

Base independence in the analysis of tax policy effects: with an application to Norway 1992–2004*

by

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Abstract

The analysis contrasts results of two recently expounded micro-level data approaches to derive robust intertemporal characterizations of redistributive effects of income tax schedules; the fixed-income procedure of Kasten, Sammartino and Toder (1994) and the transplant-and-compare method of Dardanoni and Lambert (2002). Our study is normative in that the Blackorby and Donaldson (1984) index of tax progressivity is employed. This enables contributions from vertical redistribution and horizontal inequity also to be assessed, using for the latter one classical measure and one no reranking measure. When the competing methodologies are applied to Norwegian data for 1992–2004, their respective strengths and weaknesses are revealed. The transplant-and-compare procedure is found to have a number of advantages.

Keywords: Income tax; Tax progressivity; Horizontal inequity

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1. Introduction

One main advantage of considering information from empirical measures of tax progressivity lies in its twofolded informational content. With micro-level data information at hand, descriptions of pre-tax income distribution can be analyzed jointly with either the distribution of tax burdens, as according to the disproportionality measure (Kakwani, 1977), or the distribution of income after shaping from tax schemes, as in redistributive effect procedures (beginning with Musgrave and Thin, 1948). The tax policymaker can then assess to what extent the tax system decreases income disparities, by addressing information on distributions of tax burdens and of post-tax and pre-tax incomes. As tax policies often are discussed in an intertemporal perspective, for instance because governments are evaluated with respect to their efforts to redistribute income, information from tax progressivity measures is advantageous: trends in inequality of pre-tax income distributions can be held up against changes in distributions of taxes or post-tax income.

Trends in tax progressivity are obviously influenced by a number of factors. Not the least from a policymaking perspective, it is of great interest to isolate the effects of tax policies in order to obtain a better understanding of the driving forces behind a particular observed pattern. The literature has offered some suggestions to obtain more detailed information on tax policy effects, and two such efforts are the approaches proposed by Kasten, Sammartino and Toder (1994) and Dardanoni and Lambert (2002). Kasten, Sammartino and Toder (1994) suggest to identify effects of tax policy changes through a "fixed-income" approach, which means that pre-tax income distributions are kept fixed, a base year being chosen and exposed to taxation as per the various tax schemes of the period. Dardanoni and Lambert (2002) propose to compare post-tax distributions that have been adjusted to a common base regime, in which differences in pre-tax income inequality have been eliminated: this is the so-called "transplant-and-compare" approach.

The main contribution of this paper comes by applying each approach to the evaluation of tax policy in Norway 1992–2004. In so doing, we expose their respective strengths and weaknesses. A main criterion is that results should *not* be sensitive to the choice of base year, when the methodologies are employed in order to rank tax progressivity effects of tax schemes. On this ground, as well as for ease of application, transparency and informational requirements generally, we find that the transplant-and-compare procedure has a number of advantages.

Our study employs the Blackorby-Donaldson index of redistributive effect (Blackorby and Donaldson, 1984) in order to measure global tax progressivity. This measure is normative in the sense that it is rooted in social welfare function reasoning, in line with Atkinson (1970). The overall redistributive effect is decomposed into a vertical effect and a horizontal inequity effect, under the view that horizontal equity violations are separated and controlled for in order to describe the vertical

performance of the tax system. Estimates of horizontal inequity are derived according to two main perceptions of tax systems' vertical effects; the "classical view" (Musgrave, 1990) and the "no reranking perspective" (Atkinson, 1980; Plotnick, 1981). The former identifies vertical redistribution as the *average effect* on relative income differentials, while the latter defines the vertical effect as the transformation from the given pre-tax income distribution to the given post-tax income distribution that would not create *procedural unfairness*, in terms of rank reversals. The indices of horizontal inequity that are employed rest upon the same social welfare function foundation as the Blackorby-Donaldson index, described for the classical approach by Duclos and Lambert (2000) and for the no reranking approach by King (1983) and Jenkins (1994).

The structure of the paper is as follows. In section 2, we present the Blackorby-Donaldson index of tax progressivity, and we introduce the associated measures of vertical redistribution and horizontal inequity. In section 3 we present a series of tax progressivity measures for Norway 1992–2004, each conditioned by that year's income distribution, in order to set the scene for our subsequent isolation and analysis of tax policy effects. The two methods to distinguish the effects of tax policies from those of distributional change over the period, the fixed-income and transplant-and-compare methods, are presented and contrasted in Section 4. Section 5 concludes the paper with a discussion of its main findings.

