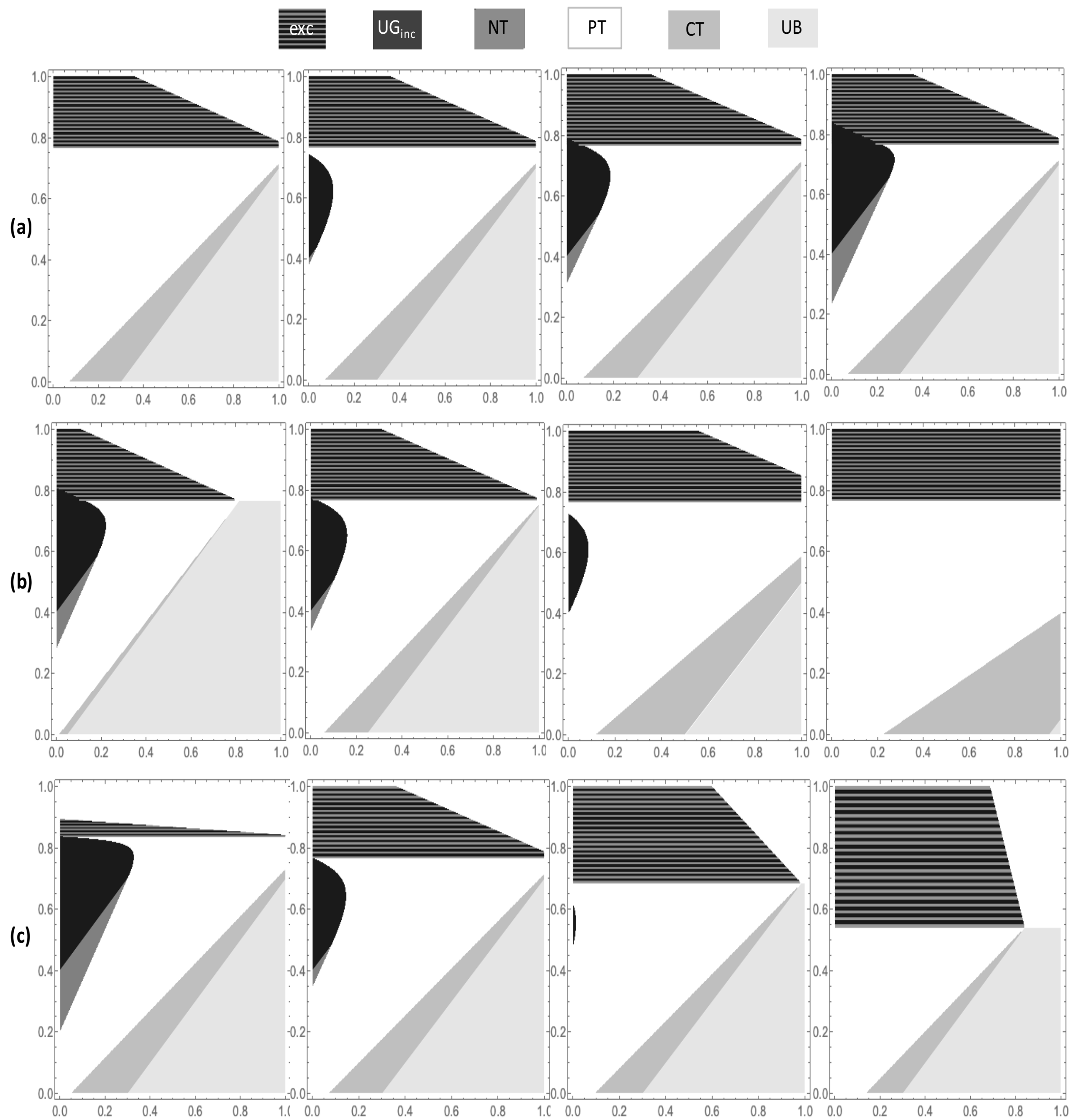


FIGURE 1. Optimal policy maps and sensitivity analysis

is prohibitively high that, the only profit opportunity rests with the sales of the green variant as it offers material cost savings while offsetting the adverse effects of diseconomies-in-scope. In such cases, the firm sells only the green variant to both consumer segments, i.e., the UG_{inc} scenario, or only to the naturalites, i.e., UG_{exc}. (We highlight the latter scenario in regions marked "exc" in Figure 1.)

Of managerial importance, note that our findings in Proposition 3 also highlight the fallacy of criticism some firms with diverse product lines receive, when consumers and non-profit organizations criticize them for having green product variants, which are not fully recyclable. Similarly, firms' reluctance to focus on green products due to not having enough naturalites in their consumer base is unfounded. We show that a environmental and traditional bottom line objectives align for a monopolist, i.e., "green only" product line is profitable, when the green variant contains limited recycled content, and thus is acceptable to "all" consumer segments, not just the naturalites. If this were not the case, then a profit-driven monopolist is better-off by segmenting the market with two distinct product variants.

When the Monopolist Can Optimize Quality

In this section, we consider the scenario where the monopolist can influence the quality of the green variant by choosing the optimal recycled content fraction β. As each consumer segment's valuation of the green variant is influenced by β, the monopolist's choice affects how vertically differentiated the product variants would be. As is typical in the operations and marketing literature, we assume that the quality decision is strategic, whereas the pricing decisions are tactical. Thus, we assume the monopolist's quality optimization precedes price optimization, yielding the following non-linear objective function:

$$\begin{aligned}\Pi(\beta) &= \textstyle\sum_s I_s(\beta)\pi_s^*(\beta) \\ &= \textstyle\sum_s I_s(\beta)\Big((p_b^*(\beta)-c_b)(D_{bc}^s(p_b^*,p_g^*)+D_{bn}^s(p_b^*,p_g^*)) \\ &+ (p_g^*(\beta)-c_g)(D_{gc}^s(p_b^*,p_g^*)+D_{gn}^s(p_b^*,p_g^*))\Big).\end{aligned} \tag{2.3}$$

We can then express the monopolist's quality optimization problem by the following mixed binary-integer non-linear program:

$$\begin{aligned}\max_{\beta} \quad & \Pi(\beta) \\ \text{subject to} \quad & 0 \leq \beta \leq \beta_{\text{max}} \\ & \textstyle\sum_s I_s(\beta) = 1 \qquad I_s(\beta) = \{0,1\}\,,\ \forall s\end{aligned} \tag{2.4}$$

In program (2.4), $I_s(\beta)$ returns 1 if optimal prices $p_b^*(\beta)$ and $p_g^*(\beta)$ for a given β yields demand scenario $s \in \{UG, NT, PT, CT, UB\}$ as defined in Table 1; and 0 otherwise. Evidently, there may be technological limits on how high β can be, which we highlight via the constraint on β. Without loss of generality, we will assume that $\beta_{\text{max}} = 1$, as program (2.4) returns $\beta^* = \beta_{\text{max}}$ only when $\Pi(\beta)$ is increasing at $\beta = \beta_{\text{max}}$.

Our analysis of the structural properties of objective function (2.3) reveals that $\Pi(\beta)$ ($\equiv \sum_s I_s(\beta)\pi_s^*(\beta)$ is not well-behaved for all $\beta \in [0,1]$. More specifically, even though $\pi_s^*(\beta)$ is concave for β values that yield demand scenario s for all s, the dynamics of $\pi_s^*(\beta)$ for two different demand scenarios do not always yield a predictable relationship for a wide range of problem parameters, which we illustrate via a numerical example in Figure 2. In this figure, we present the objective function $\Pi(\beta)$, the resulting demand scenarios, and optimal recycled content percentage β^* when $c_r \in \{0, 0.05, 0.10, 0.15, 0.20, 0.25\}$, $\alpha_c = \alpha_n = 0.2$, $c_v = 0.275$, $\omega = 0.5$, and $k = 0.05$.

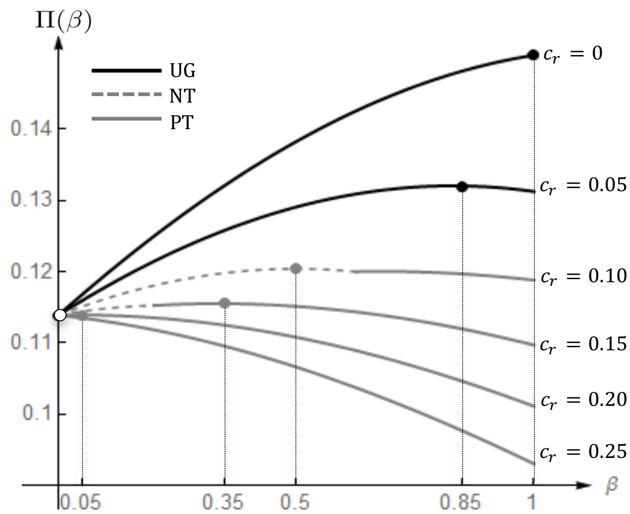


FIGURE 2. Objective function and the optimal recycled content percentage

Note in Figure 2 that $\Pi(\beta)$ is sensitive to unit material cost differential $c_v - c_r$, which we denote by Δ_c. When the recycled material is free, i.e., $c_r = 0$, the monopolist only considers demand scenario UG, for which $\Pi(\beta)$ is monotone-increasing in β, yielding $\beta^* = 1$. In contrast, when $c_r = 0.25$, unit cost differential $c_v - c_r$ is very small, and thus $\Pi(\beta)$ is monotone-decreasing in β as the monopolist utilizes strategy PT for all $\beta > 0$. Consequently, it is optimal for the monopolist to sell only the base product, i.e., $\beta^* = 0$ implying demand scenario UB. In contrast, for $c_r \in \{0.05, 0.10, 0.15, 0.20\}$ the optimal recycled fraction for the green variant is neither 0 or 1, yielding various demand scenarios UG, NT, or PT. For some parameter combinations, it is possible to exploit some structure on how $\Pi(\beta)$ transitions from one demand scenario to another. For example, note the curve with $c_r = 0.15$ in Figure 2 where $\pi^*_{NT}(\beta)$ is monotone-increasing in β. This implies that the local optimum for $\pi^*_{PT}(\beta)$ is the global optimum for $\Pi(\beta)$, i.e., $\beta^* = 0.35$. In contrast, consider the curve with $c_r = 0.10$ where the only way to find the global optimum $\beta^* = 0.50$ is to compare the local optima for $\pi^*_{NT}(\beta)$ and $\pi^*_{PT}(\beta)$.

Obviously, for parameter combinations that permit more than two demand scenarios as β changes from 0 to 1, it is even harder to detect any structure. As a result, we will refrain from fully characterizing the solution of the monopolist's quality and price optimization. Instead, we will only characterize the scenarios wherein the monopolist optimal quality choice is extreme: The cases with $\beta^* = 1 (= \beta_{\max})$, which may permit either a uniformly green product line or a targeted product line yielding demand scenario PT; and the case with $\beta^* = 0$, which imply strategy UB with the monopolist marketing only the base product. We formalize our statements for these scenarios in the next proposition:

Proposition 4

Define the following constant parameter combinations:

$$\Delta_c^{UB} \equiv -\alpha_n$$

$$\omega_\beta^{PT} \equiv \begin{cases} 1 - \frac{8k}{2\Delta_c + \alpha_n(1+\frac{c_v+k}{1+\alpha_n})} & \text{if} \quad \Delta_c < 2k \\ 1 - \frac{8k}{2\Delta_c + \alpha_n(1+\frac{4k(c_v+\Delta_c)-\Delta_c^2}{4k(1+\alpha_n)})} & \text{if} \quad 2k \leq \Delta_c \leq 4k \\ 1 - \frac{8k}{2\Delta_c + \alpha_n(1+\frac{c_r+4k}{1+\alpha_n})} & \text{if} \quad \Delta_c > 4k \end{cases}$$

$$\omega_\beta^{UG} \equiv \frac{\alpha_c}{\Delta_c - 4k}$$

Then, we have the following statements:

(i) When the unit input material cost differential Δ_c satisfies $\Delta_c < \Delta_c^{UB}$, the monopolist's optimal product line strategy is UB with $\beta^* = 0$.

(ii) Function $\pi^*_{PT}(\beta)$ is increasing in β for $\omega < \omega^{PT}_{\beta}$ when product line strategy PT is optimal.

(iii) Function $\pi^*_{UG}(\beta)$ is increasing in β for $\omega > \omega^{UG}_{\beta}$ when product line strategy UG is optimal.

Next, we provide representative examples from an extended numerical study to draw more general managerial insights. In Figure 3, we illustrate how the monopolist firm's optimal product line strategy transitions as the unit virgin-to-recycled material cost differential Δ_c changes. In other words, this figure shows the transitions for the monopolist firm's optimal product line strategy and the optimal recycled content β^* of the green variant as ω changes. In each panel, c_v (c_r) varies between 0 and 1 within the vertical (horizontal) axis; $k = 0.05$ and $\alpha_c = \alpha_n = 0.2$. (a) $\omega = 0.1$, (b) $\omega = 0.5$, (c) $\omega = 0.9$. The numbers for each (c_v, c_r) pair highlight the optimal recycled content β^* of the green variant. In cases marked by "exc," the optimal strategy is UG_{exc} with $\beta^* = 1$.

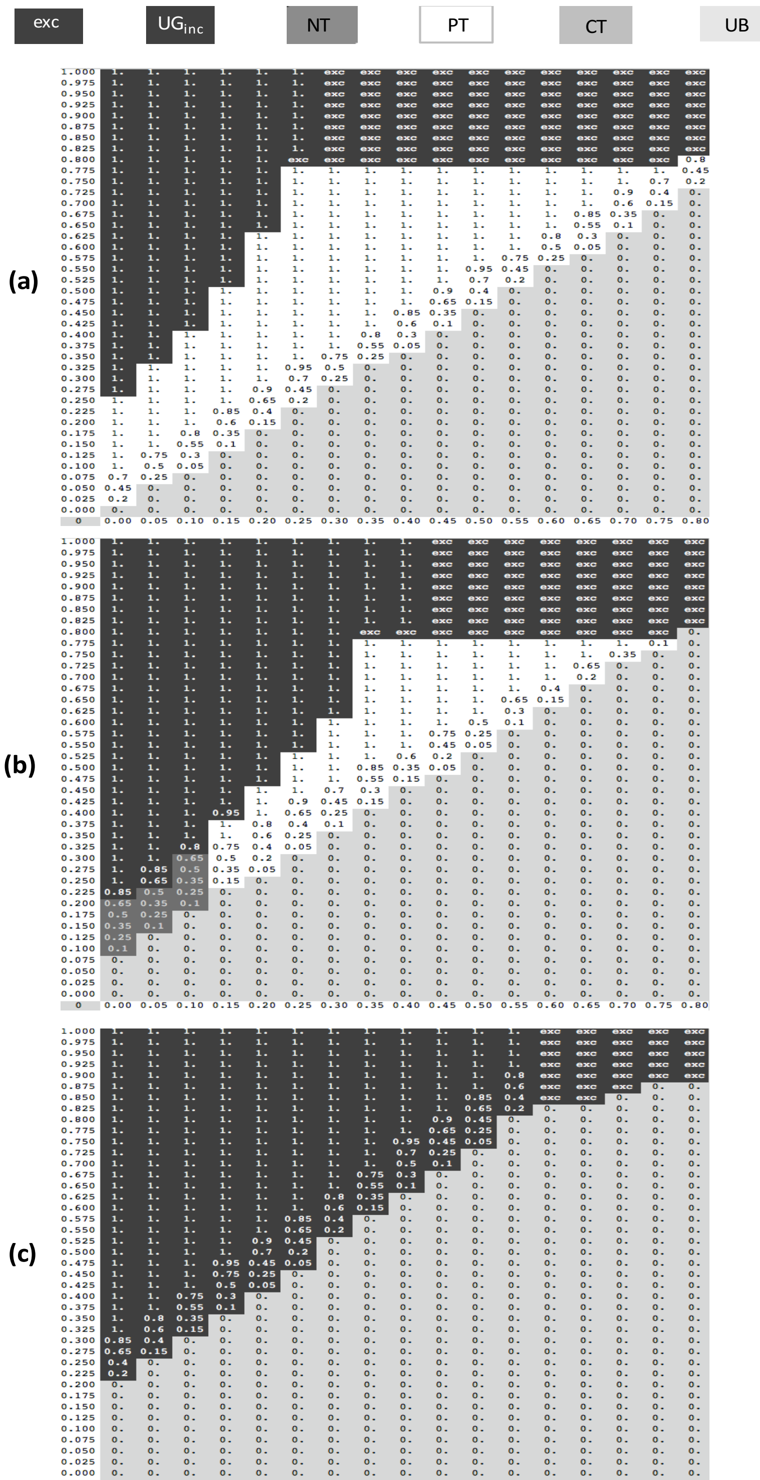


FIGURE 3. Optimal recycled content percentage

We can make several observations regarding Figure 3. As is the case in the prior section where the quality (i.e., β) was fixed, an optimal green-only product line (i.e., strategy UG) sustains when unit material cost differential Δ_c is high. Furthermore, our prior finding that strategy UG is more likely to be optimal when selling to a consumer base with fewer naturalites remains true. (This latter observation follows from region UG expanding as ω increases from panel (a) to panel (c) in Figure 3.)

The most striking difference from the results when the quality is fixed is that, a quality-optimizing monopolist should maintain a uniformly green product line with significantly high recycled content. More specifically, consider a scenario with problem parameters (e.g., high Δ_c, low k, low α_c, low α_n) that induce optimal strategy UG when the quality is fixed. As per Proposition 1(a)(iii), this must be a scenario wherein the recycled content percentage for the green variant is not high, i.e., low β. In this case, an increase in β, ceteris paribus, transitions the monopolist's optimal strategy to PT unless Δ_c is very high. The monopolist, in this case, should introduce the base product to obtain a more vertically-differentiated product line to capture the consumers' distinct valuations, which becomes increasingly diverse as β increases.

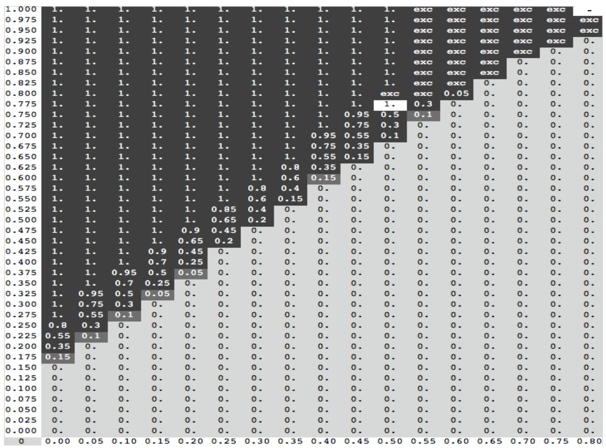
In comparison, when quality-optimization is possible, the monopolist prefers to increase β to its upper limit, which yields strategy UG if Δ_c is high enough, and strategy PT otherwise. In the former scenario, the firm maintains a uniformly green product line while charging a premium price to capture the naturalites' high willingness-to-pay while enjoying material cost savings. In the latter case, as the material cost savings are not high enough to justify a uniformly green product line, the monopolist can offer its consumer base two high-margin products, each of which uniquely targets one consumer type. In both scenarios, the green variant's recycled content is higher than it would be when the quality is fixed. Thus, we can conclude that quality optimization enables the monopolist to optimally maintain a more environmentally-conscious product line.

The implications of underestimating consumer segments' sensitivity for more recycled content

As evidenced by our findings in prior sections, the monopolist firm's ability to segment its consumer base in the most profitable way heavily depends on various cost and demand parameters. On the market side, consumers types' opposing (dis)like for how much recycled content the green variant has is of significant importance. As such, in this subsection, we assess the potential profit loss for a firm that underestimates the naturalites' marginal utility or the conventionals' marginal dis-utility from a green variant with more recycled content. We highlight a number of managerial insights by presenting findings from a numerical study, which we summarize in Figure 4. This figure has five parts, which we present within three panels. Panel (b) is the same as Figure 3(b), and will be used as a benchmark. In this problem instance we set $k = 0.05$, $\omega = 0.5$, $\alpha_n = \alpha_c = 0.2$, and vary unit material costs in virgin and recycled forms, i.e., c_v and c_r, from 0 to 1.

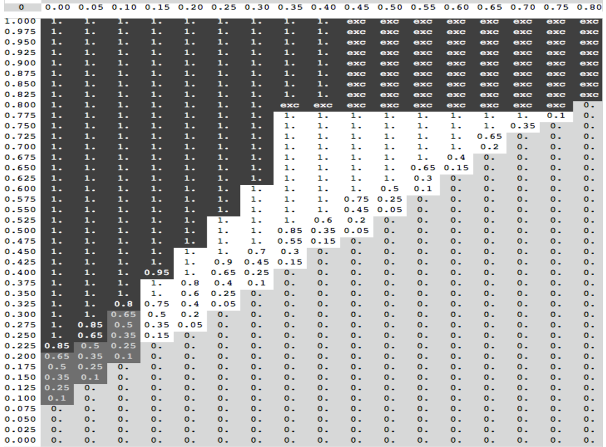
On the left-side of panel (a), we present the ensuing demand scenario realizations when the firm optimizes price and quality having incorrectly assumed $\alpha_n = 0$. On the right-side of the same panel, we highlight the firm's percentage profit loss for not choosing prices and β which would yielded the optimal demand scenario realization in panel (b). As evident in the left-side of panel (a), failing to recognize naturalites' higher willingness-to-pay for a green product variant, the firm forgoes opportunities to effectively segment its customer base by targeting each consumer type with an appropriate product, yielding only the UB demand scenario

(a)

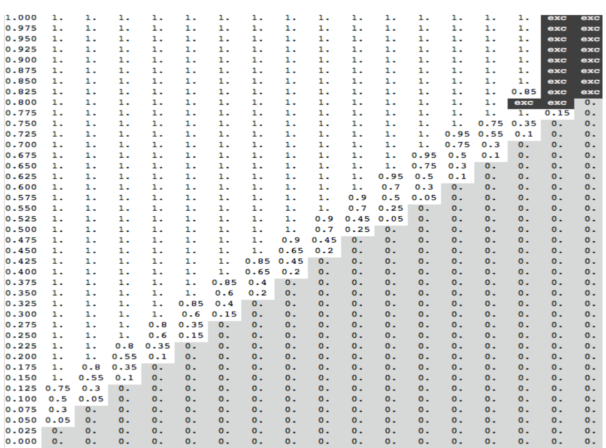


	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
1.000	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	44.44	64.00	100.00
0.975	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	44.44	64.00	**97.52**
0.950	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	44.44	64.00	92.75
0.925	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	44.44	68.67	96.25
0.900	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	44.44	**90.40**	85.00
0.875	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	**85.00**	78.40	66.25
0.850	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	32.65	73.33	61.60	40.00
0.825	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.00	3.90	24.75	37.45	19.75	25.00	**69.38**	58.33	40.00	6.25
0.800	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.52	13.51	24.75	16.00	19.75	**64.09**	55.91	40.00	13.60	0.00
0.775	0.87	1.00	1.16	1.35	1.61	1.94	2.38	3.69	13.69	24.94	15.95	**51.37**	54.27	40.36	19.00	0.15	0.00
0.750	0.87	1.00	1.16	1.35	1.61	1.94	2.38	4.20	14.22	26.16	**40.65**	49.02	41.10	23.50	2.40	0.00	0.00
0.725	0.87	1.00	1.16	1.35	1.61	1.94	2.38	5.04	15.10	28.56	39.98	41.93	27.13	6.73	0.00	0.00	0.00
0.700	0.87	1.00	1.16	1.35	1.61	1.94	2.38	6.18	17.01	**29.65**	38.56	30.06	12.79	1.02	0.00	0.00	0.00
0.675	0.87	1.00	1.16	1.35	1.61	1.94	2.38	7.62	20.24	29.64	32.46	17.96	3.46	0.00	0.00	0.00	0.00
0.650	0.87	1.00	1.16	1.35	1.61	1.94	2.38	9.31	22.09	28.73	22.02	7.06	0.47	0.00	0.00	0.00	0.00
0.625	0.87	1.00	1.16	1.35	1.61	1.94	2.38	13.00	**22.80**	25.27	11.59	2.03	0.00	0.00	0.00	0.00	0.00
0.600	0.87	1.00	1.16	1.35	1.61	1.94	4.02	15.28	22.55	16.11	4.54	0.20	0.00	0.00	0.00	0.00	0.00
0.575	0.87	1.00	1.16	1.35	1.61	1.94	8.11	16.35	19.78	7.81	1.26	0.00	0.00	0.00	0.00	0.00	0.00
0.550	0.87	1.00	1.16	1.35	1.61	1.94	10.80	**17.75**	11.72	3.09	0.07	0.00	0.00	0.00	0.00	0.00	0.00
0.525	0.87	1.00	1.16	1.35	1.61	3.63	12.27	15.53	5.60	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.500	0.87	1.00	1.16	1.35	1.61	6.50	**12.69**	8.68	2.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.475	0.87	1.00	1.16	1.35	2.32	8.14	12.20	4.14	0.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.450	0.87	1.00	1.16	1.35	3.76	**9.77**	6.64	1.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.425	0.87	1.00	1.16	2.09	5.61	9.58	3.12	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.400	0.87	1.00	1.16	2.61	**6.40**	5.20	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.375	0.87	1.00	1.48	2.84	6.51	2.38	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.350	0.87	1.00	2.51	4.53	4.11	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.325	0.87	1.33	2.68	**5.15**	1.82	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.300	0.87	1.93	2.96	3.28	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.275	0.87	2.02	**3.50**	1.39	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.250	1.49	2.84	2.72	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.225	1.97	**3.32**	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.200	2.12	2.55	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.175	**2.47**	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.150	2.40	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.125	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.100	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.075	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.050	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.025	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(b)



(c)



	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80
1.000	**96.92**	**95.79**	**94.44**	**92.78**	**90.72**	**88.10**	**84.70**	**80.13**	**73.73**	**71.67**	**65.89**	**58.58**	**49.09**	**36.28**	**18.18**	0.00	0.00
0.975	93.98	92.65	91.08	89.17	86.82	83.87	80.06	75.00	68.01	66.53	60.87	53.87	44.99	33.39	17.57	0.00	0.00
0.950	91.03	89.52	87.72	85.57	82.93	79.63	75.42	69.87	62.28	61.39	55.85	49.15	40.90	30.50	16.96	0.00	0.00
0.925	88.08	86.38	84.37	81.96	79.03	75.40	70.78	64.75	56.56	56.25	50.83	44.44	36.81	27.60	16.36	0.00	0.00
0.900	85.14	83.24	81.01	78.35	75.13	71.16	66.14	59.62	50.84	51.11	45.81	39.73	32.72	24.71	15.75	0.00	0.00
0.875	82.19	80.10	77.66	74.75	71.24	66.92	61.50	54.49	45.11	45.98	40.79	35.01	28.63	21.81	15.15	0.00	0.00
0.850	79.24	76.97	74.30	71.14	67.34	62.69	56.86	49.36	39.39	40.84	35.78	30.30	24.54	18.92	14.54	0.00	0.00
0.825	76.30	73.83	70.94	67.54	63.45	58.45	52.22	44.24	33.66	35.70	30.76	25.58	20.45	16.02	10.55	0.00	0.00
0.800	73.35	70.69	67.59	63.93	59.55	54.22	47.58	39.44	35.15	30.56	25.74	20.87	16.36	13.13	0.00	0.00	0.00
0.775	70.41	67.55	64.23	60.32	55.65	49.98	42.94	34.45	30.14	25.60	20.96	16.47	12.68	10.78	0.00	0.69	0.00
0.750	67.46	64.41	60.88	56.72	51.76	45.74	38.30	29.74	25.46	21.06	16.70	12.74	9.87	10.26	1.81	0.00	0.00
0.725	64.51	61.28	57.52	53.11	47.86	41.51	33.66	25.33	21.17	16.99	13.03	9.74	8.29	3.02	0.32	0.00	0.00
0.700	61.57	58.14	54.16	49.50	43.97	37.27	29.02	21.27	17.30	13.44	9.99	7.50	4.97	1.06	0.00	0.00	0.00
0.675	58.62	55.00	50.81	45.90	40.07	33.03	24.38	17.59	13.87	10.42	7.58	5.84	1.76	0.45	0.00	0.00	0.00
0.650	55.67	51.86	47.45	42.29	36.17	28.80	19.74	14.31	10.92	7.94	5.80	2.98	1.15	0.00	0.00	0.00	0.00
0.625	52.73	48.72	44.10	38.69	32.28	24.56	15.10	11.44	8.43	6.00	4.18	1.79	0.54	0.00	0.00	0.00	0.00
0.600	49.78	45.59	40.74	35.08	28.38	20.33	11.96	8.98	6.41	4.59	2.41	1.30	0.00	0.00	0.00	0.00	0.00
0.575	46.83	42.45	37.39	31.47	24.48	16.09	9.54	6.92	4.84	3.24	1.99	0.23	0.00	0.00	0.00	0.00	0.00
0.550	43.89	39.31	34.03	27.87	20.59	11.85	7.48	5.26	3.69	2.62	0.94	0.00	0.00	0.00	0.00	0.00	0.00
0.525	40.94	36.17	30.67	24.26	16.69	8.04	5.76	3.97	3.37	1.60	0.26	0.00	0.00	0.00	0.00	0.00	0.00
0.500	38.00	33.03	27.32	20.66	12.80	6.30	4.37	3.34	2.89	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.475	35.05	29.90	23.96	17.05	8.90	4.85	3.30	3.64	1.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.450	32.10	26.76	20.61	13.44	5.37	3.69	3.39	2.49	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.425	29.16	23.62	17.25	9.84	4.13	2.84	3.25	1.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.400	26.21	20.48	13.89	6.24	3.14	3.58	2.70	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.375	23.26	17.35	10.54	3.56	2.69	3.50	1.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.350	20.32	14.21	7.18	2.71	3.79	2.29	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.325	17.37	11.07	4.54	2.70	3.72	1.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.300	14.42	7.93	3.12	4.00	2.47	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.275	11.48	5.36	3.08	3.28	1.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.250	8.53	3.94	4.30	2.63	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.225	6.04	3.80	3.48	1.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.200	4.61	4.83	2.25	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.175	4.38	3.81	1.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.150	5.27	2.45	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.125	3.49	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.100	2.13	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.075	1.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.050	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.025	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

FIGURE 4. Potential profit loss due to underestimating α_n or α_c

realization when the input cost differential is positive-but-low or negative. In all other cases, the UG demand scenario realizes. As highlighted in the right-side of panel (a), we find that the aforementioned underestimation hurts firm profits the most when the virgin and recycled input material costs are comparable. In this case, even though a more environmentally-friendly outcome with a uniformly green product line results, firm profit suffers as the firm fails to segment the consumer base by underpricing the green variant too much, which in effect cannibalizes all demand for the base product.

In panel (c), we present the ensuing demand scenario realizations when the firm optimizes price and quality having incorrectly assumed $\alpha_c = 0$ and the resulting profit loss for the firm relative to the benchmark case highlighted in panel (b). We find that when the firm underestimates conventionals' dislike for more recycled content in the green variant, the decline in the firm's profit increases with unit virgin input material cost. In instances with such high virgin material costs case, the firm forgoes an opportunity to maintain a uniformly green product line by overpricing the green variant. The firm does so in hopes of extracting more surplus from the naturalites while satisfying conventionals' demand with the base product. As a result, each demand segment self-selects to purchase only the product variant targeting that consumer type, yielding the PT demand scenario. When this demand realization ensues not only does the firm accrue significant profit loss, but also a less environmentally-friendly demand segmentation results yielding more virgin material consumption.