2. The Blackorby-Donaldson tax progressivity measure and associated measures

The redistributive effect measured by the Blackorby-Donaldson index

Decompositions of redistribution (*RE*) into vertical effects (*VR*) and horizontal inequity (*HI*) have been undertaken in terms of various inequality indices. In each case, the decomposition takes the form

$$(1) \quad RE = VR - HI .$$

See Aronson, Johnson and Lambert (1994) for the Gini coefficient, Lambert and Ramos (1997) for the mean logarithmic deviation and Duclos and Lambert (2000) for the Atkinson index. Note that horizontal inequity enters as a deduction from the vertical redistribution component, which signifies that the overall redistributive effect is reduced by presence of horizontal inequity, or in other words: measures of the vertical stance of the tax system (*VR*) usually have a higher value than the overall redistributive effect (*RE*), since the horizontal effect (*HI*) reduces the actual redistributive performance of the tax system. In this perspective the tax breaks that cause horizontal inequity are measured as costs of foregone redistribution, see Ramos and Lambert (2003) for more on this.

Here we derive RE in terms of the Blackorby-Donaldson index (Blackorby and Donaldson, 1984, henceforth BD). It measures progressivity as the proportionate increase in equality relative to the initial level of equality. Its dependency on the initial level of equality (one minus inequality in pre-tax income) implies that it will evaluate a tax system as more progressive, when for the same improvement from the pre-tax income distribution to the post-tax income distribution, the initial pre-tax income distribution is more unequal. Subscripts x and y denote pre-tax and post-tax income respectively.

$$(2) \quad \Pi_{BD}(e) = \frac{I_x(e) - I_y(e)}{1 - I_x(e)}.$$

The BD index is founded on a social welfare function reasoning, as it employs the Atkinson (1970) index $I(e)$ as inequality measure. We let e be the (relative) inequality aversion parameter, capturing the concavity of the assumed utility function of a social decision-maker (henceforth SDM). The more concave this utility function is, the greater the net utility gain from any given rich-to-poor transfer. Higher values of e correspond to greater social valuations of equality. Let ξ define the level of income which, if equally distributed amongst all persons, would give the same level of social welfare as the original distribution. Because of the concavity, average utility of income falls short of what would pertain if the distribution were equal – the more so, the higher is e . The Atkinson measure of inequality $I(e)$, which is given as

$$(3) \quad I(e) = 1 - \frac{\xi}{\mu},$$

where μ is average income, measures the percentage of all income which could be given up, with no loss of social welfare if the remainder were distributed equally. Almost all income would be sacrificed in order to achieve equality when e approaches infinity.¹ For practical purposes, using micro data derived from a sample survey, the following measure is employed (here with respect to pre-tax income, x):

$$(4) \quad I_x(e) = 1 - \frac{\left[\sum_{i=1}^N \frac{w_i}{\sum w_i} (x_i)^{1-e} \right]^{\frac{1}{1-e}}}{\mu_x} \quad \text{if } e > 0 \wedge e \neq 1,$$

where w_i is the weight of observation i .

The BD index, as defined by equations (2) and (4), can be perceived as measuring the proportion of after-tax income the SDM would pay to convert a flat-tax system into the given one (see the appendix in Duclos and Lambert, 2000, for more on this). This definition of the BD index

¹ See *e.g.*, Cowell (2000) and Lambert (2001) for different interpretations of this approach to inequality measurement.

highlights the "cost of inequality" interpretation that usually can be given to measures based on the Atkinson index of inequality. However, the costs are here "costs of proportional taxation". This amount increases the more progressive the tax system is, and the more inequality-averse the SDM is.