Conclusion and Discussion

When firms are confronted by advocates of the environment regarding why their product offerings include no, or only a few, environmentally-friendly product variants, the typical responses cite high costs and/or lack of consumer demand due to higher prices to offset costs or some consumers' negative perception of such "green" products. As such, many firms either do not market any green variants at all, or, even when they do, they include limited number of green variants in their product lines only as complements to a wider array of less environmentally-friendly product offerings. Nevertheless, recent consumer surveys highlight an ever increasing fraction of consumers who favor green consumption to the extent that some of these consumers are willing to pay a premium for green product variants. In addition, reduced collection costs and improvements in recycling/reuse technologies provide firms with the opportunity to acquire recycled materials at a lower cost, thus encouraging less virgin material use. As a result, even firms with green product offerings are accused of "green-washing," because environmentally-conscious consumers believe that green products should include more—preferably 100%—recycled/reused content. This gives rise to the managerial questions regarding *whether it is feasible for firms to profitably maintain a product line with only green product variants with high recycled/reused content*, and, if so, *what economic factors related to product cost and consumer demand induce such environmentally-friendly outcomes*.

Seeking explanations regarding the aforementioned managerial issues, we studied product line design (i.e., both price and quality optimization) problem of a monopolist selling one or both of two product variants, i.e., a base product and a green variant with recycled/reused content, to a consumer base with two distinct customer types, i.e., conventionals and naturalites. Using an endogenous demand model that captures these two customer types' dislike and extra utility, respectively, from consuming the green product variant, and accounting for cost implications for a firm of expanding its product line to include green variants, we characterize optimal strategies for scenarios wherein the firm can or cannot influence product quality. In the former case, the firm can be a manufacturer, which also sells directly to consumers, whereas in the latter case, the firm can be a retailer reselling products with set quality levels.

Our findings indicate that some passionately-advocated product and demand characteristics such as green product variants with high recycled content, consumer bases with more environmentally-conscious customers, and a higher willingness-to-pay by such customers for green product variants are barriers preventing firms from maintaining uniformly green product lines. This is because when the aforementioned conditions hold, the firm is better-off targeting each unique customer type with a product that is positioned more closely to that customer type. We also find that firms with the ability to influence quality (as measured in our model by the amount of recycled/reused content of the green variant) can profitably maintain a product line with only a maximally green variant, e.g., one with fully recycled/reused content, when the consumer base has limited amount of naturalites who can derive low extra-utility from green variants. This finding has the managerial implication that manufacturers of green product variants with design capabilities are more likely to profitably maintain a maximally green product line than firms that only manufacture or resell. The latter set of firms should offer its customers a more diverse product line unless the green variant is not too vertically-differentiated from the base product.

Our analysis also highlights that a firm's profitability significantly hinges on its understanding of the (dis)utility its customers associate with green product variants. We show, via numerical examples, that underestimating the naturalites' (conventionals') marginal utility (dis-utility) for a green variant with more recycled/reused content yield unintended demand segmentation consequences with adverse implications for both firm profits and the environment.

Bridge to Next Chapter

In this chapter, we looked at the role of consumer's contrasting perception on quality and how that affects a firm's decision on what product(s) to offer and at what quality and price points to ultimately increase profits. Thus the optimal prices and marketing strategy are a direct function of consumer's viewpoint of (green) quality. In next chapter, we keep the quality fixed and instead study the pricing problem of a firm from a different angle: *timing*. Specifically, we analyze two pricing tactics that differ from each other only in timing of offering the rewards and we investigate how rational customers react to the time of receiving a discount and how that influences the ultimate profitability.

CHAPTER III

DELAYED VS. IMMEDIATE PRICE DISCOUNTS

This work was submitted to the journal of Management Science with co-authorship of Dr. Michael Pangburn.

Introduction

Retail promotion tactics come in many forms, including examples such as coupons, quantity discounts, and bundling. Promotions typically relate not only to *what* is purchased, but also *when*. Regarding the timing of discounts, the most common practice is to offer limited-duration sales, or incremental markdown prices over time (e.g., for clearance items). Another promotion scheme that we see in practice is to give consumers a discount that is not limited to specific products and times, but rather is a long-term promotion based on consumers' spending across all products. Such so-called "rewards programs" have become prominent in the retail sector. The primary differentiating characteristics of such programs is that they are generally ongoing and relate to spending across all products rather than specific on-sale products. In practice, most rewards programs are implemented such that the calculated discounts are not redeemable immediately, but rather only against a future purchase. For example, REI offers a 10% rewards rate, but the earned credit cannot be redeemed until the following year. In contrast with such *delayed* discounts, some rewards programs offer *instantaneous* discounts, such as the Target REDCard rewards card, which gives shoppers a 5% immediate discount off their current spending, irrespective of specifics regarding item prices and sales. The fact that, in practice, most spending-level based discounts are of the delayed type begs the question: *why*? Addressing this question is our focus. One possible explanation is that consumers may be irrational and thus feel locked in once having earning credit, or ascribe value to credit they may forget to redeem. In contrast, in this paper, we seek to investigate ongoing spending-level based discounts while assuming rational consumers, and determine under what conditions delaying such discounts may be preferable.

Delayed credit is often referred to as "points" which have some dollar-value equivalence. For example, Jamba Juice customers earn one "point" per dollar of spending, where 35 points yields a $3 discount, thus reflecting an approximate 10% delayed discount. Although there are nuanced varieties of points programs (e.g., points becoming active at certain thresholds, expiring at a particular time, or applying only in trade towards selected products and services), we consider settings where customers earn credit at some rate based on their spending, with that credit being available either now (instantaneous discounts) or later (delayed discounts). We will explore whether there may exist a possible motivation, rooted in rational consumer behavior, for offering delayed discounts.

From an individual consumer's perspective, receiving a certain discount in the future is naturally less desirable than receiving it now, due to the time value of money. However, for the same reason, the firm may be willing to increase the discount when it is delayed. If consumers rationally assess the (lower) value of delayed credit, which in turn may induce the firm to set a higher discount percentage, it is unclear whether there is a net advantage to be gained by delaying the discounts. Yet, the tactic remains popular in practice. An intriguing potential source of value from delayed discounts stems from the (endogenous) flexibility of consumers' inter-purchase intervals. For example, with delayed credit, customers who opt to wait for sales will not only earn less credit, but the value of that credit will be further reduced (via time discounting) due to the time delays associated with waiting for sales. Thus, with delayed credit specifically, different customers may face distinct net prices at the same point in time, depending on their purchase history—yielding a form of behavior-based price discrimination.

As the above discussion highlights, delayed discounts can yield two interesting effects: (i) higher initial net prices (both for the firm and consumers), and (ii) consumer behavior-dependent price levels. The latter of these two effects is relevant in particular when consumers' spending varies over time. To isolate the impact of the first effect and establish a baseline case, we will first analyze a scenario in which consumers have a consistent level of spending with the retailer over time. We subsequently consider the impact of fluctuating spending over time, which could arise for example if the firm offers sporadic sales.

If spending is consistent over time, we will establish that neither the delayed discount (DD) nor instantaneous discount (ID) policy has an advantage in extracting profit from rational consumers. More specifically, we show that for any particular choice of instantaneous discount, there exists a corresponding upwardly-scaled delayed discount that yields equivalent performance. We show this is true whether the firm targets each customer with the best possible (i.e., customized) discount percentage, or, as would typically be the case in practice, the firm sets a single discount percentage which applies to a heterogeneous market.

We subsequently consider scenarios in which consumer spending fluctuates between high and low levels, which we refer to as the *regular* and *reduced* levels. Given such spending fluctuations, the DD policy expands the number of pertinent net price states from two to six, and three optimal customer segments endogenously emerge: (i) sales shoppers, (ii) regular shoppers, and (iii) transition shoppers (i.e., customers who transition from sales- to regular-shopping). With instantaneous discounts, only the first two of these optimal behaviors apply. We then prove that if a firm can target each consumer with their optimal discount percentage, then the DD and ID policies' profits are equivalent (as was true with consistent spending) except for the transition segment, from which the DD policy can extract greater profit. Given that the size of the transition segment may be small or zero, we can conclude that *if* the discount percentage can be optimally personalized (i.e., targeted) to individual consumers, there is limited or no incremental value to be gain by delaying discounts. However, such personalized pricing (a la first degree price discrimination) may not be feasible in practice.

The most applicable case in practice is therefore a setting in which consumers are heterogeneous, with varied spending, and the firm cannot personalize the discount level to each. And, interestingly, we will show that it is in precisely such settings that delayed discounts provide greatest value. We prove not only is the firm's optimal discount level higher when delayed, but also that the higher (delayed) discount level enables the firm to extract higher aggregate profit from the heterogeneous market. We also use a series of numeric experiments to illustrate the extent to which the optimal discount percentage and resulting profits increase under the DD regime. Interestingly, we find that in most cases the DD policy simultaneously increases both profit and aggregate consumer surplus.

We also assess whether the DD policy's profits are robust to changes in the firm's choice of the discount percentage. We show that even if the discount percentage is set suboptimally, delayed discounts continue to yield a profit advantage in most instances. Finally, we consider whether our key results hold if we change from stochastic to deterministic spending variations. Analyzing a particular deterministic pattern of prices, we arrive at the same findings: delayed discounts optimally yield both a higher discount level and profit, if the delayed discount applies to a heterogeneous market with varied spending.

Literature Review

As defined by Caillaud and De Nijs (2014), "Behavior-based price discrimination (BBPD) is a very simple form of price discrimination that consists of offering different prices to different customers according to their past purchase history." An overview of the BBPD literature is presented by Fudenberg and Villas-Boas (2006). In this literature, some papers study equilibrium pricing strategies in a duopoly context, while others analyze pricing for a single firm—as we do in this paper. Delayed price discounts imply a form of BBPD because the net price paid in a given period by a customer depends on the amount of their credit, which in turns depends on purchase history.

The primary focus of the BBPD literature is on stratifying consumers based on their initial purchase choices, and then adjusting prices accordingly. The BBPD literature has focused on two-periods models spanning both monopoly (Acquisti and Varian (2005), Conitzer et al. (2012), Gandomi and Zolfaghari (2013), and Ching-Jen (2014)) and duopoly (Zhang et al. (2000), Kim et al. (2001), Pazgal and Soberman (2008), Chen and Zhang (2009), Esteves (2010), Zhang (2011), and Mehra et al. (2012)) settings. In the two-period duopoly context, Singh et al. (2008) observe numerically that a firm offering a second period discount for returning customers can coexist in equilibrium with a firm that does not. Villas-Boas (2004) and Caillaud and De Nijs (2014) consider an infinite horizon and assume that each customer purchases at most twice. In contrast, to assess the value of delayed versus instantaneous discounts, we must allow for the possibility of more than a single follow-up purchase, because the value of delayed discounts accrue over an extended purchase history with the firm.

A common assumption in the BBPD literature is that a firm can charge different customers different prices for the same product (Aydin and Ziya (2009)), even in the same period. Personalized pricing (i.e., first degree price discrimination) is problematic in practice because a firm risks alienating those consumers who are targeted with higher prices. If a firm attempts to give discounts specifically to new customers, for example, then past consumers may effectively mask their identity and thus appear to be new visitors (Acquisti and Varian (2005) and Conitzer et al. (2012)). The delayed discount tactic we analyze in this paper does not suffer from this consumer anonymization concern, because consumers benefit from establishing their purchase history. And, although the resulting net prices are purchase-history dependent, the same publicly posted prices apply uniformly to all customers, which addresses fairness concerns that relate to first degree price discrimination.

Although the existing literature contrasting instantaneous with delayed discounts is quite limited, Chen et al. (2005) have shown, assuming a consumers' utility changes post-purchase, that post-sale rebates can yield higher profits than coupons. To the extent that a rebate may be viewed as a form of delayed discount, with coupons being an instantaneous discount, our analytic results provide further evidence of the value from delay. We establish the comparison, however, without limiting consumers to a single purchase, and our findings do not require the post-purchase change in consumer utility. Zhang et al. (2000) categorize coupons and rebates as "front-loaded" versus "rear-loaded" discounts, and use empirical evidence to support that up-front discounts tend to increase sales, yet, "from a profitability perspective, rear-loaded promotions may be better than front-loaded promotions." Interestingly, the theoretical results stemming from our analytic model also lend support to these two empirically-motivated points, i.e., that delaying discounts has the potential to: (i) lower sales, while (ii) increasing profit. Moreover, our results support these findings for any exogenous (high and low) values for consumers' varied spending levels (or, equivalently, for a single-product firm, *any* given high/low price levels that define the firm regular/sale prices).

Delayed and Instantaneous Rewards with Static Pricing

Consider a market of consumers who may revisit a firm across successive time periods to make a series of purchases of its service (e.g., lunch or dinner at a restaurant) or product (e.g. a perishable good that a customer might repurchase). In any time period t that a purchase occurs, the customer gains some utility value v. We assume each consumer will make at most one purchase (i.e., one unit) in any given period. Let R denote the full retail (pre-discount) price for the firm's service or product. We consider R to be exogenous (e.g., MSRP), but ultimately the prices paid by customers will be endogenous, as the retailer sets its applicable discount.

Let $x_t \in \{0, 1\}$ denote a consumer's binary purchase decision in period t. The (endogenous) net purchase price is R minus any applicable discount. We express the firm's variable cost parameter c as a fraction (WLOG) of the firm's regular price, yielding a unit cost of cR. Similarly, we express the firm's discount level, which we denote as α, as a fraction (again, WLOG) of the price R. Therefore, a customer receives a discount equal to αR when making a purchase in the amount of R. We will often refer to the discount as a "spending reward" rather than price discount, because we contrast the alternatives of simply applying αR immediately with the alternative of giving αR in credit to apply against a future purchase. We refer to these alternatives, which are the focus of our study, as the delayed and instantaneous discounting policies, defined in Table 1. In practice, some retailers base delayed rewards on post-discount spending (i.e., $\alpha(R - \alpha R) = (\alpha - \alpha^2)R$, whereas others (e.g., Orbitz) use pre-discount spending (i.e., simply αR). Given that α is small, these two are nearly equivalent, and so we apply the latter. Under

TABLE 3. Delayed versus instantaneous discount policies

Discounting policy	Notation	Discount form
Instantaneous	ID	Immediate discount $\alpha_I R$ applied to customer's current purchase.
Delayed	DD	"Reward" (credit) $\alpha_D R$ applied to customer's next purchase.

the immediate discounts (ID) policy a customer enjoys an α-percent discount on each purchase, whereas under the delayed discounts (DD) policy a customer gains α-percent of their current purchase amount in credit towards their next purchase as is not uncommon in practice, the firm automatically logs and applies the credit). For example, with a fixed discount percentage $\alpha = 10\%$, an $R = \$150$ purchase yields \$15 in credit, where under ID that credit applies immediately, but under DD it applies to the consumer's *next* purchase—i.e., a subsequent purchase is required to redeem credit. For this baseline model with consistent spending, we will prove that equivalent profits and consumer surplus are achievable, irrespective of whether the consumer market is homogeneous or heterogeneous, provided the firm offers a suitably larger delayed discount percentage.

Optimal consumer shopping behaviors

Tackling the DD policy first, consider a representative consumer with valuation v corresponding to per-period spending level R. Such a customer realizes a net surplus of $v - R$ initially, earning credit in the amount of $\alpha_D R$, where we let α_D denote the firm's choice of (delayed) discount percentage. That discount will apply to any subsequent purchase, which will thus yield an associated net surplus of $v - R + \alpha_D R$. Given the two feasible credit states ($\alpha_D R$ and 0) and two actions (buy or not) at each, there are four potential shopping behaviors. Two of these behaviors are that a customer may be a "regular shopper" (i.e., $x_t = 1 \; \forall$t) or not shop at all ($x_t = 0 \; \forall$t). The other two potential behaviors are, respectively, suboptimal and infeasible: buying only when having zero credit, or only when having credit (this is infeasible because credit is earned via buying).

For the DD policy, the following two simultaneous equations define the net present value $V(\alpha)$ of surplus for a regular shopper who experiences a discount level of α. We denote the time discount factor as $\beta \in (0, 1)$, and $V(0)$ expresses a customer's surplus NPV from the time of initial purchase, at which point there is no earned credit.

$$\left.\begin{aligned} V(0) &= v - R + \beta V(\alpha_D) \\ V(\alpha_D) &= v - R + \alpha_D R + \beta V(\alpha_D) \end{aligned}\right\} \Rightarrow V_D(\alpha_D) \equiv V(0) = \frac{v - R(1 - \alpha_D \beta)}{1 - \beta} \quad (3.1)$$

As shown at right within (3.1), we denote the consumer surplus NPV as $V_D(\alpha_D)$. Similarly, under the ID policy, with its (potentially) distinct discount level α_I, we let $V_I(\alpha_I)$ denote the corresponding consumer surplus NPV. For the ID policy the α_I discount applies to each purchase, yielding the following result.

$$V_I(\alpha_I) = v - R + \alpha_I R + \beta V_I(\alpha_I) \Rightarrow V_I(\alpha_I) = \frac{v - R(1 - \alpha_I)}{1 - \beta} \quad (3.2)$$

The assumption that "regular shopping" corresponds to purchase in *every* period, as opposed to sporadically, is WLOG. More specifically, the following lemma proves that to accommodate sporadic periods of shopping inactivity (or disinterest, e.g., $v = 0$), we need only downwardly adjust the time discount factor accordingly.

Lemma 3

If a consumer has probability $\lambda < 1$ of visiting the firm in a period, then to account for the possibility of inactivity periods the discount factor β must be adjusted as $\hat{\beta} \equiv \beta\lambda/(1-(1-\lambda)\beta)$.

For all proofs of propositions and lemmas, see Appendix B. Although we will continue to use the terminology *regular shopping*, we can simply set $\beta = \hat{\beta}$ (decreasing in $1-\lambda$) to allow for sporadic periods in which regular shoppers are unavailable to shop.

Profit comparison for the DD and ID policies

We now wish to assess whether instituting a delayed discount on consumer spending should enable a firm to extract greater value than the more straightforward alternative of offering an instantaneous discount. For a representative customer and discount level α, $v - R + \alpha R$ expresses the surplus resulting from a purchase, whereas the firm's corresponding profit gain is $R - cR - \alpha R$. Analyzing profits for the DD and ID policies results in two simultaneous recursive equations for the DD policy, as in (3.1), and one for the ID case, as in (3.2) above. Letting $\pi_D(\alpha_D)$ and $\pi_I(\alpha_I)$ denote the respective profit NPVs for the DD and ID policies, we thus derive the DD and ID policies' profit NPVs $\pi_D(\alpha_D) = R(1-\alpha_D\beta - c)/(1-\beta)$ and $\pi_I(\alpha_I) = R(1-\alpha_I - c)/(1-\beta)$. We next establish an equivalence between these profits.

Proposition 5

When consumer spending is consistent over time, delayed and instantaneous discounts yield equivalent value to both the firm and consumers: $V_D(\alpha_D) = V_I(\alpha_I)$ and $\pi_D(\alpha_D) = \pi_I(\alpha_I)$, if (and only if) the firm sets $\alpha_D = \alpha_I/\beta$.

Proposition 5 proves that both a (rational) consumer and firm are indifferent between an immediate discount percentage α_I and a correspondingly larger delayed discount $\alpha_D = \alpha_I/\beta$. We assume equal time discounting for the firm and consumer, because if consumers were relatively insensitive to time delays (i.e., had higher β than the firm), then naturally delayed discounts would be advantageous. Given that α_I is typically much less than one, and β is near one, we generally have $\alpha_D < 1$ in practice, although it is not a requirement. (For example, $\alpha_D > 1$ is akin to a bank giving back principal plus interest on loaned money, which is viable only given a sufficiently low time discount factor and firm costs.)

Although Proposition 5 was derived for a representative consumer with valuation v, its equivalence result extends naturally to a setting in which consumers are heterogeneous in v, such as, without loss of generality, $v \in [0, 1]$. Given heterogeneity, consumers will self-select (optimally) to become regular shoppers if and only if $V_D(\alpha_D) \geq 0$ and $V_I(\alpha_I) \geq 0$, which implies valuations satisfying $v \geq R(1 - \alpha_D\beta) = v_D(\alpha_D)$ and $v \geq R(1 - \alpha_I) = v_I(\alpha_I)$, respectively. Leveraging $\alpha_D = \alpha_I/\beta$ from Proposition 5, we see that the resulting consumer purchasing thresholds $v_D(\alpha_D)$ and $v_I(\alpha_I)$ coincide. As the following proposition shows, the aggregate profits $(\pi_D(\alpha_D)[1 - v_D(\alpha_D)]$ and $\pi_I(\alpha_I)[1 - v_I(\alpha_I)])$ also match.

<u>Proposition 6</u>

Given static pricing, the correspondence $\alpha_D = \alpha_I/\beta$ also implies equivalent profits for a market of heterogeneous consumers.

Taken together, the above two propositions show that irrespective of whether the consumer market is homogeneous or heterogeneous, if consumers are forward looking and have consistent spending over time, then delayed and instantaneous discounts achieve equal performance. Note that the mapping between α_I and α_D holds for arbitrary values (i.e., $\alpha_D = \alpha_I/\beta$) and therefore also applies at optimal values of both. Similarly, we need not optimize over the (MSRP) price level R, as this comparison of instantaneous and delayed discounts holds generally for any R, not only at some specific (e.g., optimized) value. Intuitively, their equivalence holds because delaying discounts yields higher initial profits by (optimally) luring consumers with the promise of subsequent higher discounts—which in turn reduce later net revenues. However, in the next section, we will show that the two policies are *not* equivalent when individual consumers' spending fluctuates in time.

Delayed and Instantaneous Rewards Given Sporadic Sales

In this section we consider a generalized setting that exemplifies practical settings in which the firm's prices and thus consumers' spending varies over time. In practice, spending per customer may fluctuate due to having consumers with varying service needs (e.g., visiting a restaurant for both lunch and dinner over time), or the same service or product with varying high/low (sale) prices over time. We consider the latter setting, denoting the firm's sale frequency as $1-\gamma$, and the regular and sale prices as R and r, respectively. Thus, in period t, the price P_t is as follows.

$$P_t = \begin{cases} R & \textit{with prob.} \quad \gamma \\ r & \textit{with prob.} \quad 1-\gamma. \end{cases} \tag{3.3}$$

While we take the policy of fluctuating between regular and sale prices as given, motivated by practice, such a policy also finds support in the literature (Cachon and Feldman 2015). Moreover, we later show in section 6 that the insights we next derive for a stochastic spending sequence are consistent with those stemming from a deterministic sequence. As in the prior section, our goal is to establish a comparison of delayed versus instantaneous discounts not only for particular (e.g., optimized) values of R or r, but rather more generally for *any* R and sale price $r (< R)$.

We denote the realization of P_t at time t as p_t, so p_t is the posted price (i.e., pre-discount) that is in effect in period t. Naturally, if the firm had the flexibility to optimize its prices $\{p_t\}_{t=1}^{\infty}$, then there would be no purpose for further discounts. Therefore, we take the "posted prices" R and r as given, and then apply either the delayed or instantaneous discount tactic. The firm optimizes the corresponding discount level (α_D^* or α_I^*), thus making the final prices that consumers pay over time endogenous.

In each period, consumers face the choice of deciding whether to purchase at R or r (depending on whether a sale is in effect), or to not purchase. If sales are sufficiently frequent (i.e., with low enough γ), a customer may strategically wait for sales whenever the regular price R is in effect. Consistent with the prior section, and as may apply when purchasing a service or perishable product which cannot be stockpiled, we assume a consumer's purchase decision is binary in each period, $x_t \in \{0, 1\}$. The three potential per-period spending levels are thus R, r, or zero. Consistent with the assumption that the reduced spending level r reflects a price reduction rather than the firm lowering its service- or product-quality, we assume that each consumer's type $v \in [0, 1]$ is stable over time and the firm's variable cost is cR. (Assuming instead that the firm's variable cost varies between cR and cr would add complexity without additional insight.)

Under the DD policy, if k_t denotes the spending-reward discount applied to a purchase at time t, then $k_1 = 0$ and each subsequent k_t stems from period $t - 1$. For the ID policy, the discount k_t at time t is simply αp_t. For a representative consumer with purchase decisions $\{x_t\}_{t=1}^{\infty}$, the evolution of discounts for the DD and ID policies proceed, for $t = 1, 2, 3...$, as follows.

$$k_{t+1} = \begin{cases} k_t & x_t = 0 \\ \alpha p_t & x_t = 1 \end{cases} \text{ for DD} \quad (3.4) \qquad k_t = \begin{cases} 0 & x_t = 0 \\ \alpha p_t & x_t = 1 \end{cases} \text{ for ID} \quad (3.5)$$

Conveniently, using the appropriate definition for k_t (delayed versus instantaneous) from (3.4) or (3.5), the following formulation defines the firm's decision problem, which is to determine the optimal corresponding discount level α—i.e., α_D^* and α_I^*, respectively.

$$\max_{0 \leq \alpha \leq 1} E_{P_t} \ \pi(\alpha, k_t)$$

where

$$\pi(\alpha, k_t) = x_t(p_t - k_t - cR) + \beta E_{P_t} \pi(\alpha, k_{t+1}) \quad (3.6)$$

$$s.t. \ \{x_t\}_{t=1}^{\infty} = \operatorname*{argmax}_{x_t \in \{0,1\}} V(k_t) = x_t(v - p_t + k_t) + \beta E_{P_t} V(k_{t+1}) \quad (3.7)$$

This formulation extends our analyses from the prior section to accommodate sporadic sales. The firm thus maximizes the expected profit (3.6) it can extract from a representative consumer with type v—we later extend to heterogeneous consumer types—who makes purchase decisions in each period to maximize future expected surplus (3.7). Although we denote the consumer's initial arrival period as time $t = 1$ for convenience, the consumer could arrive in any arbitrary period without impacting our subsequent analysis. If multiple consumers arrive across distinct time periods, then this formulation reflects the optimal per-customer NPV the firm can extract from each, discounted back to their respective arrival times.

To determine whether delayed discounts can benefit the firm, we first must solve the consumer subproblem (3.7), which requires analyzing the potentially distinct optimal shopping behaviors that result from the DD and ID credit evolutions (3.4) and (3.5). In the next three subsections, we: (i) analyze consumers' optimal shopping behaviors, (ii) understand how consumers self-select between those behaviors based on their specific type v, and (iii) identify the firm's corresponding optimal choice of α. Later, in Section 3.5, we will leverage these analyses to consider the firm's optimal α decision for a market of *heterogeneous consumers* with variable spending, where the compelling benefits of delayed discounting ultimately emerge.

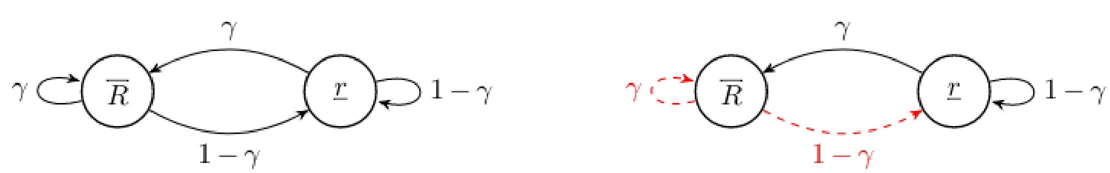
Net price states and shopping behaviors

An important implication of delaying discounts is that a greater number of net-price states result. This implication holds the potential to enhance market segmentation—as we will show. Given two price levels (R and r) in the current period, coupled with three possible credit levels (corresponding to high, low, or zero prior spending), *six* distinct net price states appear under DD, as detailed in Table 4. In contrast, the only two net price states in Table 4 that occur under ID are $\overline{R}$ and $\underline{r}$.

TABLE 4. The list of net spending states

Notation	Effective price (spending)	Condition
$\underline{\underline{R}}$	R	Zero discount, regular price
$\underline{\underline{r}}$	r	Zero discount, reduced price
$\overline{r}$	$r - \alpha R$	High discount, reduced price
$\underline{r}$	$r - \alpha r$	Low discount, reduced price
$\overline{R}$	$R - \alpha R$	High discount, regular price
$\underline{R}$	$R - \alpha r$	Low discount, regular price

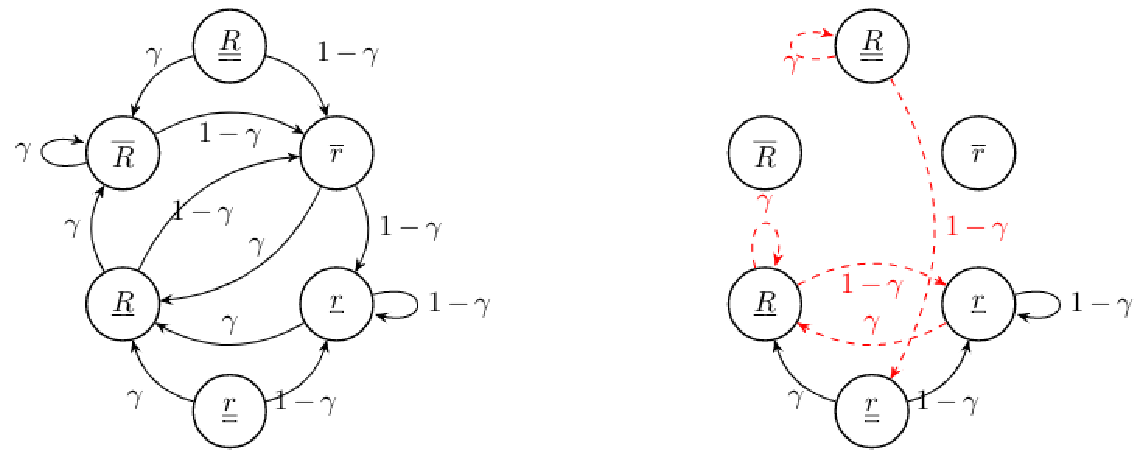
Given the ID and DD policies' respective two and six feasible net-spending levels, a variety of feasible shopping behaviors can arise. Some are clearly suboptimal, such as the behavior of shopping at regular but not at sale prices. Two intuitive behaviors that we will establish as optimal at low and high ranges for the consumer-type parameter v are *sales shopping* and *regular shopping*, respectively. The former corresponds to shoppers who optimally only purchase at price $r(< R)$; the latter corresponds to a consumer who is also willing to shop at price R. These two fundamental behaviors correspond to the Markov chains in Figures 5 and 6 for the ID and DD policies, respectively. In both figures, we use dashed rather than solid transition arrows from non-purchasing states. In Figure 5(b), dashed transition arrows stem from $\overline{R}$, highlighting that sales shoppers choose (optimally) to not purchase until a sale period occurs; similarly, dashed arrows exit states $\underline{R}$ and $\underline{\underline{R}}$ in Figure 6(b). Figures 6(a) and 6(b) also highlight the transient nature of states $\underline{\underline{R}}$ and $\underline{\underline{r}}$—after a consumer makes a purchase those zero-credit states no longer arise. We also see in Figure 6(b) that the high-credit states $\overline{R}$ and $\overline{r}$ never occur for sales shoppers.



(a) Regular shopping (b) Sales shopping

FIGURE 5. Purchase state evolution under ID

As these Markov chains imply, there are a number of potential shopping behaviors beyond the aforementioned sales- and regular-shopping behaviors. In general, given that a fully specified shopping behavior must prescribe an $x_t \in \{0, 1\}$ purchase decision in each net price state, there are 2^n possible shopping behaviors,



(a) Regular shopping (b) Sales shopping

FIGURE 6. Purchase state evolution under DD

where n is the number of such states. Thus, $2^2 = 4$ behaviors are possible for ID, versus $2^6 = 64$ for DD. The sales- and regular-shopping behaviors represent two of these many options. If we were to model two distinct sales levels (e.g., normal versus clearance sales), then twelve net price states would result under DD, implying 2^{12} possible behaviors—albeit, the overwhelming majority of which would be suboptimal.

Under ID, the 2^2 feasible behaviors are: (i) buy at neither $\overline{r}$ nor $\underline{R}$, (ii) buy at both $\overline{r}$ and $\underline{R}$ (regular shopping), (iii) buy at $\overline{r}$ but not $\underline{R}$ (sales shopping), and (iv) buy at $\overline{R}$ but not $\underline{r}$. It is clear that the last of these (buying only in the higher price state) is suboptimal. Thus, aside from the non-buying option (i) which we do not refer to as a buying behavior, we see that the only two non-dominated behaviors are those that correspond to sales- and regular-shopping. For ID, we denote sales-shopping consumers as the segment $\mathcal{S}_I$, and regular-shopping consumers as the segment $\mathfrak{R}_I$.

Under DD, we next establish that beyond the aforementioned sales- and regular-shopping behaviors, a third optimal buying behavior emerges. This third behavior corresponds to consumers who initially wait for a sale but subsequently will also purchase at price R, using their earned credit. We refer to this as *transition-shopping* behavior, because the customer transitions after their first purchase from sales-shopping to regular-shopping. For DD, we denote sales-, regular-, and transition-shopping consumers as the segments $\mathcal{S}_D$, $\mathfrak{R}_D$, and $\mathcal{T}_D$. The following proposition formalizes this result.

Proposition 7

With delayed discounts, in addition to the regular- and sales-shopping behaviors (yielding segments $\mathcal{S}_D$ and $\mathfrak{R}_D$), a third optimal behavior emerges: (segment $\mathcal{T}_D$) consumers who transition after their initial purchase from sales- to regular-shopping.

Thus, we find that whereas two shopping segments pertain to the ID policy, three segments can (optimally) emerge under delayed discounts. We next consider how a given consumer of type v optimally self-selects between these segmentation options.

Consumer segmentation and self-selection

As highlighted by the above discussion and the Markov chains in Figure 6, characterizing a shopping behavior under delayed discounts requires specifying a consumer's purchase choice (x_t) for the full set of six states in Table 4. So, we introduce the notation x_{kp} to denote a consumer's decision corresponding to the credit-price pair (k, p), where $k \in \{0, \alpha r, \alpha R\}$ and $p \in \{r, R\}$. Employing the x_{kp} notation, we express the consumer expected surplus NPV (3.7) as:

$$V(k_t) = x_{kp}(v - p_t + k_t) + \beta E_{P_t} V(k_{t+1}). \tag{3.8}$$

A full specification of a shopping behavior under DD thus corresponds to an allocation of binary values to the elements of the matrix:

$$x_{kp} \equiv \begin{pmatrix} x_{0,r} & x_{\alpha r,r} & x_{\alpha R,r} \\ x_{0,R} & x_{\alpha r,R} & x_{\alpha R,R} \end{pmatrix}$$

For behaviors $B \in \{\mathfrak{R}_D, \mathcal{S}_D, \mathcal{T}_D\}$, the matrices $x_{kp}^{\mathfrak{R}}$, $x_{kp}^{\mathcal{T}}$, and $x_{kp}^{\mathcal{S}}$ are as follows.

$$x_{kp}^{\mathfrak{R}} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad x_{kp}^{\mathcal{T}} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad x_{kp}^{\mathcal{S}} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.9}$$

By applying each of these matrices to (3.8), we solve for the corresponding surplus NPV $V(0)$, which we denote respectively as $V_B(0)$, or simply V_B for brevity. (See Appendix B for details.) Under DD we thus obtain:

$$V_{\mathfrak{R}_D} \equiv V_{\mathfrak{R}_D}(0) = [v - (1-\alpha\beta)(\beta r + \beta\gamma(R-r) + (1-\beta)R]/(1-\beta), \tag{3.10}$$

$$V_{\mathcal{T}_D} \equiv V_{\mathcal{T}_D}(0) = \underline{\beta}[v - (1-\alpha\beta)(r + \beta\gamma(R-r)]/(1-\beta), \text{ and} \tag{3.11}$$

$$V_{\mathcal{S}_D} \equiv V_{\mathcal{S}_D}(0) = \underline{\beta}[v - r(1-\alpha\underline{\beta})]/(1-\underline{\beta}), \tag{3.12}$$

where $\underline{\beta} = \beta(1-\gamma)/(1-\beta\gamma)$. We then employ (3.10), (3.11) and (3.12) to determine the valuation thresholds $v_{\mathfrak{R}_D}(\alpha), v_{\mathfrak{R}_D}(\alpha)$ and $v_{\mathcal{T}_D}(\alpha)$ which determine consumers' self-selection of a shopping behavior $B \in \{\mathfrak{R}_D, \mathcal{S}_D, \mathcal{T}_D\}$. We thus find, as we summarize next, how consumers self-select their shopping behavior based on their type (v).

– $B = \mathfrak{R}_D, \forall v : V_{\mathfrak{R}_D} \geq \max\{V_{\mathcal{T}_D}, V_{\mathcal{S}_D}\}$, yielding $v \geq R(1-\alpha\beta) = v_{\mathfrak{R}_D}(\alpha)$.

We can intuitively explain this threshold by noting that a regular shopper's initial outlay is R and the resulting credit gain αR is discounted by β because it applies in the next period.

– $B = \mathcal{T}_D, \forall v : V_{\mathcal{T}_D} \geq \max\{V_{\mathfrak{R}_D}, V_{\mathcal{S}_D}\}$, yielding $R - \alpha r - \alpha\beta(R - r) = v_{\mathcal{T}_D}(\alpha) \leq v \leq v_{\mathfrak{R}_D}(\alpha)$. We can explain this threshold by recognizing that when transitioning to regular shopping, a customer's outlay is $R - \alpha r$ and the resulting credit gain $\alpha(R - r)$ has an associated discount factor β as it applies in the next period.

– $B = \mathcal{S}_D, \forall v : V_{\mathcal{S}_D} \geq \max\{V_{\mathcal{T}_D}, 0\}$, yielding $r - \alpha\underline{\beta}r = v_{\mathcal{S}_D}(\alpha) \leq v \leq v_{\mathcal{T}_D}(\alpha)$. This result also has an intuitive interpretation. A sales shopper's initial outlay is r and the resulting credit gain αr is applicable in some uncertain future sale period. The exact discount factor between two subsequent sale periods is $\sum_{t=1}^{\infty} \beta^t(1 - \gamma)\gamma^{t-1} = \beta(1 - \gamma)/(1 - \beta\gamma) \equiv \underline{\beta}$.

Consumers of type $v < v_{\mathcal{S}_D}(\alpha)$ do not participate (shop) because for such customers even sales shopping does not generate positive expected surplus.

We now follow the same approach (details in Appendix B) for the ID policy to obtain the consumer surplus NPV functions $V_{\mathfrak{R}_I}$ and $V_{\mathcal{S}_I}$ for regular- and sales-shopping with instantaneous discounts.

$$V_{\mathfrak{R}_I} = [v - (1 - \alpha)(R - \beta(1 - \gamma)(R - r))]/(1 - \beta), \text{ and} \tag{3.13}$$

$$V_{\mathcal{S}_I} = \beta(1 - \gamma)(v - (1 - \alpha)r)/(1 - \beta). \tag{3.14}$$

In turn, depending on a consumer's type v, their optimal buying behavior will be:

– $B = \mathfrak{R}_I, \forall v : V_{\mathfrak{R}_I} \geq V_{\mathcal{S}_I}$, yielding $v \geq R(1 - \alpha) = v_{\mathfrak{R}_I}(\alpha)$, or

– $B = \mathcal{S}_I, \forall v : V_{\mathcal{S}_I} \geq \max\{V_{\mathfrak{R}_I}, 0\}$, yielding $r(1 - \alpha) = v_{\mathcal{S}_I}(\alpha) \leq v \leq v_{\mathfrak{R}_I}(\alpha)$.

Notice that under ID regular shopping requires $v \geq R(1-\alpha) = v_{\mathfrak{R}_I}(\alpha)$, whereas transition shopping (with DD) requires $v \geq R - \alpha r - \alpha\beta(R-r) = v_{\mathcal{T}_D}(\alpha)$. It is thus easy to show that $v_{\mathfrak{R}_I}(\alpha) < v_{\mathcal{T}_D}(\alpha)$ holds. For a given (fixed) α, Figure 7 illustrates the consumer segmentation structure for both the DD and ID alternatives. Given our understanding of how a consumer type v maps optimally to a particular shopping behavior, we next address the firm's problem of optimizing the reward level (α) for any specific consumer type.

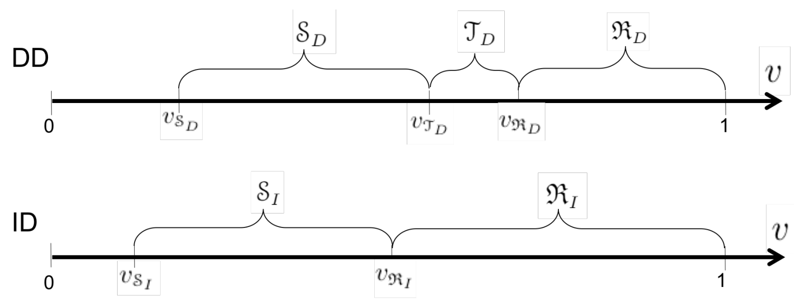


FIGURE 7. Valuation thresholds and market segmentation under DD and ID

Optimal targeted (consumer specific) spending rewards

For any shopping behavior, profit is highest for the lowest α percentage that behavior will abide, i.e., $\alpha^*_B = \text{argmin}_\alpha\{v_B(\alpha) : v_B(\alpha) \leq v\}$. Setting $v = v_B(\alpha)$ in each case yields the following optimal discount percentages for ID and DD.

$$\alpha^*_I \in \begin{cases} \alpha_{\mathfrak{R}_I} = (R-v)/R \\ \alpha_{\mathcal{S}_I} = (r-v)/r \end{cases} \qquad (3.15)$$

$$\alpha^*_D \in \begin{cases} \alpha_{\mathfrak{R}_D} = (R-v)/(\beta R) \\ \alpha_{\mathcal{T}_D} = (R-v)/(r+\beta(R-r)) \\ \alpha_{\mathcal{S}_D} = (r-v)/(\underline{\beta} r) \end{cases} \qquad (3.16)$$

Together, (3.15) and (3.16) imply $\alpha_{\mathfrak{R}_D} > \alpha_{\mathcal{T}_D} > \alpha_{\mathfrak{R}_I}$ and $\alpha_{\mathcal{S}_D} > \alpha_{\mathcal{S}_I}$. Therefore, intuitively and congruent with Proposition 5, we see that when delaying discounts the firm optimally compensates consumers by setting a higher percentage. We can also see from (3.15) and (3.16) that the optimal delayed discount is a scaling of the corresponding instantaneous percentage, such that $\alpha_{\mathfrak{R}_D} = \alpha_{\mathfrak{R}_I}/\beta$ and $\alpha_{\mathcal{S}_D} = \alpha_{\mathcal{S}_I}/\underline{\beta}$. The scaling is larger for sales shoppers because they wait longer than regular shoppers, on average, before redeeming their credit. In essence, the firm additionally compensates consumers who are less certain of redeeming their credit soon. In contrast with Proposition 5, we will show that the resulting *profits* are *not* necessarily equivalent, specifically for consumers who adopt the transition shopping behavior.

By alternately applying $x_{kp}^{\mathfrak{R}}$, $x_{kp}^{\mathcal{T}}$ or $x_{kp}^{\mathcal{S}}$ to (3.6), we obtain (details in Appendix B) the expected profit NPV for the corresponding shopping behavior $B \in \{\mathfrak{R}_D, \mathcal{S}_D, \mathcal{T}_D\}$. For each such behavior B, we denote the corresponding profit as $E\pi_B(\alpha) \equiv E\pi_B(\alpha, 0)$, yielding the following results.

$$E\pi_{\mathfrak{R}_D}(\alpha) \equiv E\pi_{\mathfrak{R}_D}(\alpha, 0) = [(\gamma R + (1-\gamma)r)(1-\alpha\beta) - cR]/(1-\beta). \tag{3.17}$$

$$E\pi_{\mathcal{T}_D}(\alpha) \equiv E\pi_{\mathcal{T}_D}(\alpha, 0) = \frac{(1-\gamma)[(\beta\gamma R + (1-\beta\gamma)r)(1-\alpha\beta) - cR]}{(1-\beta\gamma)(1-\beta)}. \tag{3.18}$$

$$E\pi_{\mathcal{S}_D}(\alpha) \equiv E\pi_{\mathcal{S}_D}(\alpha, 0) = \frac{(1-\gamma)[r(1-\alpha\underline{\beta}) - cR]}{(1-\beta\gamma)(1-\underline{\beta})}. \tag{3.19}$$

Following the same approach (details in Appendix B) , we can derive the expected profits pertaining to regular- and sales-shopping segments under the ID policy, yielding:

$$E\pi_{\mathfrak{R}_I}(\alpha) = \frac{((1-\gamma)r + \gamma R)(1-\alpha) - cR}{1-\beta}\text{, and } E\pi_{\mathcal{S}_I}(\alpha) = \frac{(1-\gamma)(r - \alpha r - cR)}{1-\beta}. \tag{3.20}$$

Applying results (3.15) and (3.16), we next establish that if a consumer is optimally targeted as either a regular or sales shopper, then equivalent profits result.

Proposition 8

If the DD and ID policies optimally target a consumer of type v, then the regular- and sales-shopping behaviors yield identical profits, i.e.,

$$E\pi_{\mathfrak{R}_D}(\alpha_{\mathfrak{R}_D}) = E\pi_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I}) = ((v/R)((1-\gamma)r + \gamma R) - cR)/(1-\beta), \text{ and}$$

$$E\pi_{\mathfrak{S}_D}(\alpha_{\mathfrak{S}_D}) = E\pi_{\mathfrak{S}_I}(\alpha_{\mathfrak{S}_D}) = (1-\gamma)(v - cR)/(1-\beta).$$

In contrast, we next show that if the consumer type v is such that transition shopping is optimal, then the delayed and instantaneous discounts *do not* generally yield equivalent profits.

Proposition 9

Given $\gamma > \bar{\gamma} = \frac{\beta R + r(1-2\beta)}{\beta(2\beta R + r(1-2\beta))}$, transition shopping increases the firm's profits for consumers of type $v \in (v_{\mathfrak{TS}}, v_{\mathfrak{TR}})$.

In the prior section we found that with consistent consumer spending delaying discounts does not increase profits. Here, we see that with fluctuating spending, the increase in segmentation options under DD can increase profits. Moreover, the sufficient condition on γ in this proposition is relatively mild, and thus holds for typical parameter settings. Note that for a typical $\beta > 1/2$, this ratio is of the form $\frac{x-a}{\beta(2x-a)}$, with $x > a > 0$; thus, given typical β close to 1, this ratio is below one half, often significantly so. The range of transition shoppers ($v \in (v_{\mathfrak{TS}}, v_{\mathfrak{TR}})$) is generally quite small or zero. For example, with $R = 0.9$, $r = 0.6$, $\beta = 0.8$, $c = 0.5$ and $\gamma = 0.7$, the range $v \in (v_{\mathfrak{TS}}, v_{\mathfrak{TR}})$ is only (0.49,0.54) and even for modest parameter perturbations will cease to exist (e.g., if $\gamma < 0.60$, $r < 0.55$, or $c > 0.55$). This result is significant primarily because it establishes that with fluctuating spending, delaying credit is not necessarily “a wash” relative to instant discounts. With this understanding, we may expect to find further support for the value of delayed discounts in practice when we extend our analysis to a heterogeneous market.

Consumer surplus analysis

If a regular or sales shopper is targeted by the firm with their respectively optimal discount percentages, we established that DD and ID profits are equivalent. The following lemma establishes a parallel result for surplus.

Lemma 4

For both the regular- and sales-shopping behaviors, the same NPV of consumer surplus results from the DD and ID policies, if each targets the consumer optimally (i.e., with the optimal discount level).

We also know from Proposition 9 that the firm can extract greater *profit* from transition shoppers. We might thus expect consumer surplus in that case to drop, but the following lemma establishes that the corresponding change in consumer surplus can also be positive.

Lemma 5

Consumer surplus from transition-shopping behavior can be higher or lower under the DD policy. Specifically, surplus increases under DD (relative to ID) for $v \in (v_{\mathfrak{T}\mathcal{S}}, v_{\mathfrak{R}_I\mathcal{S}_I})$, but decreases for $v \in (v_{\mathfrak{R}_I\mathcal{S}_I}, v_{\mathfrak{T}\mathfrak{R}})$.

In conjunction with Proposition 9, Lemma 5 shows that the DD policy can increase both profit and surplus simultaneously. As we show in the next section, this finding extends to a heterogeneous market as well.

Heterogeneous Market with Varied Spending

In subsection 3.4, we assume the firm could target each consumer type v with the correspondingly optimal (i.e., consumer-type specific) discount percentage, but in this section we consider the common practice of setting a percentage to apply consistently across all customers (such as the 5% and 10% levels in the aforementioned Target and REI examples). Therefore, we now assume a market of consumers who are heterogeneous in their type v, such that values of v are dispersed over the interval $[0, 1]$ (this interval choice is WLOG). For the heterogeneous market, we denote the optimal discount percentages for the DD and ID policies as α^*_D and α^*_I, respectively.

To determine the firm's profit we must weight the NPV of profit derived from each potential shopping-behavior by the corresponding number of shoppers who self-select into that segment. For the DD policy and a corresponding choice of α, we can thus express the total expected discounted profits under DD from the regular, transition, and sales segments as follows.

$$E\pi_D(\alpha) = (1-v_{\mathfrak{R}_D}(\alpha))E\pi_{\mathfrak{R}_D}(\alpha)+(v_{\mathfrak{R}_D}(\alpha)-v_{\mathfrak{T}_D}(\alpha))E\pi_{\mathfrak{T}_D}(\alpha)+(v_{\mathfrak{T}_D}(\alpha)-v_{\mathfrak{S}_D}(\alpha))E\pi_{\mathfrak{S}_D}(\alpha). \tag{3.21}$$

Likewise, for the ID policy and its two induced segments, we have:

$$E\pi_I(\alpha) = (1 - v_{\mathfrak{R}_I}(\alpha))E\pi_{\mathfrak{R}_I}(\alpha) + (v_{\mathfrak{R}_I}(\alpha) - v_{\mathfrak{S}_I}(\alpha))E\pi_{\mathfrak{S}_I}(\alpha). \tag{3.22}$$

Substituting the valuation thresholds from subsection 3.4 and the expected profit NPV terms from (3.17) through (3.20) in the above two functions, we can show that $d^2E\pi_D(\alpha)/d\alpha^2 < 0$ and $d^2E\pi_I(\alpha)/d\alpha^2 < 0$, and hence the aggregate expected profit functions are (strictly) concave in α. Because the profit functions (3.21) and

(3.22) are well behaved, we can apply first-order conditions ($dE\pi_I(\alpha)/d\alpha = 0$ and $dE\pi_D(\alpha)/d\alpha = 0$) to determine their respective optimal discount percentages. Denoting those optimal values as α_D^* and α_I^*, we find:

$$\alpha_I^* = \frac{\gamma R(2R-1) + (1-\gamma)r(2r-1) - cR(\gamma R + (1-\gamma)r)}{2\left(\gamma R^2 + (1-\gamma)r^2\right)}\ \text{, and} \tag{3.23}$$

$$\alpha_D^* = \frac{\gamma R(2R-1) + (1-\gamma)r(2R(1-h_1) + 2h_1 r - 1) - cR(\gamma R + (1-\gamma)r)}{2\left(\beta\gamma R^2 + \beta R h_3 r + (1-\gamma)h_2 r^2\right)}\ , \tag{3.24}$$

where $h_1 = \frac{1-\gamma}{1-\beta\gamma}$, $h_2 = h_1\frac{(1-\beta)^2\gamma}{1-\gamma} + \beta h_1^2$, and $h_3 = 2(1-\beta)\gamma h_1$. We see from the above expressions that α_I^* is independent from the time discount factor β, which is intuitive given that under ID consumers are not waiting for the discount. In contrast, α_D^* depends on β. We next establish by comparing the optimal α_I^* and α_D^* percentages that the optimal delayed discount percentage is indeed higher.

Proposition 10

When fixing a single discount percentage with which to serve the heterogeneous market, the firm optimally sets $\alpha_D^* \geq \alpha_I^*$. In the absence of time discounting ($\beta = 1$), the inequality binds.

We intuitively anticipate this ordering for the discounts, but we next also prove that the delayed spending rewards also increase *total profit*, under the sufficient condition that c exceeds a lower threshold we denote as c_M (as we later discuss, this threshold is typically negative, so $c > c_M$ is a very mild condition.)

Proposition 11

For the heterogeneous market with fluctuating consumer spending, $E\pi_D(\alpha_D^*) \geq E\pi_I(\alpha_I^*)$, for $c > c_M$. At $\beta = 1$, the expected profits match.

To better understand this result, it is helpful to first consider the case of no time discounting, as it is easy to verify that $\alpha_I^* = \alpha_D^*$ when $\beta = 1$. And, logically, in that case, all active customers should view delayed and instant credit identically—i.e., when there is no time discounting, delay can yield no segmentation power and segment $\mathfrak{T}_D$ is empty. As β drops below 1, delay begins to provide segmentation flexibility via its greater number (six versus two) of implied price states, and $\mathfrak{T}_D$ emerges as an optimal purchasing policy via consumer self-selection. DD profits then begin to dominate those of ID, and the DD policy increases α_D^* to compensate for the now-present time value of money. As α increases, under DD the relative appeal of waiting for sales (i.e., being a sales shopper) decreases, due to the larger credit amounts consumers can apply to their purchases. The relatively large α for the DD policy thus reduces dilution risk, which in turn helps support higher profit.

Assessing the magnitude of DD profit gains

We next use a set of numeric experiments to assess the *magnitude* of the profit gain implied by Proposition 11, as well as associated consumer surplus repercussions. In these experiments, we adjust each of the problem parameters both upwards and downwards, starting from the following baseline parameter values. We set the baseline regular spending level (the full price before any discounts) $R = 0.9$, with a reduced (sale) price $r = 0.6$. We set the baseline $\gamma = 0.5$, i.e., the firm will offer a sale in every other period, on average. We assume a baseline cost factor $c = 0.5$, so the regular price R reflects 100% mark-up above the firm's cost cR. We use $\beta = 0.8$ as the baseline time discount factor. For these five parameters, we alternately adjust their values by ± 0.2. In Table 5 we report the corresponding optimal results, including: (i) the α_D^* and α_I^* values from (3.23) and (3.24), (ii) segment-specific demands, denoted by D_B where $B \in \{\mathfrak{R}, \mathfrak{T}, \mathfrak{S}\}$, (iii) the profit increase from DD relative to ID, and (iv) the surplus increase from DD relative to ID. The sufficient condition $c > c_M$ in Proposition 11 holds in all the implied eleven cases, with c_M even being negative in all but two.

We previously evaluated the consumer subproblem for any given value of α, and we now leverage that analysis to compute the aggregate expected consumer surplus (EV_B) for each of the consumer segments. For the ID policy, the aggregate market consumer surplus is as follows.

$$EV_I^* = \int_{v_{\mathfrak{R}_I}(\alpha_I^*)}^{1} EV_{\mathfrak{R}_I}(\alpha_I^*)dv + \int_{v_{\mathcal{S}_I}(\alpha_I^*)}^{v_{\mathfrak{R}_I}(\alpha_I^*)} EV_{\mathcal{S}_I}(\alpha_I^*)dv$$

Assessing the aggregate market consumer surplus for the DD policy requires summing over its three relevant segments, but is otherwise identical in nature.

$$EV_D^* = \int_{v_{\mathfrak{R}_D}(\alpha_D^*)}^{1} EV_{\mathfrak{R}_D}(\alpha_D^*)dv + \int_{v_{\mathcal{T}_D}(\alpha_D^*)}^{v_{\mathfrak{R}_D}(\alpha_D^*)} EV_{\mathcal{T}_D}(\alpha_D^*)dv + \int_{v_{\mathcal{S}_D}(\alpha_D^*)}^{v_{\mathcal{T}_D}(\alpha_D^*)} EV_{\mathcal{S}_D}(\alpha_D^*)dv$$

Leveraging these two surplus expressions, the percentage surplus increase from DD relative to ID is as follows.

$$\Delta EV^* = 100\%(EV_D^* - EV_I^*)/EV_I^*$$

For consistency, we likewise present the profit increase from DD relative to ID in percentage terms as follows.

$$\Delta E\pi^* = 100\%(E\pi_D(\alpha_D^*) - E\pi_I(\alpha_I^*))/E\pi_I(\alpha_I^*)$$

We report the optimal profit and consumer surplus changes within the last two columns of Table 5, which also shows the comprehensive set of results.

TABLE 5. DD versus ID segmentation: profit and surplus comparisons

R	r	c	β	γ	Case	α^*	$D_{\mathfrak{R}}$	$D_{\mathfrak{T}}$	$D_{\mathfrak{S}}$	$\Delta E\pi^*$	ΔEV^*
0.9	0.6	0.5	0.8	0.5	DD	11%	0.18	0.013	0.25	3.85%	5.17%
					ID	7%	0.163	-	0.278		
-	-	-	**1**	-	DD	7	0.163	0	0.278	0%	0%
					ID	7	0.163	-	0.278		
-	-	-	**0.6**	-	DD	16	0.187	0.039	0.214	6.55%	6.81%
					ID	7	0.163	-	0.278		
-	-	**0.7**	-	-	DD	0	0.1	0	0.3	0%	0%
					ID	0	0.1	-	0.3		
-	-	**0.3**	-	-	DD	24	0.277	0.029	0.191	3.55%	1.2%
					ID	18	0.267	-	0.244		
-	-	-	-	**0.7**	DD	18	0.229	0.021	0.207	6.07%	2.48%
					ID	11	0.217	-	0.261		
-	-	-	-	**0.3**	DD	1	0.108	0.001	0.295	0.07%	2.93%
					ID	0	0.1	-	0.3		
-	**0.8**	-	-	-	DD	18	0.236	0.03	0.034	1.62%	-0.48%
					ID	15	0.235	-	0.085		
-	**0.4**	-	-	-	DD	7	0.152	0.005	0.461	17.44%	4.74%
					ID	2	0.125	-	0.485		

The baseline parameter settings correspond to the first row in the table. For that base case, DD outperforms ID both in terms of the resulting profit NPV increase $\Delta E\pi^* = 3.85\%$ and the consumer surplus increase $\Delta EV^* = 5.17\%$. It does so even while setting a higher discount percentage, 11% versus 7%. The first three sets of results illustrate the impact of changing β, with $\beta = \{1, 0.8, 0.6\}$. Consistent with the discussion following Proposition 11, at $\beta = 1$ we expect identical results between DD and ID, and DD should dominate as the significance of the time discount factor increases. The numeric results support this, showing both increasing profit and surplus gains from DD as β drops from 1 to 0.8 and then to 0.6. Recall from Lemma 3 that a smaller β could result from customer inactivity periods, which we see serves to increase the profit gap in favor of the DD policy. The results for $c = 0.7$ show no discounts (and hence equal profits) from the DD and ID policies, due to the cost being sufficiently high as to make additional discounts suboptimal. For the case of $r = 0.8$ (with $R = 0.9$), DD yields only minor profit increases of 1.62%, because prices change little in this case and we established in Proposition 5 that DD offers no advantage when prices are static. It is also interesting to observe that delayed discounts also induce a larger regular-shopping segment and fewer customers who adopt the sales-shopping behavior and an equal or lower *total sales* (the sum of the three demand columns) in six out of nine cases.

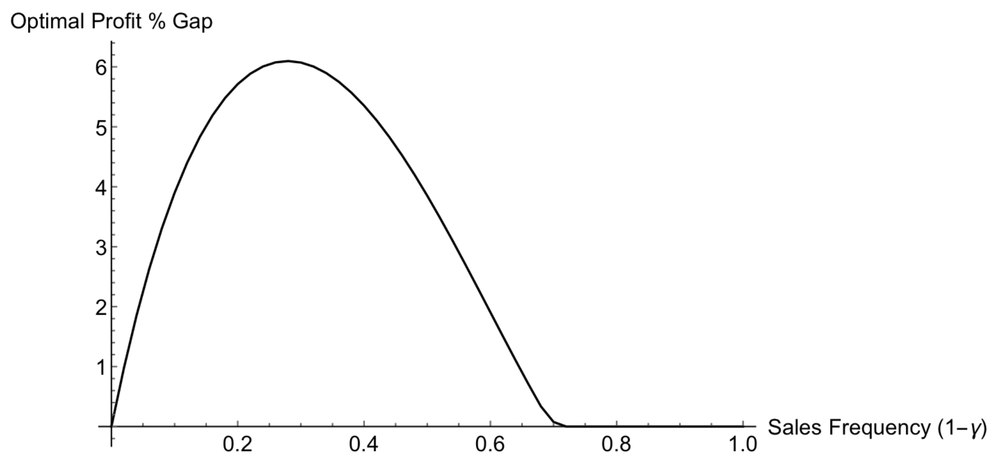


FIGURE 8. DD versus ID profit gain, versus sales frequency

Using the same baseline parameter set in Table 5, Figure 8 illustrates $\Delta E\pi^*$ for the full range of potential sales frequency $(1-\gamma)$ values between 0 and 1. From Table 5, we see three specific values from this curve: $\Delta E\pi^* = 6.07\%, 3.85\%$, and 0.07%, corresponding to $1-\gamma = 0.3, 0.5$, and 0.7. Figure 8 highlights our finding from Section 3 that when the firm's price is static (e.g., at R when $\gamma = 1$ or at r when $\gamma = 0$), equivalent profits result from delayed and instantaneous discounts. And, consistent with our analysis in this section and Section 4, we see that if there are sporadic sales, then the fluctuations in consumers' spending allow delayed discounts to be advantageous for the firm—and for consumers too in many cases as we saw in Table 5. The asymmetry in the Figure 8 curve is due to the lessening role that additional discounting (whether DD or ID) can play as sales become very prevalent. For example, when the sales frequency $1 - \gamma = 0.3$ (i.e., sales occurring 30% of the time), there is evident scope for further discounting, and we see from Table 5 that the respectively DD and ID discount levels are 18% and 11%—with the former increasing profits by about six percent. In contrast, when the sales price holds most of the time, with frequency $1 - \gamma = 0.7$, Table 5 shows the optimal DD and ID discount levels are both close to zero, since further price reductions are not optimal. For this reason, we see that the Figure 8 curve is asymmetric, with the DD policy exhibiting its most significant profit gains for low-to-moderate sales frequencies.