Horizontal inequity

Horizontal inequity can be seen as a violation of the principle of horizontal equity; the "equal treatment" of "equals", a basic command of tax design (Musgrave, 1990). We may represent horizontal inequity by employing two different suggestions seen in the literature, the "classical" and the "no reranking" approaches. Both are based on essentially the same social welfare foundation as the BD index.

a. Classical

The classical approach to horizontal inequity measurement starts from a perception of the vertical effect, VR in equation (1), as an effect on average on relative income differentials, *i.e.* among unequals (Musgrave, 1990; Aronson, Johnson and Lambert, 1994; Jenkins and Lambert, 1999). In order to identify VR one has to eliminate HI , as this precludes the measurement of VR . But who are the unequals (or the equals)? Duclos and Lambert (2000) suggest to define the equals by "banding", but instead of imposing these bands "exogenously", they let the equals be identified by statistical methods: the kernel smoothing technique, which implies that standard statistical methods for bandwidth selection can be employed. The cost of inequality within each band j defines horizontal inequity, where the cost is defined by the difference between average post-tax income and the equally distributed equivalent, ξ_j . The definition of HI is, thus, kept within the same social welfare approach as the index of redistributive effect. The amount the SDM is willing to give up in order to eliminate HI within group j is defined by:

$$(5) \quad H_j = \mu_j - \xi_j = \mu_j - \left[\sum_{k \in S_j} \frac{w_k}{\sum w_k} (y_k)^{1-f} \right]^{\frac{1}{1-f}} \quad \text{if } f > 0 \wedge f \neq 1,$$

where subscript k refers to person k within equal-group j (note that the sorting is already done through the kernel procedure), and where μ_j is average post-tax income in group j . While e represented inequality aversion in equation (4) above, now f denotes horizontal inequity aversion in equation (5).²

In order to turn this measure into a global index, we do two things. First, we summarize over all H_j 's:

² See the discussion in Auerbach and Hassett (2002) on separate social preferences toward vertical and horizontal equity.

$$(6) \quad H_C = \frac{\sum_j \hat{w}_j H_j}{\sum_j \hat{w}_j},$$

where $\hat{w}_j = \sum_k w_k \lim_{x \rightarrow \infty}$ for $k \in S_j$ and subscript C symbolizes classical approach. Second, the aggregate measure is turned into a unit free index by dividing by the overall average post-tax income, μ_y :

$$(7) \quad HI_C = \frac{H_C}{\mu_y}.$$

This index of classical horizontal inequity is invariant with respect to scale, i.e., invariant to equi-proportionate post-tax income change.

Our measure of the vertical effect under classical horizontal inequity, symbolized by VR_C , is derived by adding the horizontal effect to the overall measure of redistribution, Π_{BD} :

$$(8) \quad VR_C = \Pi_{BD} + HI_C.$$

Hence, when discussing classical vertical redistribution in the following, the term "classical" refers to the method of measuring horizontal inequity, which in turn has bearing on the measurement of vertical redistribution.

b. No reranking

The "no reranking" procedure arises from the perception that if individuals are reranked in the transformation from pre-tax income to post-tax income, this amounts to procedural unfairness (Atkinson, 1980; Plotnick, 1981). As for the classical measure, the vertical action of the tax system, VR_{NR} in this case, is defined net of horizontal inequity effects, *cf.* equation (1). The following outline is based on the modification by Jenkins (1994, p.730) of the suggestion of King (1983). The HI index according to the King/Jenkins' no reranking procedure is defined by

$$(9) \quad HI_{NR} = 1 - \left[\frac{\sum_i^N \left[w_i y_i \left(1 + (|y_i - r_i| / \mu_y) \right)^{-a} \right]^{1-e}}{\sum_i^N w_i y_i^{1-e}} \right]^{1/1-e} \quad \text{if } e > 0 \wedge e \neq 1,$$

where subscript NR symbolizes no reranking, and r_i is the post-tax income individual i would have had if he had kept the same position in the post-tax income ranking as his position in the pre-tax income ranking. HI_{NR} is a number between 0 and 1: there is no HI when HI_{NR} is zero, while maximum HI is derived for HI_{NR} equal to one. The King/Jenkins approach introduces the parameter a , which is the horizontal inequity aversion parameter, whereas e is the inequality aversion parameter as above. HI_{NR} measures the proportion of total post-tax income that the social decision maker is willing to give up in

order to eliminate horizontal inequity. The cost in absolute terms would be $H_{NR} = HI_{NR}\mu_y$, corresponding to equation (7) under the classical approach. However, in contrast to the measure of classical HI employed in this study, estimates of no reranking HI depend on the overall aversion parameter, e , as seen in equation (9). The effect of assumptions with respect to the degree of inequality aversion is complex, as variations in e influence both the evaluation of the horizontally inequitable observed distribution and the level of social welfare were there no HI (Jenkins, 1994, p. 731).