Collectively, the results in this section motivate, even with rational consumers and costless rewards redemption (as opposed to coupons, which entail inconvenience), why delayed spending rewards are generally preferable to instantaneous rewards and thus popular in practice. As we have shown, their benefits stem from the juxtaposition the heterogeneous consumers and spending variability—resulting, for example, from sporadic sales as we have considered. Notably, we have shown that these benefits hold for any arbitrary values for the regular and sale prices R and $r(< R)$, and are not tied to specific (and thus potentially unlikely, in practice) optimal values.

Delayed-Discounts Policy Robustness

In this section, we consider the robustness of delayed discounts by considering two changes in conditions. First, we consider applying a suboptimal discount, e.g., if the optimal α is 13% but a manager rounds off that value to 10%. Second, we will consider the implication of the price sequence being varied but deterministic.

Under both conditions, we show that DD continues to outperform ID and optimally set a higher reward level.

Enforcing a suboptimal discount percentage

As highlighted by the results in Table 5, the relative advantage of DD is sometimes significant. We have thusfar, however, set the discount level α optimally. We now consider how well the DD and ID policies perform if the firm were to potentially employ a suboptimal α. A firm might use a suboptimal discount percentage because: (i) the optimum is unknown, (ii) a manager applies a "round number" (e.g., 10% versus 13%), or (iii) underlying parameters (e.g., product MSRP, or cost) change over time but the firm holds its α choice steady.

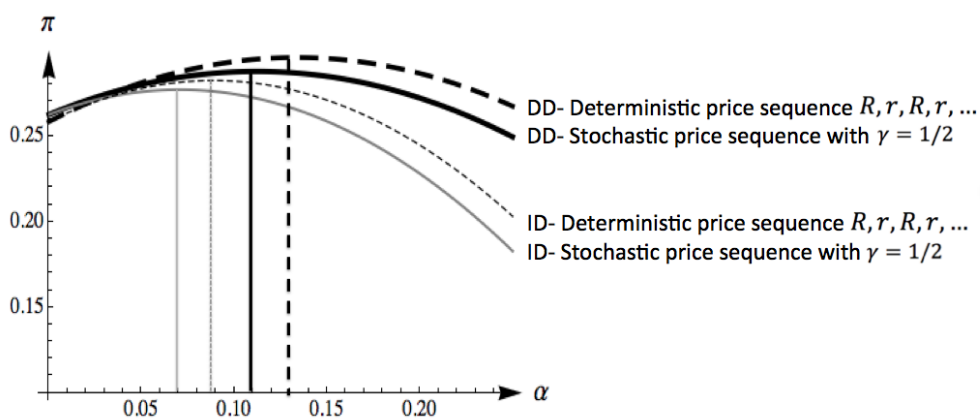


FIGURE 9. Delayed discount strategy is a more robust pricing tactic.

The solid curves in Figure 9 (we address the dashed curves in the next subsection) represent the DD and ID profits $E\pi_D(\alpha)$ and $E\pi_I(\alpha)$ from (3.21) and (3.22), over a range of α. All other parameters are at their baseline values, and we see that the solid curves reach their maximum values for α^* values of 11% and 7%, consistent with the baseline results from Table 5. Figure 9 shows the ID profit curve exhibits more curvature than the DD profit curve. The DD policy is more robust to a suboptimal α selection, yielding higher profit over the range of arbitrary (suboptimal) α values.

Enforcing a deterministic price sequence

We next consider whether our insights regarding the DD and ID policies change if we enforce a deterministic price pattern. For example, let us consider the price sequence $R, r, R, r, R, ...$, which is a deterministic analog to the stochastic sequence with $\gamma = 0.5$. It is straightforward to establish that the same three/two optimal shopping behaviors remain under DD/ID, but we must re-derive the threshold valuations that dictate consumers' self-selection choices and the resulting profits. For a regular shopper, the surplus NPV becomes $V_{\mathfrak{R}_D}(\alpha) = (v - R) + \sum_{i=0}^{\infty} \beta^{2i}(v - r + \alpha R) + \sum_{i=0}^{\infty} \beta^{2i+2}(v - R + \alpha r)$. For a transition shopper, purchasing begins in period 2 and the surplus NPV is $V_{\mathfrak{T}_D}(\alpha) = \beta(v - r) + \sum_{i=0}^{\infty} \beta^{2i+2}(v - R + \alpha r) + \sum_{i=0}^{\infty} \beta^{2i+3}(v - r + \alpha R)$. For a sales shopper, purchasing occurs in periods 2,4,6,..., and the surplus NPV is $V_{\mathfrak{S}_D}(\alpha) = \beta(v - r) + \sum_{i=0}^{\infty} \beta^{2i+3}(v - r + \alpha r)$. Via the maximization of surplus NPV, we can show that a consumer will optimally self-select to be a: (i) regular shopper if $v > R(1 - \alpha\beta)$, (ii) transition shopper if $R(1-\alpha\beta) > v > R-\alpha r-\alpha\beta(R-r)$ or (iii) sales shopper if $R-\alpha r-\alpha\beta(R-r) > v > r-\alpha\beta^2 r$. The first two of these thresholds are unchanged from the prior section; the third, corresponding to sales shoppers, has changed slightly because consumers will redeem their credit in exactly two periods. Constructing the firm's profit NPV for each of these shopping behaviors parallels the construction of the surplus for each. For example, $\pi_{\mathfrak{T}_D}(\alpha) = \beta(r-cR)+\sum_{i=0}^{\infty} \beta^{2i+2}(R-\alpha r-cR)+\sum_{i=0}^{\infty} \beta^{2i+3}(r-\alpha R-cR)$. Following the same steps that led us to expression (3.21), we can formulate the profit under DD and apply first-order conditions to find α_D^*. Similarly, we then repeat the process to formulate the ID profit for the sequence $R, r, R, r, \ldots$ and so determine α_I^*.

Lemma 6

Given deterministic and alternating spending, $\alpha_D^* > \alpha_I^*$.

We thus see that when considering a deterministic sequence, we retain our established result that discounts are optimally larger when delayed. Within Figure 9, we use dashed lines to illustrate DD and ID profits for the price sequence $R, r, R, r, \ldots$.. We see the same two profit characteristics as held for the stochastic problem: (i) optimal profits are higher with delayed discounts, and (ii) profits under DD are relatively robust if the discount percentage is specified suboptimally. Naturally, the earlier more general stochastic formulation permitted this particular deterministic sequence as well as all other feasible sequences.

Conclusion

For a firm serving repeat customers, we have explored the potential advantage of implementing delayed spending-level based discounts. In contrast with typical instantaneous discounts, delayed discounts are computed based on today's spending but applied to a subsequent purchase. The time of that next purchase may be uncertain due to a customer wishing to wait for sale, or simply having no need to shop with the firm. The uncertainty regarding when earned credit will be redeemed serves to diminish its expected value. The higher the level of earned credit, the more likely it is that a consumer will make a subsequent purchase sooner. Because the net price a consumer faces in the current period depends on previously earned credit, delayed discounts induce behavior-based price discrimination (BBPD).

From the firm's perspective, paying out a discount later rather than sooner is naturally appealing, all other things being equal. But, all other things are *not* equal, precisely because consumers adjust their shopping behavior. We find that if the firm opts to delay discounts, then rational consumers will react such that the firm must (optimally) offer a higher discount level to compensate. The firm gains initially, but over time gradually gives back that advantage over successive periods via the higher discounts. Indeed, we begin by establishing that in settings where consumer spending is consistent over time, delayed discounts offer no advantage. However, by extending our analysis to consider spending variability, we find that delayed discounts offer potential value to both the firm and consumers.

The sources of value from delayed rewards stem from the interplay of three factors: (i) consumer heterogeneity, (ii) spending fluctuations, and, naturally, (iii) time discounting. All three factors must be present for the delay to add value. As we have seen, there are two distinct sources of advantage that delayed discounts create, relative to their simpler instantaneous counterpart. First, given variable spending, delayed rewards multiply the number of subsequent net-price states (e.g., high/low and low/high combinations of price and credit, beyond the simple high/high and low/low combinations resulting from instantaneous discounts). Second, delayed rewards have distinct and higher-price states associated with initial purchase, which leads to the emergence of "transition shopping" behavior for consumers with moderate valuations. Using two price levels (regular retail versus sale prices), we showed that delayed rewards yields six net spending-level states and three consequent optimal segmentation options for consumers, versus two for each with instantaneous discounts. As we have shown, this increase in net-price states and corresponding segmentation options enables more effective price discrimination and thus greater profits. Interestingly, we also showed that the broader array of net-price states and shopping behaviors often leads to higher consumer surplus as well, yielding a win-win outcome for the firm and consumers.

Notably, these findings regarding the relative merit of delayed rewards do not hinge upon an assumption that the firm's underlying service or product prices are set optimally. Rather, our initial results showing the equivalence of delayed rewards (versus instant discounts) when consumers have consistent spending over time holds for both homogeneous and heterogeneous markets, irrespective of the firm's regular (pre-discount) price level. Moreover, when we extended the analysis to include a sporadic (lower) sales price, we proved the benefits from delayed rewards hold, again, *for arbitrary values of those sale and regular prices* (subject to a mild cost condition that we showed holds quite generally). To further demonstrate robustness of these findings, in the prior section we considered enforcing suboptimal discount levels and deterministic pricing sequences (in lieu of the earlier stochastic setting), and again find that the greater pricing flexibility due to delayed discounts continues to add value—stemming from, as highlighted above, the juxtaposition of time discounting, consumer heterogeneity, and spending fluctuations. In summary, delayed discounts offer an intriguing mechanism for leveraging behavioral based pricing to increase profits. Our findings help motivate why firms exploit time-lagged rewards in practice, even in the absence competitive pressures or irrational ("locked in") customers.

Bridge to Next Chapter

In the current chapter, the utility the customer derives from consumption of the product/service was assumed to be known to the customer and the question of interest was the effect of the timing of price discounts on the consumer's and the firm's welfare. In the next chapter, we will focus on experience goods where the customer doesn't know the true utility from the product unless the product is consumed. In such cases, we address the pricing problem of a firm on how to set the trial and selling prices for such a good and whether the firm should adjust the post-trial price to encourage the customers to upgrade to a purchase after renting the product.

CHAPTER IV

TRY BEFORE YOU BUY PRICING

This work is in preparation for submission and is co-authored with Dr. Michael Pangburn. The excerpt to be included is written entirely by me, while my coauthor provided invaluable editorial assistance.

Introduction

When a durable experience product has uncertain value, the customer may prefer to try it first–hopefully by paying a low fee–to resolve valuation uncertainty. Offering a product in a rental mode provides this opportunity for the firm to help the customer resolve uncertainty while extracting a positive surplus. Many firms rent their products, as well as sell them outright. In such cases a customer may choose to rent a product before making a potential purchase decision. After renting, a consumer may be certain that they like the product (or not). On the other hand, the consumption during the rental period may imply a reduction in the subsequent consumption utility a customer can experience from the same product. For example watching a thriller movie or a wearing a prom/wedding dress for the second time might not be as enjoyable as the first time. To alleviate such a drop in customers' willingness to pay, sellers may practice behavioral-based pricing in which they apply some or even the entire rental fee paid by the customer toward a subsequent purchase of the product. Examples include ski gear, bike rentals, and wine-tasting fees. However, it is also common in practice to see examples in which firms apply no portion of the rental price towards a subsequent purchase. For example if you rent and then purchase a movie via the iTunes Store, the purchase price is unaffected.

In this paper, we analyze the optimal pricing for a firm that can simultaneously sell and rent its product, given that consumers have uncertain product valuations. begin by analyzing a market context with homogeneous consumers (ex ante), and then later extend the analysis to consider market heterogeneity. We study all the possible product mode offerings (selling, renting or both simultaneously) and then we derive the conditions under which a firm should optimally refund some or all of its rental price to customers who subsequently convert from renters to purchasers. The firm commits to its pre and post-trial prices. However, customers with a rental history may face a different price when choosing to upgrade to purchase. We prove several interesting results. We particularly find that both price discounts and price premiums can be optimal after the first rental. One of our interesting findings indicates a necessary condition for conversion discounts to be optimal and that is the firm's rent-vs.-sell cost ratio must be less than a consumer's rent-vs.-buy value ratio (i.e., the fraction of the total product utility that a customer realizes from a single rental of the product).

Literature Review

Our paper relates to the literature on valuation uncertainty and consumer learning, renting vs. selling a durable good, behavioral based-pricing, and return policies.

Valuation uncertainty and consumer learning

We investigate a seller's optimal pricing strategy in the presence of valuation uncertainty. There is a strong stream of research surrounding valuation uncertainty, which arises due to different factors. One source of uncertainty can be explained by the gap between time of purchase and point of consumption. For example, in Courty and Li (2000), a customer commits to a contract when only having partial information about their valuation (such as buying an airline ticket). Learning occurs just prior to point of consumption and the customer can exercise the refund term of the signed contract should he decide to withdraw from potential consumption. Nocke et al. (2011) studies the profitability of advance-selling discounts when consumers face uncertainty about their valuation because the time of consumption is far ahead in the future. The monopolistic firm utilize advance-purchase discounts to achieve price discrimination among customers based on their expected valuations. In their model, consumers with a low expected valuation will wait and purchase the good at the regular (non discounted) price only in the event where their realized valuation is high. Consumers realize the true valuation when the time of consumption arrives and they do not need to try the product to resolve their valuation uncertainty. Bhargava and Chen (2012) also look at scenarios where customers have uncertain valuation at the time of purchase. They show that spot selling to informed buyers outperforms advance selling to uninformed buyers when the market is ex ante heterogeneous. The firm can choose to help the buyers to learn about their valuations by providing information to consumers, or giving them time to become better informed by coming closer to the consumption date. One of their key assumptions is that the firm (or the buyer) incurs no extra cost in providing (or obtaining) information.

A rather large body of literature evolves around the uncertain valuation of experience goods (Nelson (1970)). For example Jing (2011*a*) studies a two-period monopoly model for a durable experience good and whether it is profitable to invest in seller induced learning (SIL), i.e., actions a firm can take — such as offering a product trial — to facilitate consumer learning. This paper analyzes the trade off between selling early to an uninformed market and selling later to a better-informed market. In his model, a customer only potentially (by some probability) learns the true valuation after the first period through information dissemination via exogenous and/or seller-induced investments.In another paper, Jing (2011*c*) investigates a two-period monopoly in which late buyers potentially discover their true valuation from the product information generated by the early buyers, a phenomenon called social learning. Similarly, in Bonatti (2011)'s model on selling experience goods, learning occurs on the basis of aggregate information in the market which is increasing in the total quantity sold. Wei and Nault (2013) analyze a version-to-upgrade strategy for information goods in which the monopoly offers a lower quality version as a bridge for consumers to discover their valuation through use. In their model, the customers pay a tax for learning if they want to upgrade to the higher quality version. Xiong and Chen (2014) investigate the role of SIL in the service/product line design (quality decision) with consumer uncertainty and identify regimes under which SIL is charged at a strictly positive price. They find that inducing consumer learning is profitable as long as it completely resolves the consumers' uncertainty. They have two consumer types; either both or one type can face uncertainty. In our model, the uncertainty is resolved only via consumption and the availability of a rental option can be interpreted as sampling of a durable product which acts like a low-price SIL tool.

Selling and/or renting a durable product

There has been a long-lived debate in literature about whether, why and how a firm might benefit from offering a durable product via selling or renting (or both). Bulow (1982) proves leasing a durable product helps the firm to escape the well known time-inconsistency trap. Bhaskaran and Gilbert (2005) study the strategy of selling and/or leasing a durable product when a complementary good is available. Cachon and Feldman (2011) question the efficacy of per-use renting versus subscription selling for a consumer population who are sensitive to congestion. With and without the capacity decision, they show that despite increasing the usage rate, subscriptions may outperform rentals. In our model, we look at the simultaneous offering of a product in sales and rental modes. In one of the more recent works on co-optimality of renting and selling, Gilbert et al. (2014) highlight the trade offs between selling an information good and renting it on a per-use basis by a monopoly firm. In their model, consumers are heterogeneous according to their frequency of use and they receive a random utility at each instance of need. As a result, they establish that selling and renting serves the firm's goal of price discrimination among consumers in two ways. In particular, "renting allows the firm to price discriminate among the valuations that are realized at a particular instance of time", while "selling allows a firm to price discriminate among consumers who vary in terms of their long-run expected utility-per-unit-of time." We look at offering rentals, for a durable experience good, not only to price discriminate among consumers, but also to facilitate the resolution of fit uncertainty through a low-cost option. In their model, the firm incurs no production cost and an equal (transaction) cost to either rent or sell the product. We, on the other hand, allow for distinct per-unit rental and selling costs. Also, in their model a consumer observes his realized valuation for the product prior to each decision to rent (similar to Cachon and Feldman (2011)), whereas in our model a customer realizes their utility by trying the product.

The law of diminishing marginal utility associates a higher utility to the first unit of consumption relative to second or subsequent units. In the context of durable digital goods such as movies and books that exhibit diminishing returns to consumption, Rao (2015) empirically analyzes the coexistence of purchase and rental markets. She establishes that the consumer heterogeneity in diminishing returns motivate the co-optimality of operating in both rental and purchase markets. Calzada and Valletti (2012)'s model is also built on the assumption that the utility a customer receives after the first consumption (i.e., watching a movie in theatre) drops. They however assume that the movie quality is known and rather study the optimal release strategy (simultaneous or sequential) as a function of the distribution channel structure. In our model, we study rentals as a learning platform toward purchases, as well as a price discrimination tool among customers.

Behavioral-based price discrimination (BBPD)

Behavioral-based pricing refers to when a firm observes each consumer's purchase history and charge different prices to returning and first-time customers starting in the second period. In an early paper in the topic of BBPD with experience goods, Cremer (1984) finds the price of a non-durable experience good should be lowered for second-time buyers. Jing (2011*b*) looks at benefits of BBPD relative to price commitment for a non-durable experience good in a monopoly model.De Nijs and Rhodes (2013) show that when duopolists sell experience goods, the skewness of consumer valuations fully determines whether firms reward repeat customer or new customers. They assume perfect market coverage. Jing (2016) analyzes a duopoly two-period model for an experience good and finds that in equilibrium each firm rewards repeat purchase when the probability of a high value is relatively low and when the high-low value difference is large. Villas-Boas (2006) develops a duopoly model of non-durable experience goods to analyze the informational advantage a product gains by being picked first by customers. In our model, we look at the intersection of price commitment and behavioral-based pricing. In other words, the firm pre-announces the second period price (i.e., post-trial selling price) at the start of first period and there is no surprises there for the customers.

A commonly used alternative solution to deal with uncertain valuation is to allow for returns. The customer can return the experience good in case it doesn't meet his true preference and receive a refund (full or partial). The literature on efficacy of return policies is rich. One of the early papers, Che (1996) proves that the return policy is optimal for experience goods if the consumers are sufficiently risk averse or retail costs are high. In one of the most recent works, Shang et al. (2017) shed light on the phenomenon called "wardrobing" in which some opportunistic customers may abuse the existence of return policies to use the product for just a short period of time before returning it. In this case, the non-zero restocking fee charged for returns resembles the rental fee in our model. In our paper, we analyze resolving consumer uncertainty via pre-sales trials rather than via returns, as the latter can be problematic with opportunistic customers, with information goods (due to moral hazard) or with other products for which returns are costly to handle (e.g., due to shipping costs or damage). In a subsequent "Extensions" section in this chapter, Extensions section, we briefly compare a simple return policy with try before you buy pricing and find that the latter in fact is more efficient in extracting more consumer surplus. Therefore, our paper considers short-term rentals as a tool to address valuation uncertainty, and we analyze how the firm's corresponding optimal pricing strategy can leverage BBPD.

Model

Consider a single firm offering an experience good, for rental and/or purchase. Customers face uncertainty regarding the value of the firm's product. Let θV denote the value a consumer derives from the product, where θ represents a consumer's known *type* and V refers to the uncertain product value. The true value of V, referred to as v, is realized post-consumption. We assume a two-point discrete distribution for V. We assume that V is realized as v_h with probability of γ and v_l with probability of $1-\gamma$ (where $v_h > v_l$, WLOG) [1]. We express consumer's expected valuation as μ, where $\mu = \theta E[V] = \theta(\gamma v_h + (1-\gamma)v_l)$. We later use the notation $\bar{v} \equiv \gamma v_h + (1-\gamma)v_l$. As is typical in practice, we consider the rental period to be sufficiently short that the discount factor can be treated as 1 and thus dropped from the analysis.

An important consideration when renting and selling a product is that consumers' post-rental willingness to pay may change not only due to resolving uncertainty, but also simply because the consumer has already used the product. In the case of certain types of products, such as movies, this latter issue cannot be neglected, as a customer may have significantly reduced interest in watching a movie again, even if they found in their first viewing that they really enjoyed the movie. Therefore, to properly model the consumer behavior, beyond incorporating utility uncertainty, we must also consider the fraction of the product value, which we denote as $\alpha \in [0,1]$, consumed via the rental. High α products (e.g., a documentary movie, or a prom dress) correspond to those for which consumers derive the bulk of their utility from a single rental, whereas for products with low α (a bicycle, a watch, or a funny TV series), the fraction of (total) utility gleaned

[1] In an Extensions section later in this chapter, we consider continuous distribution over V values

from the initial use would be comparatively small. Thus, the consumer receives a priori utility of $\alpha\mu$ by renting the product and, should the consumer converts to purchase, the residual utility would be equal to $(1-\alpha)\theta v_h$ or $(1-\alpha)\theta v_l$, depending on what is the outcome of learning.

We assume that the seller and the consumers are risk neutral; i.e., they maximize their expected payoffs. For a firm that considers both renting and selling its product, the pricing decision entails three interrelated parts: (i) the selling price p, (ii) the rental price r, and (iii) a (possibly) distinct conversion price for consumers who transition from a rental to a purchase, p_c. The firm's variable costs associated with selling will typically be significantly higher than for renting, and therefore we consider distinct variable costs c_s and c_r, respectively, where $c_s > c_r$ and, as we later find, the magnitude of the cost ratio is indeed critical in selection of the optimal pricing strategy.

Our research question boils down to this: When should a firm set $p_c < p$, i.e., reimburse consumers a portion of their rental price if they convert from renting to buying. Defining $\delta \in [0,1]$ as the fraction of the rental price the firm reimburses (i.e., , $p_c = p - \delta r$), we can explore conditions under which $\delta^* > 0$, along with associated segmentation implications. It is also possible, as we see later, that $\delta^* < 0$, meaning there is a premium charged for conversion .

Given the random utility model (variable V), there is heterogeneity in consumer valuations after customers try the product (i.e., both realization of v_h and v_l can coexist). A priori, however, we model the problem under two scenarios. We first consider a scenario where consumers are homogeneous ex-ante, meaning that all consumers share the same parameters (i.e., equal θ, v_h, v_l and γ). This scenario therefore exhibits only an ex-post heterogeneity. Then we go on to extend the analysis to a scenario where customers are heterogeneous ex-ante, with respect to their type parameter, θ. This scenario exhibits both ex-ante and ex-post heterogeneity in the market. Under both scenarios, we normalize the market size to 1, without loss of generality.

Consumers with ex-ante homogeneity

For an ex-ante homogeneous market, we can equivalently analyze the decision process of a representative consumer, depicted in Figure 10.

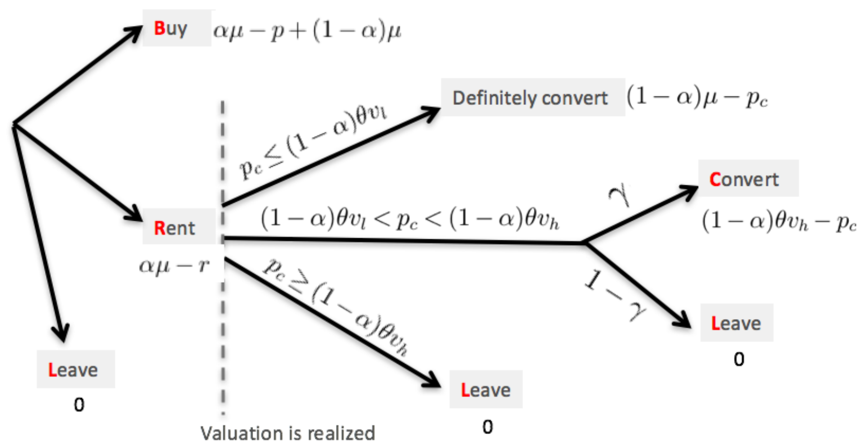


FIGURE 10. Representative consumer decision tree

The representative consumer, having observed the firm's prices r, p and p_c, first faces the decision between buying the product, trying the product, or leaving the market. Buying the product outright yields a total expected surplus of $\mu - p$, while renting (i.e., trying) yields an expected surplus $\alpha\mu - r$. After the rental and realizing the true valuation, the customer shall decide whether to purchase the product or leave[2]. If the conversion price is very low (lower than the minimum possible residual utility, i.e., $p_c \leq (1-\alpha)\theta v_l$), then upgrading to a purchase yields a definite additional (positive) surplus, regardless of the true valuation learned after

[2]One may argue that the customer may consider renting the product again. We briefly study that scenario in the later Extensions section.

trial. The total expected surplus under this branch would be equal to $\alpha\mu - r + (1-\alpha)(\gamma\theta v_h + (1-\gamma)\theta v_l) - p_c = \mu - r - p_c$.[3] On the lowest branch where the conversion price is higher than the maximum possible residual utility (i.e., $p_c \geq (1-\alpha)\theta v_h$), the consumer simply prefers to leave rather than collect a negative surplus. On the middle branch, for which $p_c \in ((1-\alpha)\theta v_l, (1-\alpha)\theta v_h)$, two possibilities arise. If the consumer learns the high valuation v_h, with probability γ, then conversion to purchase yields an additional positive surplus of $(1-\alpha)\theta v_h - p_c$; in the case of the low valuation outcome, the consumer prefers to leave rather than to convert, due to negative resulting surplus (because $(1-\alpha)\theta v_l - p_c < 0$ for this middle range on p_c).

If a consumer is indifferent between two actions, we assume the customer chooses the action that the firm prefers. Below, we explicitly analyze the firm's pricing problem regarding the middle branch in which the consumer rents and then possibly (with probability γ) converts. We later examine the corner solutions associated with the remaining branches.

$$\max_{r, p_c \geq 0} \pi_C(r, p_c) = r - c_r + \gamma(p_c - c_s) \tag{4.1}$$

$$\text{s.t.} \quad \alpha\mu - r + \gamma((1-\alpha)\theta v_h - p_c) \geq 0 \qquad \text{participation constraint}$$

$$(1-\alpha)\theta v_l < p_c < (1-\alpha)\theta v_h$$

We refer to the resulting pricing policy solving this problem, which addresses rentals with purchase conversions, as *pricing policy C*. Since the profit function π_C linearly increases in both r and p_c, the linearly decreasing participation constraint is binding at optimality with a resulting conversion price (as a function of rental price) of $p_c^*(r) = \frac{\mu-(1-\gamma)(1-\alpha)\theta v_l - r}{\gamma}$. The second constraint in the above

[3]For the consumer to prefer this action over outright buying, the firm has to refund back more than 100% of the rental fee toward conversion. It is therefore reasonable to assume that the firm would never set a conversion price strictly lower than $p - r$ (or $\delta > 1$), or otherwise under no condition would a consumer ever prefer to buy the product before trying it.

maximization problem ensures the feasibility of pricing policy C. Thus we check for this constraint at the point of $p_c^*(r)$, which simplifies to $r_{min}^* \equiv \alpha\mu < r < \mu - (1-\alpha)\theta v_l \equiv r_{max}^*$. Therefore for any $r \in (r_{min}^*, r_{max}^*)$, there is a single corresponding $p_c^*(r) \in ((1-\alpha)\theta v_l, (1-\alpha)\theta v_h)$, determined by the linear function of $p_c^*(r)$. The optimal profit under pricing policy C thus simplifies as

$$\pi_C^* = \mu - (1-\gamma)(1-\alpha)\theta v_l - \gamma c_s - c_r \tag{4.2}$$

We now analyze the corner solutions associated with other branches of the consumer tree.

– If the firm sets a high conversion price, i.e., $p_c^* = (1-\alpha)\theta v_h$ (which optimally corresponds to a low rental price, i.e., $r^* = r_{min}^*$), then the consumer rents the product and leaves afterwards. We call this case *pricing policy* R in which the optimal rental fee would be equal to $r_{min}^* = \alpha\mu$ with resulting profit of $\pi_R^* = \alpha\mu - c_r$.

– If the firm sets a low conversion price, i.e., $p_c^* = (1-\alpha)\theta v_l$ (which optimally corresponds to a high rental fee, i.e., $r^* = r_{max}^*$), then the consumer rents and definitely converts to a purchase. We call this case *pricing policy* RC, under which the optimal profit would be equal to $\pi_{RC}^* = r_{max}^* - c_r + p_c^* - c_s = \mu - c_s - c_r$.

Pricing policy RC is dominated by the only remaining policy, *pricing policy* B, where the firm sets the selling price, p^*, equal to μ and extracts the profit of $\pi_B^* = \mu - c_s$. Not surprisingly, the two policies B and RC converge if $c_r \to 0$. The three candidate dominant pricing policies, B, C and R, each can be optimal depending on the market and cost parameters. When pricing policy C outperforms the other two, the ex-ante homogeneous customers first rent the product and a fraction γ of them upgrade to purchase and the rest leave the market. To discover when offering conversion discounts (denoted by δ) is optimal, we need to therefore answer two questions: 1) when does pricing policy C outperform the other two policies (i.e., $\pi_C^* > \max\{\pi_B^*, \pi_R^*\}$), and 2) when is the optimal conversion price less than the optimal selling price. (i.e., $\delta^* = \frac{p^* - p_c^*}{r^*} > 0$). The following two propositions address these questions.

Proposition 12

The pricing policy C outperforms policies B and R iff $\frac{c_s}{v_h(1-\alpha)} < \theta < \frac{(1-\gamma)c_s - c_r}{(1-\alpha)(1-\gamma)v_l}$.

A simple pairwise comparison between optimal profits of the three pricing policies determines the optimality condition of each policy. According to Proposition 12, mid θ and mid-to-low γ corresponds to optimality of pricing policy C, where the consumers are optimally encouraged to try the product before possibly converting to buy. As either θ or γ increases, the firm prefers pricing policy B, while a lower consumer type θ leads to pricing policy R. Higher v_l, which means higher expected valuation, understandably makes the pricing policy C less attractive for the firm. A simple rearrangement of boundaries on θ (in Proposition 12) reveals that $\frac{c_r}{c_s} < \frac{(1-\gamma)(v_h - v_l)}{v_h} \equiv k$ is a necessary condition for the optimality of pricing policy C (a violation of this condition leads to an empty range for θ). An insight from this condition is that when a product is relatively expensive for the firm to sell (i.e., high c_s), letting consumers try it first is more profitable for the firm, relative to selling it outright. to support the optimality of pricing policy C, a larger dispersion of customer's perception on the valuation ($v_h - v_l$), and/or lower chance of realizing a high value (low v_h or low γ), are required. We later see that the same necessary condition applies for the existence of a potential "converter" customer segment under an ex-ante heterogeneous market [4]. We next examine the second question — when are conversion discounts optimal? — in the following proposition.