Analogously to equation (8), we propose to measure vertical redistribution for the no reranking procedure, VR_{NR} , from redistributational effect Π_{BD} and the measure of horizontal inequity HI_{NR} , by adding:

$$(10) \quad VR_{NR} = \Pi_{BD} + HI_{NR}$$

Just as for the classical measure, the no reranking measure of horizontal inequity is invariant to equi-proportionate post-tax income change.

3. Trends in tax progressivity: Norway 1992–2004

In order to set the scene for our examination of tax policy effects in Norway, we evaluate horizontal and vertical trends in tax progressivity using the measures just described and data for the period from 1992 to 2004. Such comparisons, of year-specific progressivity indices, have been the main methodological vehicle for over time studies of tax progressivity heretofore; *cf. e.g.*, Bishop et al. (1997) and Lambert and Ramos (1997) for recent analyses of data from the U.S. and Spain, respectively.

Micro level information on incomes, deductions, household composition, etc., are derived from the yearly Income Distribution Survey for the period 1992–2002, while data are projected from 2002 to 2003 and 2004 in order to establish the 2003 and 2004 income distributions.³ The Income Distribution Survey is a yearly sample-based survey comprising information from income tax returns and social security registers. The extrapolations for 2003 and 2004 are based on information from various sources, such as measures of wage growth, capital losses, etc. Measures of tax burdens are obtained by employing Statistics Norway's tax-benefit model LOTTE.⁴ The nature of the data poses a number of measurement issues. One important challenge is that data from government registers often reflect the bureaucratic purpose of collecting them. Hence, our series of data are assessed with respect to changing accounting rules. Further, income is measured by equivalent income, which is derived by

³ These data projections highlight the advantage of employing tax-benefit model technology in such analyses. In principle, the series of data could have been further extended.

⁴ In general, there is close correspondence between what the tax-payers actually pay according to the Income Distribution Survey and the simulated tax burdens.

aggregating income over household members, weighing with an equivalence scale, and letting each household be represented with as many persons as there are household members. This procedure is in accordance with what Ebert (1997) describes as method 3.⁵ A small number of observations have been removed each year, due to negative pre-tax incomes.⁶

The tax progressivity index that is employed in the current analysis and its decomposition into vertical and horizontal effects involves selecting values for the inequality aversion parameter, e , and for the horizontal inequity parameters, f and a , representing aversion to horizontal inequity under the classical approach and the no reranking procedure, respectively. In the following we present results for the values $e = 0.25$ and $e = 0.75$, which replicate parameter choices of Duclos and Lambert (2000) with respect to Canadian data. In order to allow for an alternative that puts a higher value on rich to poor transfers in the case of Norway, we also present results for $e = 1.25$.⁷ Parameter choices for overall inequality aversion must, of course, be combined with choices of horizontal inequity aversion in order to invoke our measurement system. After some experimentation and with the benefit of hindsight, we made the following choices: For $e = 0.25$, we chose $f = 0.25$ and $a = 0.025$; for $e = 0.75$, $f = 0.50$ and $a = 0.05$; and for $e = 1.25$, $f = 0.75$ and $a = 0.05$. We discuss the sensitivity of horizontal inequity rankings with respect to parameter values in Appendix B.

The kernel procedure of the classical approach involves computational choices, even if it can be argued that this procedure to identify equals is less restrictive than other suggestions (Duclos and Lambert, 2000). The "close equals groups" are defined by employing a Gaussian kernel function⁸ where the optimal bandwidth is approximated by Silverman's rule of thumb (Silverman, 1986), and sample weights are utilized in order to control for representativity. The number of grid points is set to 500. Appendix B also reports results of altering the number of grid points.

In order to approach the normative and decompositional characteristics of tax progressivity, Table 1 presents figures for some of the components of the progressivity indices. Information for three years in the period under consideration is given; 1992, 1998 and 2004; calculations for each year are presented for the combinations ($e = 0.25$, $f = 0.25$, $a = 0.025$), ($e = 0.75$, $f = 0.50$, $a = 0.05$) and ($e = 1.25$, $f = 0.75$, $a = 0.05$). As described by equation (4), the Atkinson indices for pre-tax and post-tax income, I_x and I_y , are constructed from estimates of average

⁵ Ebert's method 4 uses the "equivalent adult" as the unit of analysis. We return to this later in the paper.

⁶ Negative incomes mainly stem from capital loss realizations. The procedure deletes not more than 0.3 percent of the original samples. One extreme observation at the high end of the 1998 post-tax income distribution has also been taken out, see footnote 21.

⁷ In accordance with the impression that there may be a high degree of inequality aversion in Scandinavian countries. Such attitudes, however, remain to be identified; see Lambert, Millimet and Slottje (2003) on inequality aversion across countries.

⁸ One should be aware that a Gaussian kernel is not the only kernel available, see Chapter 3 in Silverman (1986).

incomes and of the equally distributed equivalents (ξ). The Atkinson index increases monotonically with e ; for instance the inequality in post-tax income in 1992 increases from 2.4 percent for $e = 0.25$ to 12.1 percent for $e = 1.25$. If there is more inequality, the Atkinson index increases, the SDM becoming willing to accept a lower level of equally distributed income in order to eliminate income differences. The estimate of 12.1 percent means that, if incomes were equally distributed, only 87.9 percent ($100\% - 12.1\%$) of total (equivalent) income in 1992 would be required to achieve exactly the same level of total welfare. Inequality in both post-tax and pre-tax income increases over the time period. This is clearly not explained by uniform income growth, as the index is invariant with respect to scale.

Table 1. Estimates of some of the components of tax progressivity measures

	1992			1998			2004		
μ_x	198,961			261,414			359,476		
μ_y	153,316			194,807			272,355		
e	0.25	0.75	1.25	0.25	0.75	1.25	0.25	0.75	1.25
f	0.25	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
a	0.025	0.05	0.05	0.025	0.05	0.05	0.025	0.05	0.05
ξ_x	192,537	179,910	166,786	251,380	233,091	211,784	337,900	305,563	261,770
ξ_y	149,651	142,398	134,702	188,874	178,351	165,650	256,434	235,253	206,812
I_x (%)	3.229	9.575	16.171	3.838	10.835	18.985	6.002	14.998	27.180
I_y (%)	2.391	7.121	12.141	3.046	8.447	14.967	5.846	13.622	24.065
H_C	79	158	236	115	230	345	323	634	940
H_{NR}	225	399	361	296	467	398	683	775	573
HI_C (%)	0.051	0.103	0.154	0.059	0.118	0.177	0.119	0.233	0.345
HI_{NR} (%)	0.147	0.260	0.236	0.152	0.240	0.204	0.251	0.284	0.211

Table 1 presents cost measures of horizontal inequity, *cf.* equation (7),⁹ both for the classical approach and the no reranking procedure. We see that for $e = 0.25$ and $f = 0.25$ the SDM is willing to pay a "fee" of NOK79 of post-tax income per person in order to eliminate classical horizontal inequity in 1992, which means that the equal welfare level without horizontal inequity is rather close to average post-tax income. There is less classical horizontal inequity for smaller fees. In other words, a small fee indicates that there is good correspondence between tax burdens of persons

⁹ The same relationship between absolute costs of horizontal inequity and the index, as seen with respect to the classical approach in equation (7), also applies to the no reranking approach.

with similar pre-tax incomes.¹⁰ Table 1 also confirms, as we would expect, that the SDM is willing to pay more to eliminate classical HI as f increases. Note that classical HI, as defined in this study, is independent of the choice of overall inequality aversion.

This is in contrast to the no reranking measure employed in this study, which depends upon values of both e and a , see equation (9). According to the no reranking approach, the estimated costs in 1992 for $e = 0.25$ and $a = 0.025$ are NOK225, which means that the SDM is willing to pay NOK225 per person in order to abolish reranking of individuals in the transition from the pre-tax to the post-tax income distribution.¹¹ HI_{NR} , which is independent of scale, measures these costs in proportion to mean post-tax income. As noted above, horizontal inequity according to the no reranking criterion is associated with procedural unfairness of the tax system, while classical horizontal inequity is rooted in the unfair treatment of persons within groups, *cf. e.g.*, Jenkins and Lambert (1999).