<u>Proposition 13</u>

Depending on the choice of $(r, p_c^*(r))$, an optimal conversion discount (δ^*) can fall anywhere in the range of $(\frac{1-\frac{(1-\alpha)v_h}{\bar{v}}}{\alpha}, 1)$.

- When choosing the pair $r = r_{min}^*$ and $p_c = p_c^*(r_{min}^*)$, it is optimal to give a discount to converters only if $\alpha > k$. On the contrary, when $\alpha < k$, the firm should optimality raise the conversion price, i.e., $\delta^* < 0$ or equivalently $p_c^* > p^*$.
- As the rental price increases toward r_{max}^*, a larger conversion discount becomes optimal. At r_{max}^*, it is optimal to refund back 100% of the rental fee to converters.

As there are alternative optima for (r, p_c), depicted in Figure 11, we should expect to also find an optimal range for δ, which is defined by end points of $\delta_{min}^* = \frac{p^*-p_c^*(r_{min}^*)}{r_{min}^*}$ and $\delta_{max}^* = \frac{p^*-p_c^*(r_{max}^*)}{r_{max}^*}$. It turns out that δ_{max}^* is calculated to be 100%, always. This result is consistent with our discussion of the consumer decision tree, specifically the upper branch of the second layer. It essentially means that if the firm choose to charge a high rental fee, then consumers need to be promised a full refund toward conversion. At the other end of the spectrum, we have $\delta_{min}^* = \frac{1-\frac{(1-\alpha)v_h}{\bar{v}}}{\alpha} > 0$ iff $\alpha > \frac{(1-\gamma)(v_h-v_l)}{v_h} \equiv k$. For products with large α (i.e., $\alpha > k$), the conversion discount is *always* optimal and the higher the rental price advertised, the higher the optimal conversion discount should be.

[4] For products with $c_s = c_r$, pricing policy C will never be optimal (θ range would be empty). This can have implication for digital products with zero marginal cost.

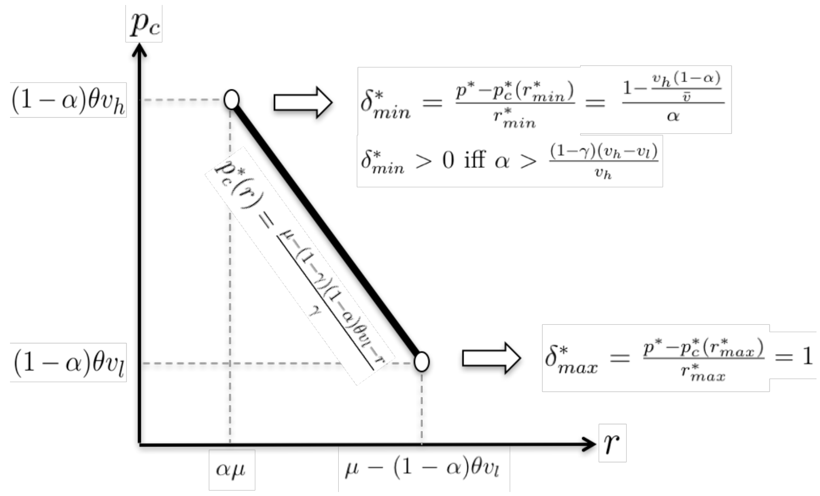


FIGURE 11. Optimal prices under pricing policy C

Although we do not account for any type of irrational consumer behavior regarding the mentality toward gain and loss, the existence of alternative optima gives a firm the flexibility to choose the right policy that can play out well in the marketing and psychological aspects. The magnitude of δ^* increases non-linearly with α (i.e. $\frac{d\delta^*_{min}}{d\alpha} > 0$). However, for a product with $\alpha < k$, the optimal discount size depends on the size of rental fee and conversion price. If the firm chooses the alternative optima pair of (r, p_c) closer to the endpoint of $(\mu-(1-\alpha)\theta v_l, (1-\alpha)\theta v_l)$, then offering a relatively large conversion discount is optimal such that $\delta^* \to 1$. However, if the choice of (r, p_c) pair falls closer to the endpoint of $(\alpha\mu, (1-\alpha)\theta v_h)$, no conversion discount is optimal.

The results also show that it is possible that the firm should charge a premium (over the selling price if it were to sell the product outright) after the rental. Consider a product for which $\alpha < k$ and $(r, p_c) = (\alpha\mu, (1-\alpha)\theta v_h)$. In this case, the conversion price optimally exceeds the optimal selling price of $p^* = \mu$ (i.e., $\delta^* < 0$).

Consumers with ex-ante heterogeneity

In this section, we consider the case where consumers are heterogeneous in terms of their *type* parameter θ. Let θ be uniformly dispersed between $\underline{M}$ and $\overline{M}$ across the market. Recall that $\overline{v} \equiv \gamma v_h + (1-\gamma)v_l$ and thus the expected product valuation for consumer type θ is $\mu \equiv \theta\overline{v}$. Depending on their type, a consumer will self select from the action set $\{B, C, R, L\}$ where B refers to outright buying, C refers to potentially converting after renting, R refers to only renting once, and L stands for leaving. Consumer surplus maximization dictates that a high type consumer will prefer outright buying to renting, while a lower θ prefers to rent first while being open to convert to purchase, especially when some refund toward purchase is available. An even smaller θ forces the consumer to select one-time renting, or not participating as a customer at all. Looking back to Figure 10, we can define the three participating customer segments by forming the selection/participation constraints in sets $\Lambda_i(\theta)$ for $i \in \{B, C, R\}$:

- Outright buyers: $\Lambda_B(\theta) = \{\theta|\theta\bar{v} - p \geq \alpha\theta\bar{v} - r + \gamma((1-\alpha)\theta v_h - p_c)\}$
- Potential converters: $\Lambda_C(\theta) = \{\theta|\theta\bar{v} - p \leq \alpha\theta\bar{v} - r + \gamma((1-\alpha)\theta v_h - p_c) \geq \alpha\theta\bar{v} - r\}$
- One-time renters: $\Lambda_R(\theta) = \{\theta|\alpha\theta\bar{v} - r + \gamma((1-\alpha)\theta v_h - p_c) \leq \alpha\theta\bar{v} - r \geq 0\}$

Figure 12 showcases an instance of market segments for a range on p_c. In this example, the firm should offer some discount i.e., $p_c \leq p$, to encourage the existence of potential converters segment. We can thus formulate the firm's decision problem as follows:

$$\max_{r,p,p_c \geq 0} \pi(p, r, p_c) = \frac{1}{\overline{M} - \underline{M}}[\Lambda_B(\theta)(p - c_s) + \Lambda_C(\theta)(r - c_r + \gamma(p_c - c_s)) + \Lambda_R(\theta)(r - c_r)] \tag{4.3}$$

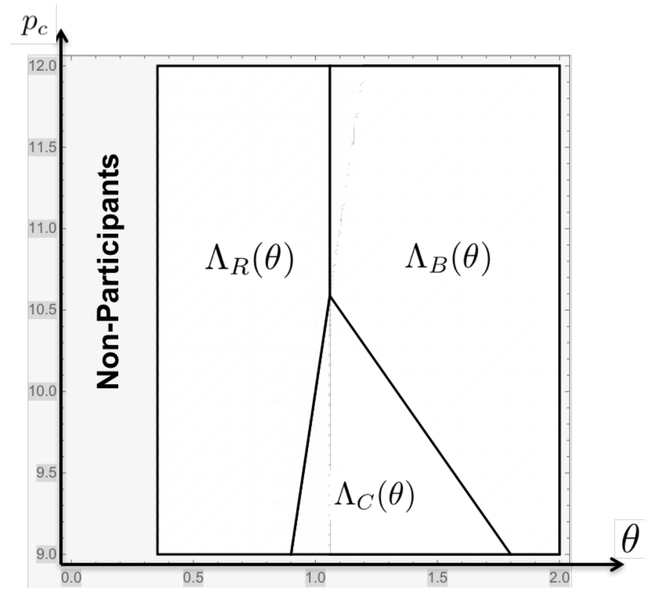


FIGURE 12. An instance of market segmentation

Beyond a unique interior solution to this decision problem, there are five potential corner solutions, each corresponding to a particular market segmentation policy. For a detailed solution, please refer to the Appendix for this chapter. The following proposition summarizes the results:

Proposition 14

Depending on the market and cost parameters, one of the six pricing policies, denoted by $S \in \{BCR, BC, BR, CR, B, R\}$, are optimal.

The analytic expressions of optimality conditions, denoted by Γ_S sets, are presented in the Appendix. Figure 13 illustrates the firm's optimal policy map with respect to the changes in two parameters of α and c_r (for parameter set of $v_h = 20$, $v_l = 5$, $\gamma = 0.5$, $c_s = 5$, $\underline{M} = 0$ and $\overline{M} = 2$) Depending on the parameter values, one of the six pricing policies dominates the rest. Each policy is named such that it presents which customer segments will be active in the market. For example, BR corresponds to a pricing scheme that stimulates outright buyers and one-time

TABLE 6. Optimal pricing policies for a heterogeneous market

S	r^*	p^*	p_c^*	Optimality Condition: Γ_S
BCR	$\frac{\alpha\overline{M}\overline{v}+c_r}{2}$	$\frac{\overline{M}\overline{v}+c_s}{2}$	$\frac{(1-\alpha)\overline{M}v_h+c_s}{2}$	Γ_{BCR}
BC	$\frac{\alpha p_c^*\overline{v}}{(1-\alpha)v_h}$	$\frac{\overline{M}\overline{v}+c_s}{2}$	$\frac{(1-\alpha)v_h(\gamma(c_s+\overline{M}(v_h-\alpha v_l))+c_r+\alpha\overline{M}v_l)}{2(\gamma v_h+\alpha v_l(1-\gamma))}$	Γ_{BC}
BR	$\frac{\alpha\overline{M}\overline{v}+c_r}{2}$	$\frac{\overline{M}\overline{v}+c_s}{2}$	$\frac{1}{2}v_h\left(\frac{c_s-c_r}{\overline{v}}+\overline{M}(1-\alpha)\right)$	Γ_{BR}
CR	$\frac{\alpha\overline{M}\overline{v}+c_r}{2}$	$r^*+p_c^*\gamma+$ $\overline{M}v_l(1-\alpha)(1-\gamma)$	$\frac{(1-\alpha)\overline{M}v_h+c_s}{2}$	Γ_{CR}
B	αp^*	$\frac{\overline{M}\overline{v}+c_s}{2}$	$\frac{p^*v_h(1-\alpha)}{\overline{v}}$	Γ_B
R	$\frac{\alpha\overline{M}\overline{v}+c_r}{2}$	$r^*+\overline{M}(1-\alpha)\overline{v}$	$\overline{M}v_h(1-\alpha)$	Γ_R

renters. As expected, low values of c_r induce the optimality of pricing policies

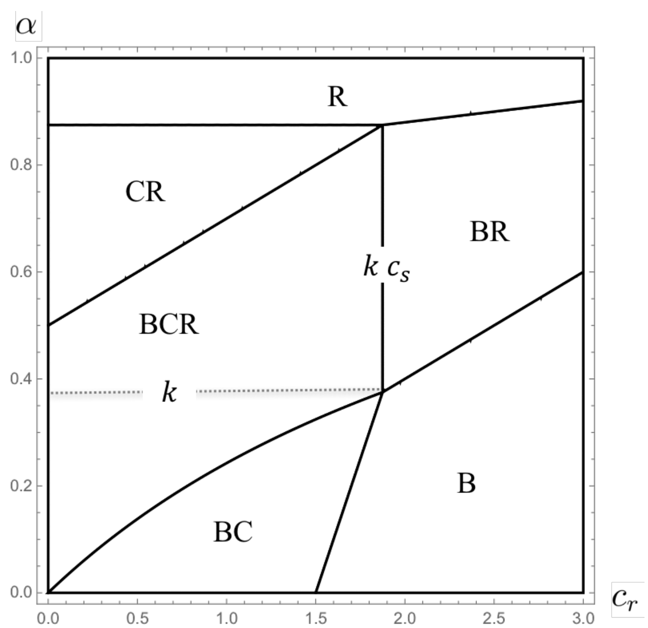


FIGURE 13. Optimal pricing policies in the plane of $\alpha - c_r$

BC, BCR, CR and R, under which one-time renting (segment R) and/or trying (segment C) are voluntarily active. When the utility gain from the first usage (α) is very low, it is difficult to optimally encourage one-time renting and hence pricing scheme BC dominates the rest. In the opposite direction, α values close to 1 induce policy R where all participating customers optimally are encouraged to be a one-time renter. Let's look at optimal prices for $c_r = 1$ over a range on α in Figure 14. Low values of α drive the one-time renters out of the market. Moderate values of

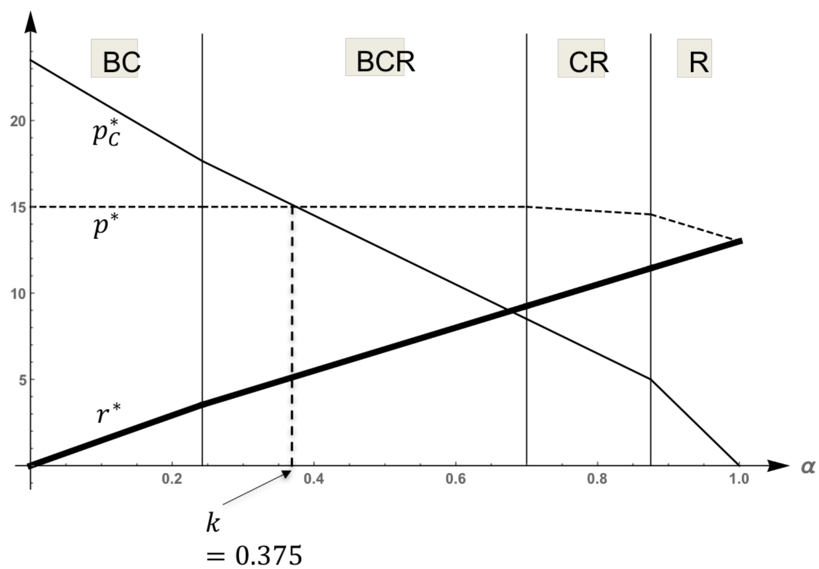


FIGURE 14. Optimal prices with respect to α

α activate all three possible segments. A product with higher α calls for a higher rental fee and a relative lower p_c. We can see that post-rental conversion discounts are optimal for $\alpha > k = 0.375$. The following proposition derives this condition formally.

Proposition 15

$\frac{c_r}{c_s} < k < \alpha$ (where $k \equiv \frac{(v_h - v_l)(1-\gamma)}{v_h}$) is a necessary and sufficient condition for the co-optimality of encouraging purchase upgrades with conversion discounts, under pricing policy BCR. If $\alpha < k$, it is optimal to charge a premium conversion price (i.e., $p_c^* > p^*$) under policy BCR or BC.

The first inequality, i.e., $\frac{c_r}{c_s} < k$, derives the optimality of pricing policy BCR over BR pricing in which the converter segment is priced out of the market. The inequality $\alpha > k$ ensures the optimality of conversion discounts (i.e., $p^* < p_c^*$). We can further compute the optimal conversion discount as $\delta^* = \frac{p^*-p_c^*}{r^*} = \frac{\overline{M}(\bar{v}-(1-\alpha)v_h)}{\alpha\overline{M}\bar{v}+c_r}$. Figure 15 showcases the sensitivity of δ^* wrt to α and γ (for parameter set of $v_h = 20$, $v_l = 5$, $c_r = 1$, $c_s = 5$, $\underline{M} = 0$ and $\overline{M} = 2$). The optimal discount increases in γ and in α. When there is a higher probability of conversion (higher γ), the optimal conversion discount increases. For example, consider a product with $\alpha = 0.4$. For the parameter set of Figure 15 and $\gamma = 0.5$, we have $r^* = \$5.5$, $p^* = \$15$ and $p_c^* = \$14.5$ i.e., it is optimal for the firm to refund back 10% of the rental fee to converters. However, if γ decreases to 40%, then we have $r^* = \$4.9$, $p^* = \$13.5$ and $p_c^* = \$14.5$, i.e., the firm should charge a premium of \$1 to converters i.e. $\delta^* < 0$.

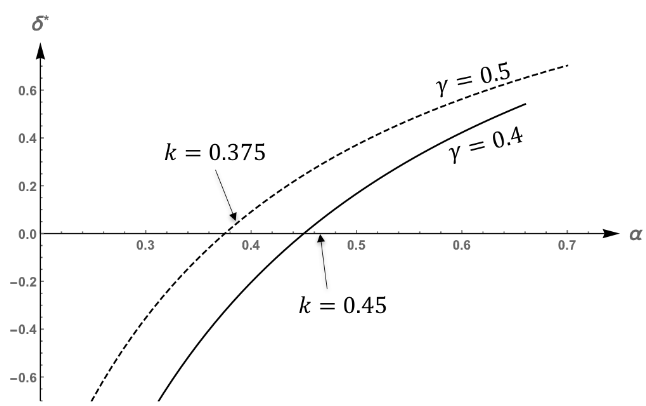


FIGURE 15. Optimal conversion discount with respect to α and γ

Extensions

In this section, we: (i) discuss the implications of second rentals, (ii) contrast trials versus returns, and (iii) consider stochastic valuations that follow a continuous distribution.

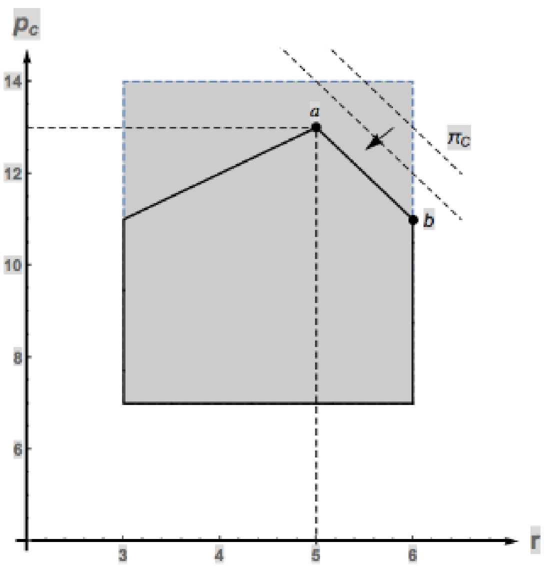
We now assume that the first rental utility proportion equals α_1 and the second rental utility proportion equals α_2. We focus our attention to cases where $\alpha_2\theta v_l < r < \alpha_2\theta v_h$ (condition 1) and $(1-\alpha_1)\theta v_l < p_c < (1-\alpha_1)\theta v_h$ (condition 2) hold. Condition 1 assures that a second rental can be an optimal decision for the customer. If the customer learns v_h, then the second rental yields positive surplus. If $r < \alpha_2\theta v_l$ (or $r > \alpha_2\theta v_h$) second rental is optimal (not optimal) regardless of the outcome of the learning. Similar reasoning holds for justifying the second condition. Outside these ranges, the problem is trivial. We are still interested to find when the firm wants to optimally encourage a customer to try once before committing to purchase. We can find the optimal prices under policy C (i.e., encouraging the rep customer to rent first and then possibly convert (with probably γ)) by solving the following problem:

$$\max_{r,p_c\geq 0} \pi_C(r,p_c) = r - c_r + \gamma(p_c - c_s) \tag{4.4}$$

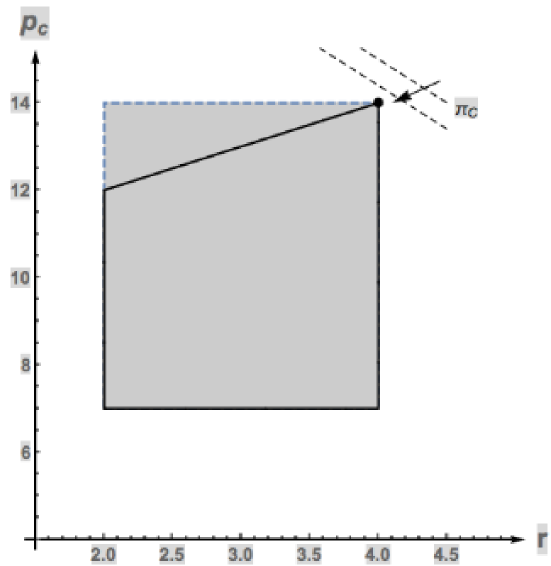
$$\text{s.t.} \quad \alpha_1\mu - r + \gamma((1-\alpha_1)\theta v_h - p_c) \geq 0 \qquad \text{participation constraint}$$

$$(1-\alpha_1)\theta v_h - p_c > \alpha_2\theta v_h - r \qquad \text{selection constraint}$$

Figure 16(a) showcases the scenario with multiple optima lying on the participation constraint, bounded by two end points, determined by $[r^a = \frac{\theta}{1-\gamma}((\alpha_1+\alpha_2)v_h + \alpha_1(1-\gamma)v_l),\ p_c^a = \frac{\theta}{1-\gamma}((1-\alpha_1-\alpha_2+\gamma)v_h + \alpha_1(1-\gamma)v_l)]$ and $[r^b = \alpha_2\theta v_h,\ p_c^b = \frac{\theta}{\gamma}((\gamma-\alpha_2)v_h + \alpha_1(1-\gamma)v_l)]$. Optimal profit under this case equals to $\pi_C^* = \alpha_1\theta v_l - c_r + \gamma(\theta(v_h - \alpha_1 v_l) - c_s)$. The parameters underlying this figure are $v_h = 20$, $v_l = 10$. $\gamma = 0.5$, $c_s = 3$, $c_r = 1$, $\theta = 1$, and $\alpha_1 = 0.3$. Note that when point (a) falls outside the rectangle specified by conditions (1) and (2), we should refer to Figure 16(b). Thus Figure 16(a) is viable



(a) $\alpha_2 = 0.3 > \alpha_1\bar{v}/v_h = 0.225$



(b) $\alpha_2 = 0.2 < \alpha_1\bar{v}/v_h = 0.225$

FIGURE 16. Feasible price region when the customer considers a second rental

as long as $r^a < \alpha_2\theta v_h$ and $p_c^a < (1-\alpha_1)\theta v_h$. These simplify to $\alpha_2 > \alpha_1\bar{v}/v_h$. Figure 16(b) shows the feasible region under $\alpha_2 < \alpha_1\bar{v}/v_h$, for which case we hit a corner solution at $[r^* = \alpha_2\theta v_h,\, p_c^* = (1-\alpha_1)\theta v_h]$ with resulting profit of $\pi_C^* = \alpha_2\theta v_h - c_r + \gamma(\theta(1-\alpha_1)v_h - c_s)$. Two other candidate pricing policies are 1)to set a selling price $p = \mu$ with resulting profit of $\pi_B^* = \mu - c_s$ or 2) to rent once at $r^* = \alpha_1\mu$ with resulting profit of $\pi_R^* = \alpha_1\mu - c_r$. Focusing on scenarios like Figure 16(a), we can see that $\pi_C^* > \max\{\pi_R^*, \pi_B^*\}$ holds iff $\frac{c_s}{v_h(1-\alpha_1)} < \theta < \frac{c_s(1-\gamma)-c_r}{(1-\alpha_1)(1-\gamma)v_l}$. We can further see that $p^* > p_c^a$ iff $\alpha_1(v_h - v_l(1-\gamma)) + \alpha_2 v_h > (v_h - v_l)(1-\gamma^2)$. This condition implies that for conversion discounts to be optimal, both α_1 and α_2 should be relatively large. This result is consistent with Proposition 13 and 15.

Comparison of trials with a simple return policy

In this section we analyze whether there are conditions under which consumer uncertainty is more efficiently (i.e., profitably) resolved via trials (rentals) than by allowing for post-purchase product returns. One scenario of a return policy can be to let the customer try the product by paying a fee (=r). If the customer likes the product (realizing v_h), he can upgrade to purchase by paying $p_u - r$, which means a full refund of the fee. If the product doesn't meet the expectation (realizing $v_l = 0$ for simplicity), the customer can return the product, despite having used it, and receive 100% refund (equal to $+r$). Thus the expected surplus for consumer type θ under this return policy would be equal to

$$ES_u = \alpha\mu - r + \gamma \max\{(1-\alpha)\theta v_h - p_u + r, r\} + (1-\gamma)r$$

After trying the product and liking it (realizing v_h), the customer will choose to upgrade if $(1-\alpha)\theta v_h + r - p_u > r$. To encourage upgrades, then the firm should set $p_u^* = (1-\alpha)\theta v_h - \epsilon$. At this upgrade price, total expected surplus the customer receives at time zero will net to $\alpha\mu - r + \gamma((1-\alpha)\theta v_h - p_u^* + r) + (1-\gamma)r = \alpha\mu > 0$. Considering an ex-ante homogeneous market, the firms profit $= r - c_r + \gamma(p_u^* - r - c_s) + (1-\gamma)(-r) = \gamma(1-\alpha)\theta v_h - \gamma c_s - c_r$. This expression can be compared to equation 4.2. One can see that such a policy always underperforms the Pricing Policy C for an ex-ante homogeneous market, in terms of optimal profit. It is not a surprising result because when allowing for returns and full refund, the customer optimally leaves with positive surplus (which could be turned into profit for the firm).

Modelling valuation uncertainty with a continuous distribution

We now let the customer's uncertain valuation V follow a uniform distribution, i.e., $V \sim U[\mu - \epsilon, \mu + \epsilon]$, rather than a discrete distribution (as was the assumption so far). Similar to before, a customer may choose to buy the product outright (B), rent first to possibly convert (C) or be a certain one-time renter (R). The expected surplus of buying outright would be

$$ES_B(\mu) = E[\alpha V - p + (1-\alpha)V] = \mu - p. \tag{4.5}$$

Renting the product first yields the expected surplus of

$$ES_R(\mu) = E[\alpha V - r] = \alpha\mu - r. \tag{4.6}$$

Calculating expected surplus associated with a product trial before possibly buying, which we denote as $ES_C(\mu)$, is now more involved. By trying the product, the consumer learns the true v somewhere in the range of $[\mu - \epsilon, \mu + \epsilon]$ and faces a subsequent decision between converting to purchase or leaving. Through conversion, an extra surplus of $(1-\alpha)v - p_c$ can be earned, so conversion yields a nonnegative surplus iff $v > p_c/(1-\alpha) \equiv v_c$. The probability of conversion then can be calculated as $\int_{v_c}^{\mu+\epsilon} 1/(2\epsilon)dv = \frac{\mu+\epsilon-v_c}{2\epsilon}$ denoted as $Pr(\mu, p_c)$. The probability of conversion ($Pr(\mu, p_c)$) is linearly increasing (decreasing) in μ (p_c). However it is nonlinearly increasing (decreasing) in ϵ for $\mu < v_c$ ($\mu > v_c$). Checking for $0 \leq Pr(\mu, p_c) \leq 1$ results in $\underline{\mu} \equiv v_c - \epsilon \leq \mu \leq v_c + \epsilon \equiv \overline{\mu}$. This implies that for a relatively low consumer type (i.e., $\mu < \underline{\mu}$), the conversion is not going to be optimal, i.e., the chance of conversion is pre-known to that consumer to be zero. However, for a high consumer type (i.e., $\mu \geq \overline{\mu}$), the conversion is unquestionably optimal. We now are ready to calculate the expected surplus for a consumer with type μ who tries the product with the positive chance of conversion as:

$$ES_C(\mu) = \alpha\mu - r + Pr(\mu, p_c)((1-\alpha)\frac{v_c + \mu + \epsilon}{2} - p_c) \tag{4.7}$$

Let us consider the setting in which customers are heterogeneous in term of the parameter μ with $\mu \sim U[\underline{M}, \overline{M}]$, similar to our assumption on θ as the consumer type distribution main part of the paper. From the equations (4.5), (4.6) and (4.7), we can form consumer selection constraints to determine the μ range for each consumer segment. The consumer type μ selects to be:

- an outright buyer if $\mu \in \{\mu | ES_B(\mu) > ES_C(\mu)\} \equiv (\mu_{bc}, \overline{M}]$ where $\mu_{bc} = \frac{p_c - 2\sqrt{\epsilon(1-\alpha)(p_c+r-p)}}{1-\alpha} + \epsilon = \overline{\mu} - 2\sqrt{\frac{\epsilon(p_c+r-p)}{1-\alpha}}$ [5]
- a potential converter if $\mu \in \{\mu | ES_B(\mu) \leq ES_C(\mu) \geq ES_R(\mu)\} \equiv [\underline{\mu}, \mu_{bc}]$
- a one-time renter if $\mu \in \{\mu | ES_C(\mu) < ES_R(\mu) \geq 0\} \equiv [\frac{r}{\alpha}, \underline{\mu})$

The firm's profit maximization problem can be formulated as

$$\max_{p,r,p_c \geq 0} \pi(p, r, p_c) = \frac{1}{\overline{M} - \underline{M}}((\overline{M} - \mu_{bc})(p - c_s) + \int_{\underline{\mu}}^{\mu_{bc}} (r - c_r + Pr(\mu, p_c)(p_c - c_s))d\mu + (\underline{\mu} - \frac{r}{\alpha})(r - c_r))$$

s.t. $\quad \overline{M} \geq \mu_{bc} \geq \underline{\mu} \geq \frac{r}{\alpha} \geq \underline{M} \geq 0$

Clearly $\underline{\mu} < \mu_{bc}$ is a necessary and sufficient condition for the potential converter segment to be non-empty [6]. This inequality simplifies to $p_c \leq p - r + \epsilon(1-\alpha) \equiv \bar{p}_c$. Therefore $p_c^* < \min\{\bar{p}_c, p^*\}$ is the necessary and sufficient condition

[5] $ES_B(\mu) == ES_C(\mu)$ have two roots. The second root is always larger than $\overline{\mu}$ and thus ruled out

[6] $\underline{\mu} < \mu_{bc}$ is simplified to $\sqrt{(p_c - p + r)(1-\alpha)\epsilon} < (1-\alpha)\epsilon$.

for optimality of conversion discounts. The following proposition addresses our question of interest: under what condition are the conversion discounts optimal?

Proposition 16

Conversion discounts are optimal for large α:

$$p_c^* < p^* \leftarrow \alpha > \frac{\sqrt{\epsilon\left(8c_r\overline{M}^2 + \epsilon(c_r + \overline{M})^2\right)} + (\overline{M} - c_r)\epsilon}{2\overline{M}(2\overline{M} + \epsilon)}$$

Consistent with our previous results, we observe that there should exist a lower bound on α for conversion discounts to be optimal. Thus we see that the assumption of discrete distribution on V drives us the same qualitative results.

Conclusion

In this paper, we address the popular pricing technique for experience goods under which customers are encouraged to try a product to resolve the valuation uncertainty. Should they decide to upgrade from a rental to a purchase, a partial or full refund of the rental fee may be applicable, depending upon the firm's pricing policy. We study the firm's optimal pricing problem for consumers who are, alternatively, either ex-ante identical (i.e., homogeneous) and ex-ante heterogeneous. For the case of the ex-ante homogeneous market, we proved that if the representative customer type belongs to a middle range and the ratio of the rental cost to selling cost is small enough, then the firm implements the try before you buy pricing policy. In that case, the firm can choose the rental fee and the conversion price from a range of alternative optima. Depending on the magnitude of rental fee charged, the conversion discount might or might not be optimal. Depending on the desired marketing strategy, the firm could choose to raise the rental fee to thus optimally afford offering conversion discounts. This may be valuable to the firm in presence of irrational consumer behavior where the refunds may be perceived differently than being charged a lower price in the first place. For the case of the heterogeneous market, we showed that for conversion discounts to be optimal, rental-to-selling cost must be smaller than a threshold (k), and the degree of diminishing marginal utility (α) must be larger than the same threshold. In other words, for a product to be qualified for a conversion discount, the portion of the utility that is derived from the first usage should be considerable and it should be specifically large than the rent-to-sell cost ratio. As the dispersion between the two possible learning outcomes ($v_h - v_l$) (or the probability of realizing a low value $= 1 - \gamma$) increases, more customers choose to try the product first (rather than to buy outright). However, that reduces the chance of optimal offering of conversion discounts. Indeed, when the dispersion surpasses a threshold, then the firm should increase the conversion price rather than reducing it. In that case, the customers who end up liking the product and want to upgrade to a purchase are subjected to a tax on their learning. If the customers have access to ways to hide their rental history, they can escape this higher post-trial price by acting like new customer, and pay the selling price that is lower in this case.

CHAPTER V

CONCLUSION

This dissertation consists of three essays addressing problems that lie at the interface of marketing and operations management with a focus on pricing. The *first* essay (chapter II) investigated the economics of optimally going green by solely selling a product that has some degree of recycled content in it. When the profit maximization problem results in a green-only product line, we showed that the firm profitably omits the non-green version from the product line and serve the customers - regardless of what group they belong to - with a single green product and thus can honestly claim as a green firm. However, the conditions should be right so that the firm can optimally adopt a green-only strategy. For instance, recycled material should be cheaper than virgin raw material and production of a green product shouldn't be very costly, or otherwise green products cannot be justified from a cost perspective. Also the customers shouldn't strongly prefer one of the products over the the other one. Because once they do, then the profits can increase by maintaining both product types in the line and serve each group of customers by their own targeted product and a uniformly green line cannot sustain. A rather more surprising result relates to whether the firm can or cannot choose the degree of recycled content or in other words, optimize the green quality. At some cases, the seller does not have the flexibility to decide on the degree of recycled content, such as retailers who just buy the product and not manufacture it. We found that under this case, the green-only product can sustain when the product is not very green. Only then the endogenous price of green product can be attractive to both consumer types. However if the firm can influence the design and thus quality of the product, we found that a maximally green product sustains as the result of price and quality optimization.

In the *second* essay (chapter III), we explored the time-related dynamics of price discounts. In particular, we analyzed whether it is optimal to shift discounts in time rather to offer immediate lower prices? Delayed discounts — a percentage of today's spending applied to future purchases — are quite common in practice. It is quite reasonable to quickly conclude that a firm should prefer delayed discounts if (i) customers may forget to redeem their credits or (ii) if customers irrationally weigh the credit more than real money or (iii) if the firm has a higher time value of money relative to the customers. However, we have explored whether delayed discounts are beneficial, if the firm's consumers are fully rational. We established that delayed discounts are equivalent in performance to immediate discounts when the posted pre-discounted prices (i.e., consumer spending) are stable over time, as long as the firm scales up the delayed reward rate to compensate the loss in consumer surplus for the wait the customer incurs, before credit redemption. This result is independent of whether the rewards are personalized for each customer or a single reward percentage is applied to a range of heterogeneous customers. If the posted prices, and consequently the customer spending, vary over time, the results change. Specifically when prices fluctuate over time, the customer faces a larger number of net prices (price minus credit) over time with delayed discounts — relative to immediate discounts — and thus has more flexibility to self select the shopping pattern that maximizes her surplus. With a personalized reward percentage, we find that delayed discounts potentially increase profits if a customer's valuation falls into a particular range. Regardless, the optimal discount percentage should be higher when discounts are delayed. When the firm has to choose a single discount percentage and must apply that uniformly to a range of customers, we prove that delayed discounts segment the market more efficiently and turn more of the customers into frequent shoppers. By inducing more consumption, the delayed discount accumulate more profits over time. Hence the delayed discounts are proven to be more successful in the more realistic setting in which prices fluctuate and rewards are not customized per customer.

In the *third* essay (chapter IV), we studied “try-before-you-buy” pricing, which applies in practice to goods with valuation uncertainty. We allow for both selling and renting a durable good and consider renting as a learning tool for customers who prefer to resolve their uncertainty — by paying a fee – before committing to purchase a product. Inspired by practice, we examine when it is optimal to stimulate post-rental conversions to purchase by offering a refund of the rental fee —partial or full— as a discount applied to the selling price. This incentive might be required for those products whose value drop significantly after the first usage. Our findings show that a relatively small ratio of the unit rental vs. selling cost is necessary to profitably encourage customers to try the product. To optimally giving a rental rebate toward conversion, a relatively large portion of the overall product utility must have been derived during the trial or rental period.

APPENDIX A

CHAPTER 2: TECHNICAL PROOFS

Proof of Lemma 1: Given prices p_b and p_g, and the green variant's recycled content β, each consumer with base valuation v (i.e., valuation for the base product) assesses her surplus from buying the base product, the green variant, and the no-purchase option. If a type-i ($i \in \{n, c\}$) consumer's participation constraint for product variant j ($j \in \{b, g\}$) is satisfied, and the same customer's surplus from purchasing product variant j is more than her surplus from the other product variant (i.e., her self-selection constraint for product j is satisfied), then that consumer is included in demand D_{ji}. We highlight these participation and self-selection constraints for each demand segment in the second and third columns of Table 7. As consumer valuations follow uniform distribution over the unit interval, the aforementioned constraints yield the demand quantities highlighted in the fourth column of Table 7. Imposing the price restrictions outlined in the "Price Range" column of Table 1 on the aforementioned demand quantities yield the remaining entries in Table 1.

TABLE 7. The participation and self-selection constraints

Demand Segment	Participation	Self-Selection	Demand
D_{bc}	$v - p_b \geq 0$	$v - p_b \geq (1-\alpha_c\beta)v - p_g$	$\omega(1 - \max\{\min\{\frac{p_b-p_g}{\alpha_c\beta}, 1\}, p_b\})$
D_{gc}	$(1-\alpha_c\beta)v - p_g \geq 0$	$(1-\alpha_c\beta)v - p_g \geq v - p_b$	$\omega(\max\{0, \min\{\frac{p_b-p_g}{\alpha_c\beta}, 1\} - \frac{p_g}{(1-\alpha_c\beta)}\})$
D_{gn}	$(1+\alpha_n\beta)v - p_g \geq 0$	$(1+\alpha_n\beta)v - p_g \geq v - p_b$	$(1-\omega)(1 - \max\{\min\{\frac{p_g-p_b}{\alpha_n\beta}, 1\}, \frac{p_g}{1+\alpha_n\beta}\})$
D_{bn}	$v - p_b \geq 0$	$v - p_b \geq (1+\alpha_n\beta)v - p_g$	$(1-\omega)(\max\{0, \min\{\frac{p_g-p_b}{\alpha_n\beta}, 1\} - p_b\})$

Proof of Lemma 2: We will prove the Lemma in five parts, each corresponding to one demand scenario s when $I_s = 1$, $\forall s \in \{UB, CT, PT, NT, UG\}$. In each part, for brevity, we will denote $\pi_s(p_b, p_g|\beta)$ as π_s.

When $I_{UB} = 1$, the demand expressions of Table 1 yield $\pi_{UB} = (p_b - c_b)(1 - p_b)$. Taking the first and the second order derivatives with respect to p_b yields $\frac{d^2\pi_{UB}}{dp_b{}^2} = -2 < 0$. Furthermore, π_{UB} does not depend on p_g. Therefore, function π_{UB} is jointly concave for prices that yield $I_{UB} = 1$.

When $I_{CT} = 1$, the demand expressions of Table 1 yield $\pi_{CT} = (p_b - c_b)(\omega(1 - p_b) + (1-\omega)(\frac{p_g - p_b}{\alpha_n\beta} - p_b)) + (p_g - c_g)((1-\omega)(1 - \frac{p_g - p_b}{\alpha_n\beta}))$, which has the following Hessian:

$$H(\pi_{CT}) = \begin{bmatrix} \frac{-2(1+\alpha_n\beta-\omega)}{\alpha_n\beta} & \frac{2(1-\omega)}{\alpha_n\beta} \\ 0 & \frac{-2(1-\omega)}{\alpha_n\beta} \end{bmatrix}$$

As $\frac{\partial^2\pi_{CT}}{\partial p_b{}^2} = \frac{-2(1+\alpha_n\beta-\omega)}{\alpha_n\beta} < 0$ and $\frac{\partial^2\pi_{CT}}{\partial p_g{}^2} = \frac{-2(1-\omega)}{\alpha_n\beta} < 0$, and the determinant of $H(\pi_{CT})$ is positive, i.e., $\frac{-2(1+\alpha_n\beta-\omega)}{\alpha_n\beta} \cdot \frac{-2(1-\omega)}{\alpha_n\beta} - 0 > 0$ function π_{CT} is jointly concave for prices that yield $I_{CT} = 1$.

When $I_{PT} = 1$, the demand expressions of Table 1 yield $\pi_{PT} = (p_b - c_b)(\omega(1 - p_b)) + (p_g - c_g)((1-\omega)(1 - \frac{p_g}{1+\alpha_n\beta}))$, which has the following Hessian:

$$H(\pi_{PT}) = \begin{bmatrix} -2\omega & 0 \\ 0 & \frac{-2(1-\omega)}{\alpha_n\beta} \end{bmatrix}$$

As $\frac{\partial^2\pi_{PT}}{\partial p_b{}^2} = -2\omega < 0$ and $\frac{\partial^2\pi_{PT}}{\partial p_g{}^2} = \frac{-2(1-\omega)}{\alpha_n\beta} < 0$, and the determinant of $H(\pi_{PT})$ is positive, i.e., $-2\omega \cdot \frac{-2(1-\omega)}{\alpha_n\beta} - 0 > 0$, function π_{PT} is concave for prices that yield $I_{PT} = 1$.

When $I_{NT} = 1$, the demand expressions of Table 1 yield $\pi_{NT} = (p_b - c_b)(\omega(1 - \frac{p_b - p_g}{\alpha_c\beta})) + (p_g - c_g)((1-\omega)(1 - \frac{p_g}{1+\alpha_n\beta}) + \omega(\frac{p_b - p_g}{\alpha_c\beta} - \frac{p_g}{1-\alpha_c\beta}))$, which has the following

Hessian:

$$H(\pi_{NT}) = \begin{bmatrix} \frac{-2w}{\alpha_c\beta} & \frac{2w}{\alpha_c\beta} \\ 0 & \frac{-2(1-w)}{1+\alpha_n\beta} + \frac{-2w}{\alpha_c\beta(1-\alpha_c\beta)} \end{bmatrix}$$

As $\frac{\partial^2\pi_{NT}}{\partial {p_b}^2} = \frac{-2w}{\alpha_c\beta} < 0$ and $\frac{\partial^2\pi_{NT}}{\partial {p_g}^2} = \frac{-2(1-w)}{1+\alpha_n\beta} + \frac{-2w}{\alpha_c\beta(1-\alpha_c\beta)} < 0$, and the determinant of $H(\pi_{CT})$ is positive, i.e., $\frac{-2w}{\alpha_c\beta} \cdot (\frac{-2(1-w)}{1+\alpha_n\beta} + \frac{-2w}{\alpha_c\beta(1-\alpha_c\beta)}) - 0 > 0$, function π_{NT} is jointly concave for prices that yield $I_{NT} = 1$.

When $I_{UG} = 1$, the demand expressions of Table 1 yield $\pi_{UG} = (p_g - c_g)(\omega(1 - \frac{p_g}{1-\alpha_c\beta}) + (1-\omega)(1 - \frac{p_g}{1+\alpha_n\beta}))$. Taking the first and the second order derivatives with respect to p_g yields $\frac{d^2\pi_{UG}}{d{p_g}^2} = \frac{-2(1-\omega)}{1+\alpha_n\beta} + \frac{-2\omega}{1-\alpha_c\beta} < 0$. Furthermore, π_{UG} does not depend on p_b. Therefore, π_{UG} is jointly concave for prices that yield $I_{UG} = 1$.

To prove that function $\pi = \sum_{\forall s} I_s\pi_s$ is not jointly concave in p_b and p_g for all prices satisfying $p_b \leq 1$ and $p_g \leq \bar{v}_n$, we provide a numerical example as depicted in Figure 17. In this figure, we have function $\pi = \sum_{\forall s} I_s\pi_s$ for $p_b \in \{0.85, 0.90, 1.00\}$ and $p_g \in [0.6, \bar{v}_n]$, where $\omega = 0.7$, $c_v = 0.7$, $c_r = 0.2$, $k = 0.05$, $\alpha_c = 0.3$, $\alpha_n = 0.4$, and $\beta = 0.5$, implying $\bar{v}_n = 1 + \alpha_n\beta = 1.2$.

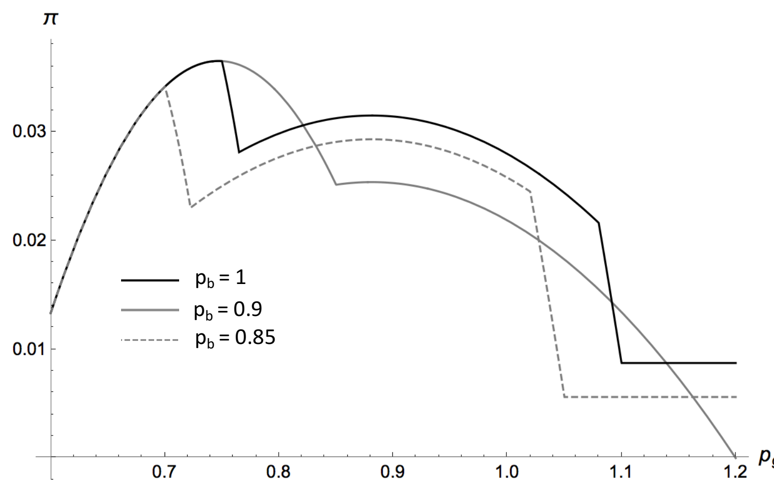


FIGURE 17. Profit function is not jointly concave over the whole region

Proof of Proposition 1: We will prove the proposition in five parts. For each part, we will specialize non-linear program (2.2) to one demand scenario s when $I_s = 1$ ($\forall s \in \{UB, CT, PT, NT, UG\}$), which implies the constraint regarding the prices in the "Price Range" column of Table 1 holds. For each demand scenario s, we will denote by ${p_b}_s^{FOC}$ and ${p_g}_s^{FOC}$ the prices that solve the firm's unconstrained optimization problem, which satisfy $\frac{\partial \pi_s}{\partial p_b}|_{p_b = {p_b}_s^{FOC}} = 0$ and $\frac{\partial \pi_s}{\partial p_g}|_{p_g = {p_g}_s^{FOC}(s)} = 0$ due to the joint concavity of π_s. We will denote by ${p_b}_s^*$ and ${p_g}_s^*$ the prices that solve the firm's optimization problem subject to $I_s = 1$. (For notational brevity, we will drop the subscript "s" when necessary.)

Uniform Base (UB): Using the constraint in Table 1 that induces the UB demand scenario, the firm solves

$$\begin{aligned} \max_{p_b} \pi_{UB} = \quad & (p_b - c_b)(1 - p_b) \\ \text{subject to} \quad & p_b \leq \min\{p_g - \alpha_n \beta,\, 1\}. \end{aligned}$$

We have $p_b^* = p_b^{FOC} = \frac{c_b+1}{2}$ and $p_g^* = \frac{c_b+1}{2} + \alpha_n\beta$ if and only if $\frac{c_b+1}{2} \leq \min\{\frac{c_b+1}{2} + \alpha_n\beta, 1\}$, i.e, $c_b \leq \min\{c_g - \alpha_n\beta, 1\}$. Using the optimal prices in π_{UB} yields $\pi_{UB}^* = \frac{(1-c_b)^2}{4}$.

If, on the other hand, $c_b \geq 1$ then we have $p_b^* = 1$ and $p_g^* = \bar{v}_n$ yielding no demand and $\pi_{UB}^* = 0$. Thus, we denote by $\Gamma_{UB} \equiv \{c_b \leq \min\{c_g - \alpha_n\beta, 1\}\}$ the parameter space over which the firm may feasibly choose prices that yield the UB demand scenario.

Conventional Targeting (CT): Using the constraint in Table 1 that induces the CT demand scenario, the firm solves

$$\begin{aligned} \max_{p_b, p_g} \pi_{CT} = \quad & (p_b - c_b)(\omega(1-p_b) + (1-\omega)(\tfrac{p_g - p_b}{\alpha_n\beta} - p_b)) + (p_g - c_g)((1-\omega)(1 - \tfrac{p_g - p_b}{\alpha_n\beta})) \\ \text{subject to} \quad & p_g - \alpha_n\beta < p_b < \min\{\tfrac{p_g}{1+\alpha_n\beta}, 1\}. \end{aligned}$$

We have $p_b^* = p_b^{FOC} = \frac{c_b+1}{2}$ and $p_g^* = p_g^{FOC} = \frac{c_g+\bar{v}_n}{2}$ if and only if $\frac{c_g+\bar{v}_n}{2} - \alpha_n\beta < \frac{c_b+1}{2} < \min\{\frac{c_g+\bar{v}_n}{2\bar{v}_n}, 1\}$, i.e., $c_g - \alpha_n\beta < c_b < \min\{\frac{c_g}{\bar{v}_n}, 1\}$. Using the optimal prices in π_{CT} yields $\pi_{CT}^* = \frac{(1-c_b)^2}{4} + (1-\omega)\frac{(c_b - c_g + \alpha_n\beta)^2}{4\alpha_n\beta}$.

If, instead, $c_g - \alpha_n\beta \geq c_b$, we have $p_g - \alpha_n\beta \geq p_b$, which yields the UB demand scenario. Similarly, if $c_b \geq \frac{c_g}{1+\alpha_n\beta}$, we have $p_b \geq \frac{p_g}{1+\alpha_n\beta}$, which yields the PT demand scenario. Thus, we denote by $\Gamma_{CT} \equiv \{c_g - \alpha_n\beta < c_b < \min\{\frac{c_g}{\bar{v}_n}, 1\}\}$ the parameter space over which the firm may feasibly choose prices that yield the CT demand scenario.

Perfect Targeting (PT): Using the constraint in Table 1 that induces the PT demand scenario, the firm solves

$$\begin{aligned} \max_{p_b, p_g} \pi_{PT} = \quad & (p_b - c_b)(\omega(1-p_b)) + (p_g - c_g)((1-\omega)(1 - \tfrac{p_g}{1+\alpha_n\beta})) \\ \text{subject to} \quad & \tfrac{p_g}{1+\alpha_n\beta} \leq p_b \leq \min\{\tfrac{p_g}{1-\alpha_c\beta}, 1\}. \end{aligned}$$

We have $p_b^* = p_b^{FOC} = \frac{c_b+1}{2}$ and $p_g^* = p_g^{FOC} = \frac{c_g+\bar{v}_n}{2}$ if and only if $\frac{c_g+\bar{v}_n}{2\bar{v}_n} < \frac{c_b+1}{2} < \min\{\frac{c_g+\bar{v}_n}{2\bar{v}_c}, 1\}$, i.e., $\frac{c_g}{\bar{v}_n} < c_b < \min\{\frac{c_g}{\bar{v}_c} + (\frac{\bar{v}_n}{\bar{v}_c} - 1), 1\}$. Using the optimal prices in π_{PT} yields $\pi_{PT}^* = \frac{\omega(1-c_b)^2}{4} + (1-\omega)\frac{(\bar{v}_n-c_g)^2}{4\bar{v}_n}$.

If, instead, $\frac{c_g}{\bar{v}_n} \geq c_b$, then the firm solves $\max_{p_b} \pi_{PT}$ over the line satisfying $p_b = \frac{p_g}{\bar{v}_n}$. As π_{PT} is concave, we must have $\frac{d\pi_{PT}}{dp_b}|_{p_b=p_b^*} = 0$, which yields $p_b^* = 0.5\frac{(1-w)(c_g+\bar{v}_n)+(c_b+1)w}{(1-w)\bar{v}_n+w}$ and $p_g^* = \bar{v}_n p_b^*$. Similarly, if $c_b \geq \frac{c_g}{\bar{v}_c} + (\frac{\bar{v}_n}{\bar{v}_c} - 1)$, then the firm solves $\max_{p_b} \pi_{PT}$ over the line satisfying $p_b = \frac{p_g}{\bar{v}_c}$. As π_{PT} is concave, we must have $\frac{d\pi_{PT}}{dp_b}|_{p_b=p_b^*} = 0$, which yields $p_b^* = 0.5\frac{\bar{v}_c(1-w)(c_g+\bar{v}_n)+\bar{v}_n w(c_b+1)}{(1-w)\bar{v}_c^2+w\bar{v}_n}$ and $p_g^* = \bar{v}_c p_b^*$. Thus, we denote by $\Gamma_{PT}^1 \equiv \{\frac{c_g}{\bar{v}_n} \leq c_b \leq \min\{\frac{c_g}{\bar{v}_c} + (\frac{\bar{v}_n}{\bar{v}_c} - 1), 1\}\}$ the parameter space over which the firm may feasibly choose prices that yield the PT demand scenario.

Naturalite Targeting (NT): Using the constraint in Table 1 that induces the NT demand scenario, the firm solves

$$\begin{aligned} \max_{p_b, p_g} \pi_{NT} = \quad & (p_b - c_b)(\omega(1 - \tfrac{p_b-p_g}{\alpha_c\beta})) + (p_g - c_g)((1-\omega)(1 - \tfrac{p_g}{1+\alpha_n\beta}) + \omega(\tfrac{p_b-p_g}{\alpha_c\beta} - \tfrac{p_g}{1-\alpha_c\beta})) \\ \text{subject to} \quad & \tfrac{p_g}{1-\alpha_c\beta} < p_b < p_g + \alpha_c\beta \end{aligned}$$

We have $p_b^* = p_b^{FOC} = \frac{c_b+\alpha_c\beta+z}{2}$ and $p_g^* = p_g^{FOC} = \frac{c_g+z}{2}$ if and only if $\frac{c_g+z}{2\bar{v}_c} < \frac{c_b+\alpha_c\beta+z}{2} < \frac{c_g+z}{2} + \alpha_c\beta$, i.e., $\frac{c_g}{\bar{v}_c} + z\frac{\alpha_c\beta^2(1-w)(\alpha_c+\alpha_n)}{\bar{v}_n\bar{v}_c} < c_b < c_g + \alpha_c\beta$. Using the optimal prices in π_{NT} yields $\pi_{NT}^* = \frac{(z-c_g)^2}{4z} + \omega\frac{(c_g-c_b+\alpha_c\beta)^2}{4\alpha_c\beta}$.

If, instead, $\frac{c_g}{\bar{v}_c} + z\frac{\alpha_c\beta^2(1-w)(\alpha_c+\alpha_n)}{\bar{v}_n\bar{v}_c} \geq c_b$, we have $\frac{p_g}{1-\alpha_c\beta} \geq p_g$, which yields the PT demand scenario. Similarly, if $c_b \geq c_g + \alpha_c\beta$, we have $p_b \geq p_g + \alpha_c\beta$, which yields the UG demand scenario. Thus, we denote by $\Gamma_{NT} \equiv \{\frac{c_g}{\bar{v}_c} + z\frac{\alpha_c\beta^2(1-w)(\alpha_c+\alpha_n)}{\bar{v}_n\bar{v}_c} < c_b < c_g + \alpha_c\beta\}$ the parameter space over which firm may feasibly choose prices that yield the NT demand scenario.

Uniform Green (UG): Using the constraint in Table 1 that induces the UG demand scenario, the firm solves

$$\max_{p_g} \pi_{UG} = (p_g - c_g)(\omega(1 - \tfrac{p_g}{1-\alpha_c\beta})^+ + (1-\omega)(1 - \tfrac{p_g}{1+\alpha_n\beta}))$$
$$\text{subject to} \quad p_g \le \min\{p_b - \alpha_c\beta, 1 + \alpha_n\beta\}$$

We have $p_g^* = p_g^{FOC} = \frac{c_g+z}{2}$ and $p_b^* = \frac{c_g+z+2\alpha_c\beta}{2}$ if and only $\frac{c_g+z}{2} < \min\{\frac{c_g+z+2\alpha_c\beta}{2} - \alpha_c\beta, 1+\alpha_n\beta\}$, i.e., $c_g < 2\bar{v}_c - z$. Note that the minimum operator in the constraint must return $p_b - \alpha_c\beta$ when $p_b^* < 1$, and thus both consumer types purchase the green variant, yielding the UG_{inc} demand scenario. As such, we denote by $\Gamma_{UG_{inc}} \equiv \{c_g \le \min\{c_b - \alpha_c\beta, 2\bar{v}_c - z\}\}$ the parameter space over which firm may feasibly choose prices that yield the UG_{inc} demand scenario. Using the optimal prices in π_{UG} yields $\pi^*_{UG_{inc}} = \frac{(z-c_g)^2}{4z}$.

If, on the other hand, $p_b^* \ge 1$ (which, the firm would resort to only if $c_b \ge 1$), then the minimum operator in the constraint must still return $p_b - \alpha_c\beta$, as otherwise $\pi_{UG} = 0$ due to zero demand. In such a case, as $p_g^* > 1 - \alpha_c\beta$, no conventional consumer purchases the green variant, yielding the UG_{exc} demand scenario. In this case, we have $p_g^* = p_g^{FOC} = \frac{c_g+\bar{v}_n}{2}$ and $p_b^* = p_g^* + \alpha_c\beta = \frac{c_g+\bar{v}_n+2\alpha_c\beta}{2}$ is and only if $\bar{v}_c < \frac{c_g+\bar{v}_n}{2} < \bar{v}_n$, i.e., $2\bar{v}_c - \bar{v}_n < c_g < \bar{v}_n$. Thus we denote by $\Gamma_{UG_{exc}} \equiv \{\{2\bar{v}_c - \bar{v}_n \le c_g \le \bar{v}_n\} \cap \{c_b \ge 1\}\}$ the parameter space over which firm may feasibly choose prices that yield the UG_{inc} demand scenario. Using the optimal prices in π_{UG} yields $\pi^*_{UG_{exc}} = \frac{(1-\omega)(\bar{v}_n - c_g)^2}{4\bar{v}_n}$.

We next characterize the parameter spaces over which the firm adopts a particular demand scenario, i.e., Γ^*_s, $\forall s \in \{UB, CT, PT, NT, UG\}$. We will prove the corresponding results in a series of lemmas.

Lemma 7

$\Gamma_{UB} = \Gamma^*_{UB}$ and $\Gamma_{CT} = \Gamma^*_{CT}$.

Lemma 7 follows from $\Gamma_{UB} \cap \Gamma_s = \emptyset, \, , \Gamma^*_s, \, \forall s \in \{CT, PT, NT, UG\}$ and $\Gamma_{CT} \cap \Gamma_s = \emptyset, \, , \Gamma^*_s, \, \forall s \in \{UB, PT, NT, UG\}$.

Lemma 8

$\Gamma^*_{NT} \subset \Gamma_{NT}$. Furthermore, $\Gamma^*_{NT} = \{\frac{c_g + \sqrt{y}(\bar{v}_n - \bar{v}_c)}{\bar{v}_c} < c_b < c_g + \alpha_c \beta\}$.

As $\frac{c_g}{\bar{v}_n} < \frac{c_g}{\bar{v}_c} + z\frac{\alpha_c \beta^2 (1-w)(\alpha_c + \alpha_n)}{\bar{v}_n \bar{v}_c}$ and $\min\{\frac{c_g}{\bar{v}_c} + (\frac{\bar{v}_n}{\bar{v}_c} - 1), 1\} > c_g + \alpha_c \beta$, we have $\Gamma_{NT} \subset \Gamma_{PT}$. Therefore, to characterize Γ^*_{NT}, we investigate the dynamics of $\pi^*_{NT} - \pi^*_{PT}$, which is a convex quadratic function of c_b as $\frac{d^2\{\pi^*_{NT} - \pi^*_{PT}\}}{d{c_b}^2} = \frac{\omega \bar{v}_c}{2(1-\bar{v}_c)} > 0$. As $\pi^*_{NT} - \pi^*_{PT} = 0$ yields $c_b = \frac{c_g \mp (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c}$, we have $\pi^*_{NT} - \pi^*_{PT} > 0$ for parameter spaces satisfying $\{c_b < \frac{c_g - (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c}\}$ and $\{c_b > \frac{c_g + (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c}\}$. As the intersection of the former set and Γ_{NT} is the empty set, $\Gamma^*_{NT} \subseteq \{\frac{c_g + (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c} < c_b < c_g + \alpha_c \beta\}$ must hold. Furthermore, simple algebra dictates $\frac{c_g}{\bar{v}_c} + z\frac{\alpha_c \beta^2 (1-w)(\alpha_c + \alpha_n)}{\bar{v}_n \bar{v}_c} < \frac{c_g + (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c}$, which yields $\Gamma^*_{NT} \subset \Gamma_{NT}$, and further implies $\Gamma^*_{NT} = \{\frac{c_g + (\bar{v}_n - \bar{v}_c)\sqrt{y}}{\bar{v}_c} < c_b < c_g + \alpha_c \beta\}$.

Lemma 9

$\Gamma_{UG^1_{inc}} \bigcap \Gamma^1_{PT} \neq \emptyset$. Furthermore, $\Gamma^*_{UG} = \Gamma^*_{UG^1_{inc}} \bigcup \Gamma^*_{UG^2_{inc}} \bigcup \Gamma^*_{UG_{exc}}$ where

$$\begin{aligned}
\Gamma^*_{UG^1_{inc}} &= \{c_b < 1\} \bigcap \{c_g \leq \min\{\bar{v}_c - \sqrt{x}, c_b - \alpha_c \beta\}\}, \\
\Gamma^*_{UG^2_{inc}} &= \{c_b \geq 1\} \bigcap \{c_g \leq \bar{v}_c - \sqrt{\bar{v}_c \left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)}\}, \\
\Gamma^*_{UG_{exc}} &= \{c_b \geq 1\} \bigcap \{\bar{v}_c - \sqrt{\bar{v}_c \left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)} < c_g < \bar{v}_n\}.
\end{aligned}$$

As $\frac{\bar{v}_n}{\bar{v}_c} - 1 > \alpha_c \beta$ always holds, we have $\frac{c_g + \alpha_c \beta}{\bar{v}_c} < \frac{c_g + \bar{v}_n - \bar{v}_c}{\bar{v}_c}$, which implies we have $\Gamma_{UG^1_{inc}} \bigcap \Gamma^1_{PT} \neq \emptyset$. Therefore, to characterize $\Gamma^*_{UG^1_{inc}}$, we investigate the dynamics of $\pi^*_{UG} - \pi^*_{PT}$, which is a convex quadratic function of c_g as $\frac{d^2\{\pi^*_{UG} - \pi^*_{PT}\}}{d{c_g}^2} =$

$\frac{\omega}{2\bar{v}_c} > 0$. As $\pi^*_{UG} - \pi^*_{PT} = 0$ yields $c_g = \bar{v}_c \mp \sqrt{x}$, we have $\pi^*_{UG} - \pi^*_{PT} > 0$ for parameter spaces satisfying $\{c_g > \bar{v}_c + \sqrt{x}\}$ and $\{c_g < \bar{v}_c - \sqrt{x}\}$. As the intersection of the former set and $\Gamma_{UG^1_{inc}}$ is the empty set, $\Gamma^*_{UG^1_{inc}} \subseteq \{c_g \leq \min\{\bar{v}_c - \sqrt{x}\,, 2\bar{v}_c - z\}$ must hold, where the minimum operator always returns $\bar{v}_c - \sqrt{x}$ as simple algebra dictates $\bar{v}_c - \sqrt{x} < 2\bar{v}_c - z \Leftrightarrow \frac{-(1-w)w\bar{v}_c\bar{v}_n(\bar{v}_c-\bar{v}_n)^2}{(\bar{v}_c(1-w)+w\bar{v}_n)^2} < \bar{v}_c(1-c_b)^2$. As such, we have $\Gamma^*_{UG^1_{inc}} \equiv \{\{c_g + \alpha_c\beta \leq c_b \leq 1\} \cap \{c_g \leq \bar{v}_c - \sqrt{x}\}\}$.