Figure 1 shows the trend in tax progressivity from 1992 to 2004 for $e = 0.25$, $e = 0.75$ and for $e = 1.25$. This trend is downward, however it is less marked for the latter choice of inequality aversion (*cf.* Table A1 in Appendix A for more detailed descriptions of the estimates that Figure 1 is based upon).¹² Different values of e correspond to variations in emphasis on changes in different parts of the income distribution. For $e = 0.25$ and $e = 0.75$, both in terms of the two vertical measures, VR_C and VR_{NR} , and in terms of the overall redistributive effect, Π_{BD} , the estimates of 1992 are significantly higher than in 2004, according to 95 percent confidence intervals. For $e = 1.25$, Π_{BD} is significantly higher in 1992 than in 2004. The confidence intervals are obtained by bootstrapping procedures, drawing 200 random samples (with replacement) of the same size as the survey samples, and deriving tax progressivity estimates from each of these 200 samples. The rather low estimates of variance yield non-overlapping confidence intervals for measures of overall tax progressivity in 1992 and 2004.¹³

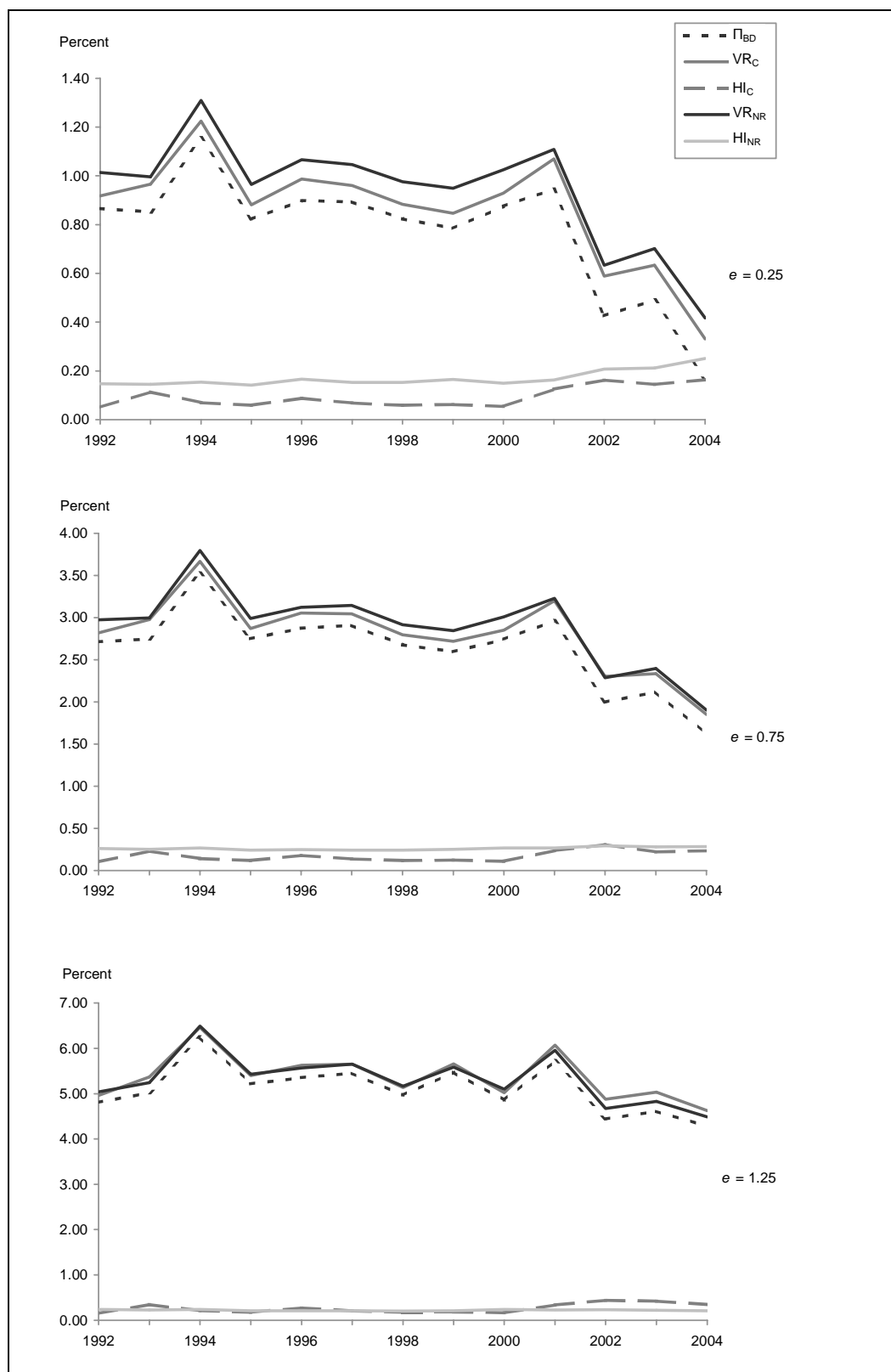
¹⁰ There is a close relationship between this procedure and income inequality measurement by the Atkinson index, as costs are derived from equally distributed post-tax incomes within equals-groups, see Section 2 above.