Note that no demand scenario other than UG is feasible when $c_b \geq 1$, and thus, scenario UG_{inc} sustains optimally over parameter space $\Gamma_{UG^2_{inc}} \backslash \Gamma_{UG_{exc}}$, and UG_{exc} sustains optimally over parameter space $\Gamma_{UG_{exc}} \backslash \Gamma_{UG^2_{inc}}$. To complete characterizing $\Gamma^*_{UG^2_{inc}}$ and $\Gamma^*_{UG_{exc}}$, we compare $\pi^*_{UG_{inc}}$ and $\pi^*_{UG_{exc}}$ over parameter space $\Gamma_{UG^2_{inc}} \bigcap \Gamma_{UG_{exc}} = \{2\bar{v}_c - \bar{v}_n \geq c_g \geq 2\bar{v}_c - z\} \bigcap \{c_b \geq 1\}$. As $\pi^*_{UG_{inc}} > \pi^*_{UG_{exc}}$ when $c_g < \bar{v}_c - \sqrt{\bar{v}_c(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))}$, we have $\Gamma^*_{UG^2_{inc}} = \{\{c_b > 1\} \cap \{c_g \leq \bar{v}_c - \sqrt{\bar{v}_c\,(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))}\}$ and $\Gamma^*_{UG_{exc}} = \{\{c_b > 1\} \cap \{\bar{v}_c - \sqrt{\bar{v}_c\,(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))} < c_g < \bar{v}_n\}\}$.

Finally, scenario PT sustains optimally when neither scenario NT nor scenario UG^1_{inc} is optimal over parameter space Γ^1_{PT}, which we state formally in the next lemma:

<u>Lemma 10</u>

$\Gamma^*_{PT} = \Gamma^1_{PT} \setminus (\Gamma^*_{NT} \bigcup \Gamma^*_{UG^1_{inc}})$.

We illustrate how all prior results are integrated in Figure 18, where we highlight parameter spaces Γ_s (in panel (a)) and Γ^*_s (in panel (b)) for each demand scenario s, where $s \in \{UB, CT, PT, NT, UG\}$. Lemmas 7 through 10 yield the full characterization of Proposition 1.

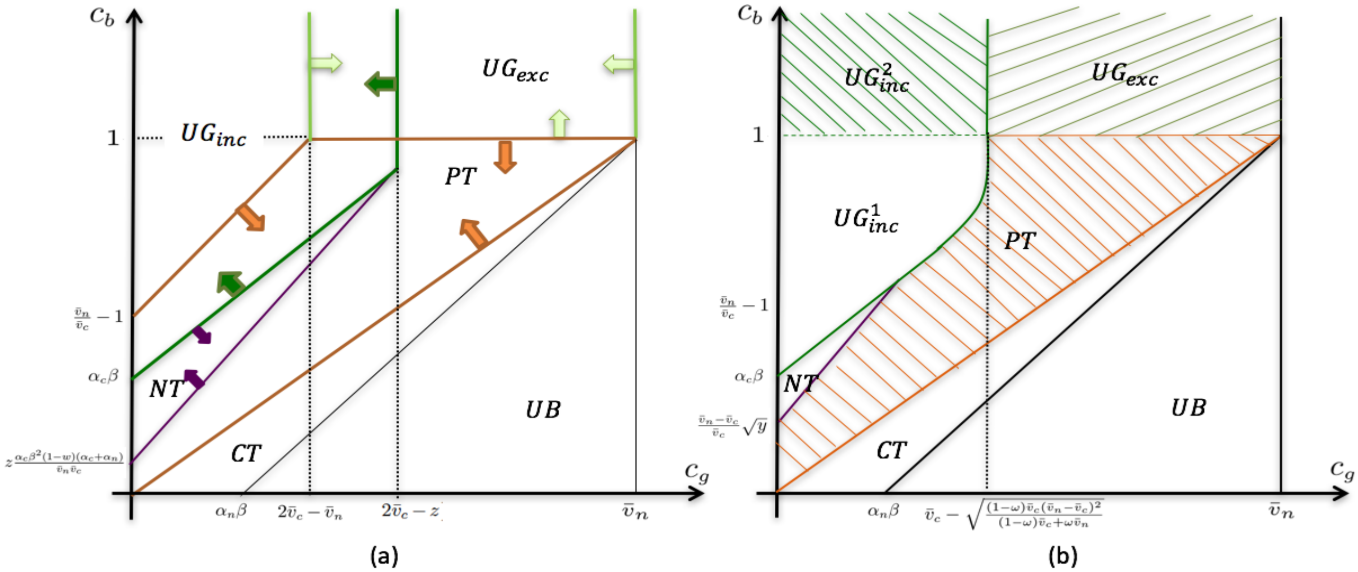


FIGURE 18. Feasible and optimal regions for each demand scenario

Proof of Proposition 2: As per Table 2 in Proposition 1, strategy NT is optimal when $\frac{c_g}{\bar{v}_c} + \sqrt{y}(\frac{\bar{v}_n}{\bar{v}_c} - 1) < c_b < c_g + \alpha_c\beta$ holds, which we can rewrite as $\frac{c_r}{1-\alpha_c} + \frac{k(1+\beta)^2\alpha_c+(\alpha_c+\alpha_n)\sqrt{y}}{1-\alpha_c} < c_v < c_r + \alpha_c$. Evidently $c_v - c_r < \alpha_c$ is necessary for Γ^*_{NT} to be non-empty.

(i) Defining $M \equiv \frac{c_v(1-\alpha_c)-c_r-\alpha_c k(1+\beta)^2}{\alpha_n+\alpha_c}$, we can re-write $\omega^{NT} = \frac{(\alpha_c\beta-M^2)\bar{v}_c}{(\alpha_c\beta-M^2)\bar{v}_c+M^2\bar{v}_n}$.

Then, we have

$$\begin{aligned}
\omega > \omega^{NT} \quad &\Leftrightarrow \quad \omega > \frac{(\alpha_c\beta-M^2)\bar{v}_c}{(\alpha_c\beta-M^2)\bar{v}_c+M^2\bar{v}_n} \\
&\Leftrightarrow \quad M^2 > \alpha_c\beta(1 - z\tfrac{\omega}{\bar{v}_c}) \\
&\Leftrightarrow \quad M > \sqrt{\alpha_c\beta(1 - z\tfrac{\omega}{\bar{v}_c})} \quad \text{(using } M > 0 \text{ as } c_b = c_v + k(1+\beta)^2 \leq \tfrac{c_v-c_r}{\alpha_c} \leq 1 \text{)} \\
&\qquad \text{(and } z\tfrac{\omega}{\bar{v}_c} \leq 1 \Leftrightarrow (1-w)\bar{v}_c > 0 \text{ via simple algebra.)} \\
&\Leftrightarrow \quad M > \sqrt{y} \quad \text{(using } y = \alpha_c\beta(1 - \tfrac{z\omega}{\bar{v}_c}).) \\
&\Leftrightarrow \quad c_v > \frac{c_r}{1-\alpha_c} + \frac{k(1+\beta)^2\alpha_c+(\alpha_c+\alpha_n)\sqrt{y}}{(1-\alpha_c)}
\end{aligned}$$

(ii) Note that, if $k < \frac{c_v(1-\alpha_c)-c_r-\alpha_c\sqrt{\alpha_c\beta(1-\omega)}}{\alpha_c(1+\beta)^2}$ holds, then $\alpha_{NT} > 0$. Furthermore

$$\begin{aligned}
\alpha_n < \alpha_{NT} \quad &\Leftrightarrow \quad \alpha_n < \frac{\sqrt{\alpha_c\beta(1-\alpha_c)(1-\alpha_c-2k(1+\beta)^2)}}{\beta} - \alpha_c \\
&\Leftrightarrow \quad k(1+\beta)^2\alpha_c + (\alpha_c+\alpha_n)\sqrt{\alpha_c\beta(1-\omega)} < c_v(1-\alpha_c) - c_r \\
&\Rightarrow \quad k(1+\beta)^2\alpha_c + (\alpha_c+\alpha_n)\sqrt{y} < c_v(1-\alpha_c) - c_r \\
&\qquad\ \text{(using } y < \alpha_c\beta(1-w) \text{ when } \bar{v}_c < z.\text{)} \\
&\Leftrightarrow \quad c_v > \tfrac{c_r}{1-\alpha_c} + \tfrac{k(1+\beta)^2\alpha_c+(\alpha_c+\alpha_n)\sqrt{y}}{(1-\alpha_c)}
\end{aligned}$$

(iii) We define $F(\beta) = (\alpha_c+\alpha_n)\sqrt{\alpha_c\beta(1-\omega)} + \alpha_c k(1+3\beta) - c_v(1-\alpha_c) - c_r$, and let β^{NT} be the largest β satisfying $F(\sqrt{\beta^{NT}}) = 0$. Then, we have

$$\begin{aligned}
\beta < \beta^{NT} \quad &\Rightarrow \quad F(\beta) < 0 \text{ and } F(0) < 0 \quad \text{(using } \alpha_c(c_v+k) < c_v - c_r.\text{)} \\
&\Leftrightarrow \quad (\alpha_c+\alpha_n)\sqrt{\alpha_c(1-\omega)}\beta_0 + \alpha_c k(1+3\beta_0^2) < c_v(1-\alpha_c) - c_r \\
&\Rightarrow \quad (\alpha_c+\alpha_n)\sqrt{\alpha_c(1-\omega)}\sqrt{\beta} + \alpha_c k(1+2\beta+\beta^2) < c_v(1-\alpha_c) - c_r \\
&\Rightarrow \quad k(1+\beta)^2\alpha_c + (\alpha_c+\alpha_n)\sqrt{y} < c_v(1-\alpha_c) - c_r \\
&\qquad\ \text{(using } y < \alpha_c\beta(1-w) \text{ as } \bar{v}_c < z.\text{)} \\
&\Leftrightarrow \quad c_v > \tfrac{c_r}{1-\alpha_c} + \tfrac{k(1+\beta)^2\alpha_c+(\alpha_c+\alpha_n)\sqrt{y}}{(1-\alpha_c)}
\end{aligned}$$

Proof of Proposition 3: For part (a), we consider the cases satisfying $c_b < 1$, i.e., $c_v < 1 - k(1+\beta)^2$, in which case strategy UG sustains optimally if problem parameters fall within $\Gamma^*_{UG^1_{inc}}$, i.e., both $c_g + \alpha_c\beta \leq c_b$ and $c_g \leq \bar{v}_c - \sqrt{x}$ hold. Using $c_v - c_r \geq \alpha_c$, we establish $c_g + \alpha_c\beta \leq c_b$ for all parts.

(i) Using $S = (\bar{v}_c - c_g)^2 - \bar{v}_c(1-c_b)^2$ we have

$$\begin{aligned}
\omega > \omega^{UG_1} &\Leftrightarrow \omega > 1 - \tfrac{S(1+\alpha_n\beta)}{\beta^2(\alpha_n+\alpha_c)^2(1-\alpha_c\beta)+S\beta(\alpha_n+\alpha_c)} \\
&\Leftrightarrow S > \tfrac{(1-\omega)\beta^2(\alpha_n+\alpha_c)^2(1-\alpha_c\beta)}{\omega(1+\alpha_n\beta)+(1-\omega)(1-\alpha_c\beta)} \\
&\Leftrightarrow (\bar{v}_c - c_g)^2 > \bar{v}_c(\tfrac{(1-\omega)(\bar{v}_n-\bar{v}_c)^2}{\omega\bar{v}_n+(1-\omega)\bar{v}_c} + (1-c_b)^2) \quad \text{(using the definition of } S.) \\
&\Leftrightarrow \bar{v}_c - c_g > \sqrt{\bar{v}_c(\tfrac{(1-\omega)(\bar{v}_n-\bar{v}_c)^2}{\omega\bar{v}_n+(1-\omega)\bar{v}_c} + (1-c_b)^2)} \quad \text{(taking square roots)} \\
&\Leftrightarrow c_g \leq \bar{v}_c - \sqrt{\bar{v}_c((1-c_b)^2 + z(1-\omega)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))} \\
&\Leftrightarrow c_g \leq \bar{v}_c - \sqrt{x} \quad \text{(using the definition of } x \text{ as in Proposition 1.)}
\end{aligned}$$

(ii) We first define $F(\alpha_n) = \alpha_n^2\bar{v}_c(1-\omega)\beta^2 + \alpha_n(2\bar{v}_c(1-\omega)\alpha_c\beta^2 - S\omega\beta) + \bar{v}_c(1-\omega)\beta^2\alpha_c^2 - S(\omega + \bar{v}_c(1-\omega))$, and let $\alpha_n^{UG_1}$ be the largest α_n satisfying $F(\alpha_n) = 0$. Furthermore, as $\omega > \frac{\bar{v}_c(\beta^2\alpha_c^2 - S)}{\bar{v}_c(\beta^2\alpha_c^2-S)+S} = \omega^{\alpha_1}$, we have $F(0) < 0$. Using $S = (\bar{v}_c - c_g)^2 - \bar{v}_c(1-c_b)^2$ and, we have

$$\begin{aligned}
\alpha_n < \alpha_n^{UG_1} &\Rightarrow F(\alpha_n) < 0 \quad (\text{as } \tfrac{d^2F(\alpha_n)}{d\alpha_n{}^2} > 0\,, \forall\alpha_n \text{ and } F(0) < 0 \text{ when } \omega > \omega^{\alpha_1}.) \\
&\Leftrightarrow (\bar{v}_c - c_g)^2 > \bar{v}_c(\tfrac{(1-\omega)(\bar{v}_n-\bar{v}_c)^2}{\omega\bar{v}_n+(1-\omega)\bar{v}_c} + (1-c_b)^2) \quad \text{(using the definition of } S.) \\
&\Leftrightarrow c_g \leq \bar{v}_c - \sqrt{x} \quad \text{(using part (a)(i) of the proof.)}
\end{aligned}$$

(iii) We define $F_L(\beta) \equiv \sqrt{\bar{v}_c((1-c_b)^2 + z(1-\omega)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2))}$ and $F_R(\beta) \equiv \bar{v}_c - c_g$, and rewrite $c_g \leq \bar{v}_c - \sqrt{x}$ as $F_L(\beta) < F_R(\beta)$. We establish the result in three steps:

1. $F_L(0) = F_R(0) = 1 - c_v - k$.

2. If $k < 1/4(1 - c_v + \sqrt{(1-\alpha_c) - \frac{(1-\omega)(\alpha_n+\alpha_c)^2}{1-\alpha_c+\omega(\alpha_n+\alpha_c)}}$, then we have

$$\begin{aligned}
F_L(1) &= \sqrt{(1-\alpha_c)((c_v + 4k - 1)^2 + \tfrac{(1-\omega)(\alpha_n+\alpha_c)^2}{\alpha_n\omega - \alpha_c(1-\omega)+1})} \\
&> 1 - \alpha_c - c_r - 4k \\
&= F_R(1).
\end{aligned}$$

3. Condition $k < 1 - c_v$ implies $1 + c_v + k < \frac{2(c_v - c_r)}{\alpha_c}$, as $c_v - c_r > \alpha_c$. Then we have

$$\begin{aligned} \frac{dF_L(\beta)}{d\beta}\Big|_{\beta=0} &= \frac{(1-c_v-k)(-\alpha_c(1-c_v-k)-4k)}{2\sqrt{(1-c_v-k)^2}} \\ &< c_v - c_r - \alpha_c - 2k \\ &= \frac{dF_R(\beta)}{d\beta}\Big|_{\beta=0}. \end{aligned}$$

As such, there must exist $\beta^{UG_1} \in (0,1)$ satisfying $F_L(\beta^{UG_1}) = F_R(\beta^{UG_1})$, and $F_L(\beta) < F_R(\beta)$ for $\beta < \beta^{UG_1}$.

For part (b), we consider the cases satisfying $c_b \geq 1$, i.e., $c_v \geq 1 - k(1+\beta)^2$, in which case only strategy UG can sustain optimally if problem parameters lie in $\Gamma^*_{UG^2_{inc}}$ or $\Gamma^*_{UG_{exc}}$; otherwise, the monopolist does not participate.

(i) Using the definition of ω^{UG_2}, we have

$$\begin{aligned} \omega < \omega^{UG_2} &\Leftrightarrow \bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right) < (\bar{v}_c - c_g)^2 \\ &\Leftrightarrow \bar{v}_c - \sqrt{\bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)} > c_g \quad \text{(taking square roots.)} \end{aligned}$$

(ii) Using the definition of α^{UG_2}, we have

$$\begin{aligned} \alpha_n < \alpha_n^{UG_2} &\Leftrightarrow \beta(\alpha_n + \alpha_c) - 1 + \alpha_c\beta + (1-\beta)c_v + \beta c_r + k(1+\beta)^2 < 0 \\ &\Leftrightarrow \beta(\alpha_n + \alpha_c) < \bar{v}_c - c_g \\ &\Rightarrow \sqrt{\frac{(1-\omega)\bar{v}_c(\beta(\alpha_n+\alpha_c))^2}{(1-\omega)\bar{v}_c + \omega\bar{v}_n}} < \beta(\alpha_n + \alpha_c) < \bar{v}_c - c_g \quad \left(\text{using } \frac{(1-\omega)\bar{v}_c}{(1-\omega)\bar{v}_c + \omega\bar{v}_n} < 1.\right) \\ &\Leftrightarrow \sqrt{\frac{(1-\omega)\bar{v}_c(\beta(\alpha_n+\alpha_c))^2}{(1-\omega)\bar{v}_c + \omega\bar{v}_n}} < \bar{v}_c - c_g \\ &\Leftrightarrow c_g \leq \bar{v}_c - \sqrt{\bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)} \end{aligned}$$

(iii) We define $F(\beta) = \beta(\alpha_n + \alpha_c) - 1 + \alpha_c\beta + (1-\beta)c_v + \beta c_r + k(1+\beta)^2$, and let β_{UG_2} be the largest β satisfying $F(\beta_{UG_2}) = 0$. Furthermore, note that $F(0) < 0$

when $c_v < 1 - k$. Then, we have

$$\begin{aligned}
\beta < \beta^{UG_2} \quad &\Rightarrow \quad F(\beta) < 0 \\
&\Leftrightarrow \quad \beta(\alpha_n + \alpha_c) < \bar{v}_c - c_g \quad \text{(Rearrangement and using the definition of } c_g) \\
&\Leftrightarrow \quad \beta(\alpha_n + \alpha_c) < \bar{v}_c - c_g \\
&\Rightarrow \quad \sqrt{\tfrac{(1-\omega)\bar{v}_c(\beta(\alpha_n+\alpha_c))^2}{(1-\omega)\bar{v}_c+\omega\bar{v}_n}} < \beta(\alpha_n + \alpha_c) < \bar{v}_c - c_g \qquad \text{(using } \tfrac{(1-\omega)\bar{v}_c}{(1-\omega)\bar{v}_c+\omega\bar{v}_n} < 1.) \\
&\Leftrightarrow \quad \sqrt{\tfrac{(1-\omega)\bar{v}_c(\beta(\alpha_n+\alpha_c))^2}{(1-\omega)\bar{v}_c+\omega\bar{v}_n}} < \bar{v}_c - c_g \\
&\Leftrightarrow \quad c_g \leq \bar{v}_c - \sqrt{\bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)}
\end{aligned}$$

(iv) Using the definition of ω^{UG_2}, we have

$$\begin{aligned}
\omega > \omega^{UG_2} \quad &\Leftrightarrow \quad \bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right) > (\bar{v}_c - c_g)^2 \\
&\Leftrightarrow \quad \bar{v}_c - \sqrt{\bar{v}_c\left(z(1-w)(\bar{v}_n/\bar{v}_c + \bar{v}_c/\bar{v}_n - 2)\right)} < c_g \quad \text{(taking square roots.)}
\end{aligned}$$

Proof of Proposition 4: For part (i), note that as $\Gamma^*_{UB} = \{\Delta_c < \Delta_c^{UB}\}$, the optimality of strategy UB is straightforward. Furthermore, as $c_b < 1$ when UB is optimal, and $\frac{dc_b}{d\beta} = 2k(1+\beta) > 0$, we must have $\frac{d\pi_{UB}}{d\beta} = -\frac{1}{2}(1-c_b)\frac{dc_b}{d\beta} < 0$, which implies $\beta^* = 0$. For part (ii), note the following regarding how c_g evolves as β changes:

- As $\frac{dc_g}{d\beta} > 0$ for all β when $\Delta_c < 2k$ holds, we have $c_g > c_g|_{\beta=0} = c_v + k$.

- When $2k \leq \Delta_c \leq 4k$, $c_g > c_g|_{\beta=\frac{c_v-c_r}{2k}-1} = 2c_v - c_r - \frac{(c_v-c_r)^2}{4k}$.

- As $\frac{dc_g}{d\beta} < 0$ for all β when $\Delta_c > 4k$ holds, we have $c_g > c_g|_{\beta=1} = c_r + 4k$.

We denote by C a lower bound for c_g in each scenario, we have

$$
\begin{aligned}
\omega < \omega_\beta^{PT} \;&\Leftrightarrow\; \omega < 1 - \tfrac{8k}{2(c_v - c_r) + \alpha_n(1 + \frac{C}{1+\alpha_n})} \quad \text{(using the definition of } C.\text{)} \\
&\Leftrightarrow\; \tfrac{8k}{1-\omega} - 2(c_v - c_r) < \alpha_n(1 + \tfrac{C}{1+\alpha_n}) \\
&\Leftrightarrow\; \tfrac{8k - 2(c_v - c_r)(1-\omega)}{\alpha_n(1-\omega)} < 1 + \tfrac{C}{1+\alpha_n} \\
&\Leftrightarrow\; \tfrac{2\frac{dc_g}{d\beta} + 2\omega(c_v - c_r)}{\alpha_n(1-\omega)} < \tfrac{2\frac{dc_g}{d\beta}|_{\beta=1} + 2\omega(c_v - c_r)}{\alpha_n(1-\omega)} < 1 + \tfrac{c_g}{1+\alpha_n\beta} \quad \text{(bcz } \tfrac{dc_g}{d\beta} \le \tfrac{dc_g}{d\beta}|_{\beta=1} \text{ and } C < c_g.\text{)} \\
&\Leftrightarrow\; \tfrac{2\frac{dc_g}{d\beta} + 2\omega(c_v - c_r)}{\alpha_n(1-\omega)} < 1 + \tfrac{c_g}{1+\alpha_n\beta} \quad \text{(using } \tfrac{dc_g}{d\beta} \le \tfrac{dc_g}{d\beta}|_{\beta=1} \text{ and } C < c_g.\text{)} \\
&\Leftrightarrow\; 2\omega(1+\alpha_n\beta)(1+\alpha_n\beta - c_g)\tfrac{dc_b}{d\beta} \\
&\qquad < (1-\omega)(1+\alpha_n\beta - c_g)(\alpha_n c_g + (1+\alpha_n\beta)(\alpha_n - 2\tfrac{dc_g}{\beta})) \\
&\Leftrightarrow\; 2\omega(1+\alpha_n\beta)^2(1-c_b)\tfrac{dc_b}{d\beta} < 2\omega(1+\alpha_n\beta)(1+\alpha_n\beta - c_g)\tfrac{dc_b}{d\beta} \quad (\;\tfrac{c_g}{1+\alpha_n\beta} < c_b \text{ for PT.)} \\
&\Leftrightarrow\; -\tfrac{1}{2}\omega(1-c_b)\tfrac{dc_b}{d\beta} + \tfrac{1}{4}(1-\omega)\tfrac{1+\alpha_n\beta - c_g}{(1+\alpha_n\beta)^2}(\alpha_n c_g + (1+\alpha_n\beta)(\alpha_n - 2\tfrac{dc_g}{\beta})) > 0 \\
&\Leftrightarrow\; \tfrac{d\Pi_{PT}(\beta)}{d\beta} > 0 \quad \text{(using the definition of } \Pi_{PT}(\beta).\text{)}
\end{aligned}
$$

Therefore, we must have $\beta^* = 1$ if $\omega < \omega_\beta^{PT}$ when the firm optimally implements implies scenario PT.

For part (iii), we have

$$
\begin{aligned}
\omega > \omega_\beta^{UG} \quad & \Leftrightarrow \quad \tfrac{\alpha_c}{\omega} < c_v - c_r - 4k \\
& \Rightarrow \quad \tfrac{\alpha_c}{\omega} < c_v - c_r - 2k(1+\beta) \quad \text{(using } \beta \leq 1.\text{)} \\
& \Leftrightarrow \quad \tfrac{\omega}{\alpha_c} > \tfrac{1}{c_v - c_r - 2k(1+\beta)} \quad \text{(taking the inverse of both sides.)} \\
& \Rightarrow \quad \tfrac{\omega}{\alpha_c} + \tfrac{(1-\omega)^2 \bar{v}_c^2}{\alpha_c \bar{v}_n^2 \omega} + \tfrac{2\bar{v}_c(1-\omega)}{\alpha_c \bar{v}_n} > \tfrac{\omega}{\alpha_c} > \tfrac{1}{c_v - c_r - 2k(1+\beta)} \\
& \Leftrightarrow \quad \tfrac{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2}{\alpha_c \bar{v}_n^2 \omega} > \tfrac{1}{c_v - c_r - 2k(1+\beta)} \\
& \Leftrightarrow \quad \tfrac{\alpha_c \bar{v}_n^2 \omega}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2} < c_v - c_r - 2k(1+\beta) \quad \text{(taking the inverse of both sides.)} \\
& \Rightarrow \quad \tfrac{\alpha_c \bar{v}_n^2 \omega}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2} 2z < 2z(c_v - c_r - 2k(1+\beta)) \\
& \qquad < \tfrac{\alpha_c \bar{v}_n^2 \omega}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2} 2z < 2z(c_v - c_r - 2k(1+\beta)) \quad (\ c_g < 2\bar{v}_c - z \text{ and } \bar{v}_c < z.) \\
& \Leftrightarrow \quad -\tfrac{\alpha_c \bar{v}_n^2 \omega}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2}(c_g + z) > -2z(c_v - c_r - 2k(1+\beta)) \\
& \qquad \text{(multiplying both sides by -1.)} \\
& \Rightarrow \quad \tfrac{-\alpha_c \bar{v}_n^2 \omega}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2}(c_g + z) + \tfrac{\alpha_n \bar{v}_c^2 (1-\omega)}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2}(c_g + z) \\
& \qquad > 2z(-c_v + c_r + 2k(1+\beta)) \quad \text{(as the newly added term is positive.)} \\
& \Leftrightarrow \quad \tfrac{-\alpha_c \bar{v}_n^2 \omega + \alpha_n \bar{v}_c^2 (1-\omega)}{(\omega \bar{v}_n + (1-\omega)\bar{v}_c)^2}(c_g + z) > 2z(-c_v + c_r + 2k(1+\beta)) \\
& \Leftrightarrow \quad \tfrac{dz}{d\beta}(c_g + z) - 2z\tfrac{dc_g}{d\beta} > 0 \\
& \Leftrightarrow \quad \tfrac{d\pi_{UG}(\beta)}{d\beta} = \tfrac{(z - c_g)}{4z^2}\left(\tfrac{dz}{d\beta}(c_g + z) - 2z\tfrac{dc_g}{d\beta}\right) > 0 \quad (\ c_g < z \text{ and the definition of } \Pi_{UG}(\beta).)
\end{aligned}
$$

Therefore, we must have $\beta^* = 1$ if $\omega > \omega_\beta^{UG}$ when the firm optimally implements implies scenario UG.

APPENDIX B

CHAPTER 3: TECHNICAL PROOFS

Proof of Lemma 3: Let $\lambda < 1$ denote the probability that a consumer is available to shop in a given period. Therefore, the probability that a consumer will *next* be available after precisely n periods is equal to $(1-\lambda)^{n-1}\lambda$, and the corresponding discount factor is β^n. Therefore, considering all possible $n = 1, 2, \ldots \infty$, the expected discount factor corresponding to the subsequent availability period is given by the infinite sum:

$$\beta\lambda + \beta^2(1-\lambda)\lambda + \beta^3(1-\lambda)^2\lambda + \beta^4(1-\lambda)^3\lambda + ... = \frac{\beta\lambda}{1-(1-\lambda)\beta} \equiv \hat{\beta}.$$

We see that $d\hat{\beta}/d\lambda > 0$, implying that infrequent visitation (low λ) translates to a lower resulting β, as we should expect. Also, $\lim_{\lambda \to 1} \hat{\beta} = \beta$, as expected.

Proof of Proposition 7: To determine the set of dominant shopping behaviors under DD, we begin by ordering the four recurrent price states in order of *increasing* appeal to a consumer, i.e., $\underline{R} \prec \overline{R} \prec \underline{r} \prec \overline{r}$ (of these, $\underline{r} \succ \overline{R}$ is non obvious but must hold because $v - r - \alpha r > v - R - \alpha R$ for these two alternatives, and neither causes a change in credit—implying no future ramifications). In Table 8, we highlight the ordering of the four recurrent price states, along with the first-purchase price states $\underline{\underline{R}}$ and $\underline{\underline{r}}$ (between which the latter is more desirable).

Individual rows of the table correspond to distinct feasible shopping behaviors. For a given behavior in the row, a check mark denotes participation (a purchase), a cross denotes non-participation, and a dash denotes that a state does not occur. The check and cross marks correspond to the preference ordering $\underline{\underline{R}} \prec \underline{\underline{r}}$

TABLE 8. Set of dominant purchase policies under DD

First purchase		After first purchase				
$\underline{\underline{R}}$	$\underline{\underline{r}}$	$\underline{R}$	$\overline{R}$	$\underline{r}$	$\overline{r}$	**Dominated?**
✓	✓	✓	✓	✓	✓	No (regular shoppers segment, $\mathfrak{R}_D$)
		×	✓	✓	✓	Yes
		×	×	✓	✓	Yes
		×	×	×	✓	Yes
×	✓	✓	✓	✓	✓	No (transition shoppers segment, $\mathcal{T}_D$)
		×	-	✓	-	No (sales shoppers segment, $\mathcal{S}_D$)
		×	×	✓	-	No (sales shoppers segment, $\mathcal{S}_D$)
		×	×	×	-	Yes
×	×	-	-	-	-	No (non-participating consumer)

and $\underline{R} \prec \overline{R} \prec \underline{r} \prec \overline{r}$. We see that if buying (in the table, a check mark) occurs in a less-favored state, then buying should occur in any feasible more-favored state; visually, this translates to check marks propagating to the right. Of the 2^6 possible rows, this consideration eliminates all but the nine rows that appear in the table. Of these nine, we see from the table that four are dominated. The dominance result for the first three of these four holds because we can show that any customer who chooses to initially buy in state $\underline{\underline{R}}$ (rather than wait for the lower price state, as in the next block of four table rows) with zero credit would also optimally purchase in state $\underline{R}$—which yields higher immediate payoff and carries forward the same (high) credit value. The dominance result for the last of the four holds because any customer who buys in state $\underline{\underline{r}}$ will also optimally purchase in state $\underline{r}$—which yields a higher immediate payoff and carries forward the same (low) credit value.

Proof of Proposition 9: Consider when $E\pi_{\mathcal{T}_D}(\alpha_{\mathcal{T}_D}) \geq \max\{E\pi_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I}), E\pi_{\mathcal{S}_I}(\alpha_{\mathcal{S}_I})\}$ holds. $E\pi_{\mathcal{T}_D}(\alpha_{\mathcal{T}_D}) \geq E\pi_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I})$ reduces to $v \leq v_{\mathcal{T}\mathfrak{R}}$ where $v_{\mathcal{T}\mathfrak{R}} \equiv \frac{\beta\gamma R(R-r)(cR+r(1-\gamma))+Rr(c\gamma R+r(1-\gamma))}{\beta\gamma(R-r)(R+r(1-\gamma))+r(\gamma R+r(1-\gamma))}$. The inequality $E\pi_{\mathcal{T}_D}(\alpha_{\mathcal{T}_D}) \geq E\pi_{\mathcal{S}_I}(\alpha_{\mathcal{S}_I})$ reduces to a lower threshold $v \geq v_{\mathfrak{TS}} \equiv \frac{\beta\gamma(\beta(R-r)(cR+r)+r((c-1)R+r))-(1-\beta)r^2}{\beta R(2\beta\gamma-1)-(1-2\beta)r(1-\beta\gamma)}$, provided that $\gamma > \bar{\gamma} \equiv \frac{\beta R+r(1-2\beta)}{\beta(2\beta R+r(1-2\beta))}$. Therefore, given sufficiently high γ and $v_{\mathfrak{TS}} < v < v_{\mathfrak{TR}}$, the firm maximizes profits via $\alpha = \alpha_{\mathfrak{T}_D}$ corresponding to the transition shopping segment.

Proof of Lemma 4: Plugging the associated optimal α functions (3.15) and (3.16), respectively, into expressions (B.1) and (B.4) for a regular shopper and expressions (B.3) and (B.5) for a sales shopper, we find that $V_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I}) = V_{\mathfrak{R}_D}(\alpha_{\mathfrak{R}_D}) = \frac{(1-\gamma)v(R-r)}{(1-\beta)R}$ and $V_{\mathfrak{S}_I}(\alpha_{\mathfrak{S}_I}) = V_{\mathfrak{S}_D}(\alpha_{\mathfrak{S}_D}) = 0$.

Proof of Lemma 5: Under ID, the inequality $E\pi_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I}) \geq E\pi_{\mathfrak{S}_I}(\alpha_{\mathfrak{S}_I})$ holds for all $v \geq v_{\mathfrak{R}_I\mathfrak{S}_I} \equiv \frac{c\gamma R^2}{(2\gamma-1)R+r(1-\gamma)}$, given $\gamma > \frac{R-r}{2R-r} \equiv \bar{\bar{\gamma}}$ (this must be satisfied under Proposition 9, because $\bar{\bar{\gamma}} < \bar{\gamma}$). Under DD, with $\gamma > \bar{\gamma}$, the firm optimally (Proposition 9) induces transition shopping if and only if $v_{\mathfrak{TS}} < v < v_{\mathfrak{TR}}$. Therefore, there are two possible cases.

Case (i) $v : v_{\mathfrak{TS}} < v < v_{\mathfrak{R}_I\mathfrak{S}_I}$. In this valuation range, the ID policy's profit is maximized by offering $\alpha_{\mathfrak{S}_I}$ to induce sales shopping, while DD profit is maximized by offering $\alpha_{\mathfrak{T}_D}$ to induce transition shopping. Given that the representative consumer earns zero expected surplus when targeted as a sales shopper, and here under DD the consumer chooses transition-shopping over sales-shopping, the expected surplus is positive. Hence, DD yields higher surplus for this valuation range.

Case (ii) $v : v_{\mathfrak{R}_I\mathfrak{S}_I} < v < v_{\mathfrak{TR}}$. In this (higher) valuation range, the ID policy's profit is maximized by offering $\alpha_{\mathfrak{R}_I}$ to induce regular shopping, while DD profit is maximized by offering $\alpha_{\mathfrak{T}_D}$ to induce transition shopping. It is easy to check that $V_{\mathfrak{R}_I}(\alpha_{\mathfrak{R}_I}) \geq V_{\mathfrak{T}_D}(\alpha_{\mathfrak{T}_D})$ holds for $v < R$ (for $v > R$, a zero discount is optimal); thus, ID yields higher expected surplus for this valuation range.

Proof of Proposition 10: Since $0 \leq h_1 \leq 1$, the numerator of α_D^* is larger than the numerator of α_I^* (i.e., $2R(1-h_1) + 2h_1 r - 1 \leq 2r - 1 \Leftrightarrow r < R$). Utilizing $0 \leq h_2 \leq 1$ and $0 \leq h_3 \leq 1$, we can show that the denominator of α_D^* is less than or equal to denominator of α_I^*. Thus, $\alpha_D^* \geq \alpha_I^*$. The inequality binds at $\beta = 1$.

Proof of Proposition 11: To establish that DD earns higher profit ($E\pi_D(\alpha_D^*) - E\pi_I(\alpha_I^*) \geq 0$), we will show that even when setting $\alpha = \alpha_I^*$ the DD policy dominates, i.e., $E\pi_D(\alpha_I^*) - E\pi_I(\alpha_I^*)$ is positive (since, from Proposition 10, we know $\alpha_D^* > \alpha_I^*$ for $\beta < 1$). We can express this profit difference as $\frac{\alpha}{(1-\beta\gamma)^2} M(\alpha)|_{\alpha=\alpha_I^*}$, where $M(\alpha)$ is a linear function of c. $M(\alpha_I^*) > 0$ simplifies to $c > c_M = K_1(\beta)/K_2(\beta)$, where $K_1(\beta)$ and $K_2(\beta)$ are lengthy third degree polynomials in β. Moreover, we can formally establish that $K_2(\beta) > 0$ holds, and, as we confirm with our analyses reported in Table 5, the condition $c > c_M$ is nonrestrictive (with even $c_M < 0$ in most cases—due to $K_1(\beta)$ being negative—implying *any non-negative cost* is sufficient).

Proof of Lemma 6: Via FOC we solve for $\alpha_D^* = \frac{(2-c)R^2 - R(\beta r(2\beta+c-2)+1)+\beta r(2\beta r-1)}{2\beta(R^2-2(\beta-1)\beta Rr+(\beta(\beta^2+\beta-2)+1)r^2)}$ and $\alpha_I^* = \frac{(2-c)R^2-\beta cRr-R+\beta r(2r-1)}{2(R^2+\beta r^2)}$. It is easy to establish that the denominator of α_D^* is smaller than the denominator of α_I^*, and the numerator of α_D^* is larger than the numerator of α_I^*. Combined, these two facts imply $\alpha_D^* \geq \alpha_I^*$, where the inequality binds at $\beta = 1$.

Deriving NPV of surplus under DD and ID: For each candidate shopping behavior B, we denote the corresponding expected surplus NPV $V(0)$ from (3.8) as $V_B(0)$, or simply V_B for brevity. These surplus NPV functions depend on the consumer type v. Using them, we derive valuation thresholds defining which behavior is optimal for a given v. To obtain the surplus NPV for each behavior, we must alternately consider the two possible first-period states $p_t = R$ or $p_t = r$, given that a consumer witnesses the outcome p_t prior to making their initial purchase decision. We begin by analyzing the case $p_t = R$ but we later find that the resulting valuation thresholds also hold when $p_t = r$. We can derive $V_{\mathfrak{R}_D}$ for $p_t = R$ by applying $x^{\mathfrak{R}}_{kp}$ to (3.8) while twice invoking its recursion to yield the following three simultaneous equations.

$$\begin{cases} V_{\mathfrak{R}_D}(0) = v - R + 0 + \beta EV_{\mathfrak{R}_D}(\alpha R) \\ EV_{\mathfrak{R}_D}(\alpha R) = \gamma(v - R + \alpha R + \beta EV_{\mathfrak{R}_D}(\alpha R)) + (1-\gamma)(v - r + \alpha R + \beta EV_{\mathfrak{R}_D}(\alpha r)) \\ EV_{\mathfrak{R}_D}(\alpha r) = \gamma(v - R + \alpha r + \beta EV_{\mathfrak{R}_D}(\alpha R)) + (1-\gamma)(v - r + \alpha r + \beta EV_{\mathfrak{R}_D}(\alpha r)) \end{cases}$$

We solve this system by initially solving the latter two equations for the two unknowns $EV_{\mathfrak{R}_D}(\alpha R)$ and $EV_{\mathfrak{R}_D}(\alpha r)$, and we then obtain the full NPV result $V_{\mathfrak{R}_D}(0)$ from the first equation, yielding:

$$V_{\mathfrak{R}_D} \equiv V_{\mathfrak{R}_D}(0) = [v - (1-\alpha\beta)(\beta r + \beta\gamma(R - r) + (1-\beta)R]/(1-\beta). \qquad \text{(B.1)}$$

Similarly, we derive $V_{\mathfrak{I}_D}$ by applying $x^{\mathfrak{I}}_{kp}$ to (3.8), which in turn leads to invoking its recursion three additional times to yield four simultaneous equations.

$$\begin{cases} V_{\mathfrak{I}_D}(0) = 0 + \beta EV_{\mathfrak{I}_D}(0) \\ EV_{\mathfrak{I}_D}(0) = \gamma\beta EV_{\mathfrak{I}_D}(0) + (1-\gamma)(v - r + \beta EV_{\mathfrak{I}_D}(\alpha r) \\ EV_{\mathfrak{I}_D}(\alpha r) = \gamma(v - R + \alpha r + \beta EV_{\mathfrak{I}_D}(\alpha R)) + (1-\gamma)(v - r + \alpha r + \beta EV_{\mathfrak{I}_D}(\alpha r)) \\ EV_{\mathfrak{I}_D}(\alpha R) = \gamma(v - R + \alpha R + \beta EV_{\mathfrak{I}_D}(\alpha R)) + (1-\gamma)(v - r + \alpha R + \beta EV_{\mathfrak{I}_D}(\alpha r)) \end{cases}$$

The latter two equations in this system have the same structure as the latter two equations within (B) above, so we know $EV_{\mathfrak{I}_D}(\alpha r) = EV_{R_D}(\alpha r)$ and $EV_{\mathfrak{I}_D}(\alpha R) =$

$EV_{\mathfrak{R}_D}(\alpha R)$ (this is also intuitive since once a transition shopper has credit, either αr or αR, their subsequent actions and surplus match those of regular shoppers). We thus solve the system for $EV_{\mathfrak{T}_D}(\alpha r)$ and $EV_{\mathfrak{T}_D}(\alpha R)$, and then $EV_{\mathfrak{T}_D}(0)$, and finally $V_{\mathfrak{T}_D}(0)$ with the following result:

$$V_{\mathfrak{T}_D} \equiv V_{\mathfrak{T}_D}(0) = \underline{\beta}[v - (1 - \alpha\beta)(r + \beta\gamma(R - r)]/(1 - \beta), \tag{B.2}$$

where $\underline{\beta} = \beta(1 - \gamma)/(1 - \beta\gamma)$. In (B.2), the expression in brackets represents the net surplus once the consumer initiates purchasing under the transition behavior (i.e., at the first reduced-price opportunity), and $\underline{\beta}$ ($< \beta$) discounts that value because the consumer may face one or more periods at R before purchasing begins. In analogous fashion we derive $V_{\mathfrak{S}_D}$ by applying $x_{kp}^{\mathfrak{S}}$ to (3.8) and invoking the recursion twice to yield three simultaneous equations with unknowns $V_{\mathfrak{S}_D}(0)$, $EV_{\mathfrak{S}_D}(0)$, and $EV_{\mathfrak{S}_D}(\alpha r)$.

$$\begin{cases} V_{\mathfrak{S}_D}(0) = \beta EV_{\mathfrak{S}_D}(0) \\ EV_{\mathfrak{S}_D}(0) = \gamma\beta EV_{\mathfrak{S}_D}(0) + (1 - \gamma)(v - r + \beta EV_{\mathfrak{S}_D}(\alpha r) \\ EV_{\mathfrak{S}_D}(\alpha r) = \gamma\beta EV_{\mathfrak{S}_D}(\alpha r) + (1 - \gamma)(v - r + \beta EV_{\mathfrak{S}_D}(\alpha r) \end{cases}$$

Solving this system yields:

$$V_{\mathfrak{S}_D} \equiv V_{\mathfrak{S}_D}(0) = \underline{\beta}[v - r(1 - \alpha\underline{\beta})]/(1 - \underline{\beta}). \tag{B.3}$$

In above analysis, we considered a first period price $p_t = R$. For the alternative case $p_t = r$, no distinction exists between regular- versus transition-shopping because both imply buying in the first period at $p_t = r$ and subsequently (by definition) transition shoppers become regular shoppers. The threshold demarcating regular-shopping from transition-shopping therefore doesn't exist for $p_t = r$ case, making it simpler to address. The V_B expressions with $p_t = r$ do change slightly (with $V_{\mathfrak{R}_D} = V_{\mathfrak{T}_D}$ now holding) such that the scalar $\underline{\beta}$ simply becomes equal to one in expressions (B.2) and (B.3), and so their pairwise comparisons yield the same $v_{\mathfrak{T}_D}(\alpha_D)$ and $v_{\mathbb{S}_D}(\alpha_D)$ expressions we derived above. Thus, our results both for DD and ID hold irrespective of whether the initial price level is low (r) or high (R).

Having derived the valuation thresholds which define optimal consumer shopping behavior under the DD policy, we must do the same for ID. With no carry-forward of credit, it is relatively easy to express (3.8) directly via its implied infinite sum. Expanding the recursion in (3.8) with $p_t = R$ at time $t = 1$ for the regular- and sales-shopping behaviors (recall that the transition segment does not apply under ID) yields the resulting surplus NPVs $V_{\mathfrak{R}_I}$ and $V_{\mathbb{S}_I}$.

$$V_{\mathfrak{R}_I} = v-R(1-\alpha)+\sum_{t=2}^{\infty}\beta^{t-1}[\gamma(v-R(1-\alpha))+(1-\gamma)(v-r(1-\alpha))] = \frac{v-(1-\alpha)(R-\beta(1-\gamma)(R-r))}{1-\beta} \tag{B.4}$$

$$V_{\mathbb{S}_I} = 0+\sum_{t=2}^{\infty}\beta^{t-1}[(1-\gamma)(v-r+\alpha r)] = \frac{\beta(1-\gamma)(v-(1-\alpha)r)}{1-\beta} \tag{B.5}$$

Deriving NPV of profit under DD and ID: The following system of equations corresponds to a regular shopper under the DD policy. By solving these simultaneous equations for $E\pi_{\mathfrak{R}_D}(\alpha,0)$, we obtain the expected NPV of profit per regular shopper under DD.

$$\begin{cases} E\pi_{\mathfrak{R}_D}(\alpha,0) = \gamma(R-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha R))+(1-\gamma)(r-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha r)) \\ E\pi_{\mathfrak{R}_D}(\alpha,\alpha r) = \gamma(R-\alpha r-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha R))+(1-\gamma)(r-\alpha r-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha r)) \\ E\pi_{\mathfrak{R}_D}(\alpha,\alpha R) = \gamma(R-\alpha R-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha R))+(1-\gamma)(r-\alpha r-cR+\beta E\pi_{\mathfrak{R}_D}(\alpha,\alpha r)) \end{cases}$$

Analogous systems of simultaneous equations yield the per-customer profit NPV for the transition-shopping and sales-shopping behaviors.

Calculating the per-customer profit NPV under the ID policy is simpler, as the result for each behavior follows from a single equation. The equation corresponding to the regular- and sales-shopping behaviors are below.

$$E\pi_{\mathfrak{R}_I}(\alpha) = (1-\gamma)(r - \alpha r - cR) + \gamma(R - \alpha R - cR) + \beta E\pi_{\mathfrak{R}_I}(\alpha)$$

$$E\pi_{\mathcal{S}_I}(\alpha) = (1-\gamma)(r - \alpha r - cR) + \beta E\pi_{\mathcal{S}_I}(\alpha).$$

We simply solve the first of these two equations for $E\pi_{\mathfrak{R}_I}(\alpha)$, and the second equation for $E\pi_{\mathcal{S}_I}(\alpha)$.

APPENDIX C

CHAPTER 4: TECHNICAL PROOFS

Proof of Proposition 14: We aim to solve the problem stated in expression 4.3.

1) Pricing Policy BCR: Each constraint set simplifies into a range on θ. We first translate the constraint sets into ranges on θ as below: $\Lambda_B(\theta) \equiv \theta \in [\frac{p-r-\gamma p_c}{(1-\alpha)(1-\gamma)v_l}, \overline{M}] \equiv [\theta_{bc}, \overline{M}]$, $\Lambda_C(\theta) \equiv \theta \in [\frac{p_c}{v_h(1-\alpha)}, \frac{p-r-\gamma p_c}{(1-\alpha)(1-\gamma)v_l}] \equiv [\theta_{cr}, \theta_{bc}]$ and $\Lambda_R(\theta) \equiv \theta \in [\frac{r}{\alpha\bar{v}}, \frac{p_c}{v_h(1-\alpha)}] \equiv [\theta_r, \theta_{cr}]$. For existence of converter segment, the inequality $\theta_{cr} < \theta_{bc}$ should hold. This inequality simplifies to $p_c < \frac{v_h(p-r)}{\bar{v}} \equiv \bar{p_c}$. Only if the conversion price be less than $\bar{p_c}$, the market consists of some potential converters. (see Figure 12). Next, we take the profit function expressed in (4.3) and replace the constraints sets with the equivalent θ ranges that we just obtained above. Through solving $\frac{d\pi_C}{dr} = \frac{d\pi_C}{dp} = \frac{d\pi_C}{dp_c} = 0$, we can obtain the sole critical point of the profit function π at the below triple:

$$r^* = \frac{\alpha\overline{M}\bar{v} + c_r}{2} \qquad p^* = \frac{\overline{M}\bar{v} + c_s}{2} \qquad p_c^* = \frac{(1-\alpha)\overline{M}v_h + c_s}{2}$$

The existence of converter segment at optimal prices can be examined via the condition $\theta^*_{bc} \geq \theta^*_{cr}$, which is simplified to $c_r \leq \frac{(v_h-v_l)(1-\gamma)}{v_h}c_s$ (i.e., $p_c^* < \bar{p_c}|_{r^*,p^*}$). The optimal conversion discount can be calculated to be $\delta^* = \frac{p^*-p_c^*}{r^*} = \frac{\overline{M}(\bar{v}-(1-\alpha)v_h)}{\alpha\overline{M}\bar{v}+c_r}$ and $\delta^* > 0$ if and only if $\alpha > \frac{(v_h-v_l)(1-\gamma)}{v_h} \equiv k$.

The feasibility region of above interior solution, as we are going to call it BCR solution - where all three segments are optimally existent - is defined via checking for non-emptiness of the customer segments at respective optimal prices:

(1) $\theta^*_r \geq \underline{M} \leftrightarrow c_r \geq -(\overline{M} - 2\underline{M})\alpha\bar{v} \quad || \quad 2\underline{M} < \overline{M}$

(2) $\theta^*_{cr} \geq \theta^*_r \leftrightarrow \frac{c_r}{c_s} \leq \frac{\alpha\bar{v}}{(1-\alpha)v_h} \leftarrow$ renter segment

(3) $\theta^*_{bc} \geq \theta^*_{cr} \leftrightarrow \frac{c_r}{c_s} \leq \frac{(v_h - v_l)(1-\gamma)}{v_h} \equiv k \leftarrow$ converter segment

(4) $\overline{M} \geq \theta^*_{bc} \leftrightarrow c_r \geq (1-\gamma)(c_s - \overline{M}v_l(1-\alpha)) \leftarrow$ buyer segment

Let's denote Γ_{BCR} as the intersection of above (non-trivial/non-binding?) conditions on cost factors.

$$\Gamma_{BCR} = \{c_r, c_s | \frac{c_r}{c_s} \leq \min\{k, \frac{\alpha\overline{v}}{(1-\alpha)v_h}\} \cap c_r \geq \max\{(1-\gamma)(c_s - \overline{M}v_l(1-\alpha)), -(\overline{M} - 2\underline{M})\alpha\overline{v}\}\}$$

Obviously, the violation of any of the above conditions derives the solution to a particular corner solution, where one segment's size shrinks to zero. We next characterize all the remaining solutions to the firm's problem.

2) Pricing Policy BC: By violation of condition (2) above, the one-time renter segment becomes non-existent (i.e., $\theta^*_{cr} \leq \theta^*_r \leftrightarrow \frac{c_r}{c_s} \geq \frac{\alpha\overline{v}}{(1-\alpha)v_h}$). We then need to resolve the firm's problem by setting $r = \frac{\alpha p_c \overline{v}}{(1-\alpha)v_h}$ (that derives $\theta_{cr} = \theta_r$) and run the FOC for the profit $\pi(r, p, p_c)$ via the two variables p and p_c. The optimal prices under BC are:

$$p^* = \frac{\overline{M}\overline{v} + c_s}{2} \quad p^*_c = \frac{(1-\alpha)v_h(\gamma(c_s + \overline{M}(v_h - \alpha v_l)) + c_r + \alpha\overline{M}v_l)}{2(\gamma v_h + \alpha v_l(1-\gamma))}$$

The feasibility of solution BC at new optimal prices can be checked via below conditions:

a) $\theta^*_r \geq \underline{M} \leftrightarrow c_r \geq -\gamma c_s - (\overline{M} - 2\underline{M})(\alpha(1-\gamma)v_l + \gamma v_h)$

b) $\theta^*_{bc} \geq \theta^*_{cr} \leftrightarrow \frac{c_r}{c_s} \leq \frac{(1-\gamma)(\gamma v_h + v_l(\alpha - \gamma))}{\overline{v}} = c_{sc}$

c) $\overline{M} \geq \theta^*_{bc} \leftrightarrow c_r \geq (1-\gamma)(c_s - \overline{M}v_l(1-\alpha))$

$$\Gamma_{BC} = \{c_r, c_s | \frac{\alpha\overline{v}}{(1-\alpha)v_h} \leq \frac{c_r}{c_s} \leq c_{sc} \cap (c_r \geq \max\{(1-\gamma)(c_s - \overline{M}v_l(1-\alpha)), -\gamma c_s -$$

$$(\overline{M} - 2\underline{M})(\alpha(1-\gamma)v_l + \gamma v_h))\}\}$$

3) Pricing Policy BR: Violation of condition (3) in the main problem (i.e., $c_r > kc_s \leftrightarrow \theta^*_{cr} > \theta^*_{bc}$) derives the converter segment out of the picture. We can resolve the firm's problem by setting $p_c = \bar{p_c}$ (that derives $\theta_{cr} = \theta_{bc}$) and re-optimize the profit $\pi(r, p, p_c)$ wrt the two variables r and p. Here are the optimal prices for pricing policy BR:

$$r^* = \frac{\alpha \overline{M}\overline{v} + c_r}{2} \quad p^* = \frac{\overline{M}\overline{v} + c_s}{2} \quad p^*_c = \bar{p_c}|_{r^*,p^*} = \frac{1}{2} v_h \left(\frac{c_s - c_r}{\overline{v}} + \overline{M}(1-\alpha) \right)$$

The feasibility of solution BR at new optimal prices can be checked via below conditions:

a) $\theta^*_r \geq \underline{M} \rightarrow c_r \geq -(\overline{M} - 2\underline{M})\alpha\overline{v}$

b) $\theta^*_{cr} \geq \theta^*_r \rightarrow c_r \leq \alpha c_s$

c) $\overline{M} \geq \theta^*_{cr} \rightarrow c_r \geq c_s - \overline{M}(1-\alpha)\overline{v}$

$$\Gamma_{BR} = \{c_r, c_s | k < \frac{c_r}{c_s} \leq \alpha \cap c_r \geq \max\{-(\overline{M} - 2\underline{M})\alpha\overline{v}, c_s - \overline{M}(1-\alpha)\overline{v}\}\}$$

4) Pricing Policy CR: Violation of condition (4) in the BCR problem (i.e., $\overline{M} < \theta^*_{bc} \leftrightarrow c_r < (1-\gamma)(c_s - \overline{M}v_l(1-\alpha))$) derives the buyer segment out of the picture. We can resolve the firm's problem by setting $p = r + p_c\gamma + \overline{M}v_l(1-\alpha)(1-\gamma)$ (that derives $\overline{M} = \theta_{bc}$) and re-optimize the profit $\pi(r, p, p_c)$ wrt the two variables r and p_c. Via FOC, we obtain $r^* = \frac{\alpha\overline{M}\overline{v} + c_r}{2}$ and $p^*_c = \frac{(1-\alpha)\overline{M}v_h + c_s}{2}$.

The feasibility of solution CR at new optimal prices can be checked via below conditions:

a) $\theta^*_r \geq \underline{M} \rightarrow c_r \geq -(\overline{M} - 2\underline{M})\alpha\overline{v}$

b) $\theta^*_{cr} \geq \theta^*_r \rightarrow \frac{c_r}{c_s} \leq \frac{\alpha\bar{v}}{(1-\alpha)v_h}$

c) $\theta^*_{bc} \geq \theta^*_{cr} \rightarrow c_s \leq \overline{M}v_h(1-\alpha)$

$$\Gamma_{CR} = \{c_r, c_s | -(\overline{M}-2\underline{M})\alpha\bar{v} < c_r < (1-\gamma)(c_s - \overline{M}v_l(1-\alpha)) \cap c_s \leq \overline{M}v_h(1-\alpha) \cap \frac{c_r}{c_s} \leq \frac{\alpha\bar{v}}{(1-\alpha)v_h}\}$$

5) Pricing Policy B: Take the solutions BR and BC. If conditions (b) (at both solutions) are violated, i.e., $\frac{c_r}{c_s} > \alpha$ and $\frac{c_r}{c_s} > \frac{(1-\gamma)(\gamma v_h + (\alpha-\gamma)v_l)}{\bar{v}}$, the participating market boils down to one segment: outright buyers. We then need to resolve the firm's problem by setting $r = \alpha p$ and $p_c = \frac{p v_h (1-\alpha)}{\bar{v}}$ (that derives $\theta_{bc} = \theta_{cr} = \theta_r$) and re-optimize the profit π via the only remaining variable p and we have $p^* = \frac{\overline{M}\bar{v}+c_s}{2}$.

$$\Gamma_B = \{c_r, c_s | \frac{c_r}{c_s} > \max\{\alpha, \frac{(1-\gamma)(\gamma v_h + (\alpha-\gamma)v_l)}{\bar{v}}\}\}$$

6) Pricing Policy R: Take the solutions CR and BR. If conditions (c) (at both solutions) are violated, i.e., $c_s > \overline{M}v_h(1-\alpha)$ and $c_r < c_s - \overline{M}(1-\alpha)\bar{v}$, the participating market boils down to one segment: one time renters. We then need to resolve the firm's problem by setting $p = r + \overline{M}(1-\alpha)\bar{v}$ and $p_c = \overline{M}v_h(1-\alpha)$ (that derives $\overline{M} = \theta_{bc} = \theta_{cr}$) and re-optimize the profit π via the only remaining variable r and we have $r^* = \frac{\alpha\overline{M}\bar{v}+c_r}{2}$

$$\Gamma_R = \{c_r, c_s | c_s > \max\{\overline{M}v_h(1-\alpha), c_r + \overline{M}(1-\alpha)\bar{v}\}\}$$

Proof of Proposition 15: Let's look back at Γ_{BCR}

$$\Gamma_{BCR} = \{c_r, c_s | \frac{c_r}{c_s} \leq \min\{k, \frac{\alpha\bar{v}}{(1-\alpha)v_h}\} \cap c_r \geq \max\{(1-\gamma)(c_s - \overline{M}v_l(1-\alpha)), -(\overline{M}-2\underline{M})\alpha\bar{v}\}\}$$

The non-binding cost condition that causes the existence of converter segment is $\frac{c_r}{c_s} \leq k$. Taking the optimal selling (p^*) and post-rental price (p_c^*) from solution BCR, we can show that $p^* > p_c^*$ iff $\alpha > k$.

Proof of Proposition 16: The profit function $\pi(p, r, p_c)$ is not jointly concave in all three decision variables at all points. However, the following function determines the best response conversion price, given a pair of (p, r):

$$\frac{d\pi(p,r,p_c)}{dp_c} = 0 \rightarrow p_c(p,r) = p - r + \frac{c_r}{2} + \frac{1}{8}(\epsilon(1-\alpha) + \sqrt{\epsilon(1-\alpha)(8c_r + \epsilon(1-\alpha))})$$

The lowest possible conversion price is naturally achieved at 100% conversion discount i.e., a full refund of rental fee toward conversion. Unless $c_r = 0$, the firm shall never prefer consumers to try and definitely convert rather to buy outright. The condition $p_c \geq p - r$ eliminates the consideration of already known dominated pricing strategy for the firm. Therefore, we take $p - r$ as the natural lower bound on p_c. Given a fixed (p, r) pair, $\pi(p, r, p_c)$ has three critical points with respect to p_c, since $\frac{d\pi(p,r,p_c)}{dp_c} = 0$ yields three solutions as 1) $p_c = \overline{p_c} \equiv p - r + \epsilon(1-\alpha)$ and 2) $p_c = p - r + \frac{c_r}{2} + \frac{1}{8}(\epsilon(1-\alpha) \pm \sqrt{\epsilon(1-\alpha)(8c_r + \epsilon(1-\alpha))})$.

Computing the derivatives at lower and upper bound on p_c, we have $\frac{d\pi}{dp_c}|_{p_c=p-r} = +\infty$, $\frac{d\pi}{dp_c}|_{p_c=\overline{p_c}} = 0$ and $\frac{d\pi^2}{d^2p_c}|_{p_c=\overline{p_c}} > 0$ for $c_r < \epsilon(1-\alpha)$. These three facts together prove that only one of the two remaining critical points could fall in the feasible range $[p - r, \overline{p_c}]$ and it should be a maximizer [1]. Another way to locate the maximizer is to examine $\frac{d\pi^2}{d^2p_c}$ evaluated at the remaining critical points. Plugging the best response function into the profit function and solving for first

[1] Both remaining critical points could not fall within the specified range, since the first critical point is a minimizer and change of curvature in this range in possible only once.

order conditions, we have:

$$\frac{d\pi(p, r, p_c(p, r))}{dp} = \frac{d\pi(p, r, p_c(p, r))}{dr} = 0$$

leads to:

$$r^* = \frac{\alpha\overline{M} + c_r}{2} \quad p^* = \frac{\overline{M} + c_b}{2} \quad p_c^* = p^* + \frac{1}{8}\left((1-\alpha)\epsilon + \sqrt{(1-\alpha)\epsilon((1-\alpha)\epsilon + 8c_r)} - 4\alpha\overline{M}\right)$$

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