¹¹ Such "abolition" would involve reassigning the given post-tax income values to persons in accordance with their pre-tax ranks. See Lambert (2001), chapter 10, for more on this.

¹² Remember that alternatives of inequality aversion parameters are combined with choices of horizontal inequity.

¹³ The standard errors are not reported, but are in general low. We find 95 percent confidence intervals smaller than ± 1 percent, as also reported by Duclos and Lambert (2000).

Figure 1. Trends in overall redistributive effects, vertical redistribution and horizontal inequity 1992–2004, combinations ($e = 0.25, f = 0.25, a = 0.025$), ($e = 0.75, f = 0.50, a = 0.05$), ($e = 1.25, f = 0.75, a = 0.05$)



According to Figure 1, maximum progressivity is reached in 1994. That year, the BD index reaches about 3.5 percent, for $e = 0.75$. This means that the SDM would pay about 3.5 percent of post-tax income to convert an equal-welfare flat tax into the existing tax schedule. So far, the literature offers rather few BD index estimates, to which these results can be compared. However, according to Canadian figures, reported in Duclos and Lambert (2000), the corresponding progressivity fee in 1994 of a Canadian SDM is nearly 50 percent.¹⁴ The lower panel of Figure 1 shows that the fee to convert an equal-welfare flat tax rate into the existing tax rises above 6 percent in 1994, when it is assumed that the SDM is more inequality-averse ($e = 1.25$). Ebert (1997) argues that one should employ artificial distributions with "equivalent adults" as the unit of analysis (not the individual, as here), as this is in accordance with a social welfare approach. However, sensitivity tests reveal that orderings of years with respect to tax progressivity do not change when employing equivalent persons as unit of analysis.¹⁵

With respect to trends in horizontal inequity, Figure 1 shows a rather stable pattern over the period, both according to the classical approach (HI_C) and especially as per the no reranking procedure (HI_{NR}). Horizontal inequity increases from 1992 to 2004 according to the classical measure (at the 95 percent confidence level), while the no reranking measure of HI shows an increase from 1992 to 2004 for $e = 0.25$ and $a = 0.025$ and for $e = 0.75$ and $a = 0.05$, and a fall for $e = 1.25$ and $a = 0.05$. As the horizontal inequity aversion parameter is kept constant at 0.05 both for $e = 0.75$ and $e = 1.25$, the divergent results according to the no reranking procedure come from variations in e . When e increases, there is more emphasis on rerankings at the lower end of the distribution, which indicates that there has been a change in the distribution of rerankings between 1992 and 2004.

Vertical redistribution according to both the classical and no reranking approaches has essentially been in decline throughout the period. Of course, these estimates conflate the effects of changing demographic and cyclical factors, that influence pre-tax distribution, with those of the tax schedule itself. One would reasonably expect that changes in tax schemes have contributed in a major way to these trends. In the rest of this paper we discuss results of two different methods to determine such contributions.

¹⁴ An important reason for the relatively low progressivity of the Norwegian tax system is the low degree of inequality of the initial pre-tax income distribution. As noted above, the BD index evaluates a tax system as more progressive when, for the same improvement from the pre-tax income distribution to the post-tax income distribution, the pre-tax income distribution is more unequal.

¹⁵ This is in contrast to a finding of Decoster and Ooghe (2003) for Belgian data. In their study using dominance analysis, these authors warn that "using the number of equivalent individuals as weights ... leads to fanciful results with respect to the choice of equivalence scale" (*ibid.*, p. 173).

4. Tax policy evaluations

Time trends in tax progressivity estimates conflate both trends in the distribution of pre-tax income and effects of tax policy changes carried out over the time period. In this section, first we describe the main features of the Norwegian tax system in the period 1992-2004, and then we apply two different methodologies to isolate the effects on progressivity, vertical redistribution and horizontal inequity of these tax changes.

The Norwegian income tax, 1992-2004

The reform of 1992 entailed the adoption of a dual income tax system (see Sørensen, 1998), with a single basic tax rate of 28 per cent for both capital and labor income. In addition, wage income and incomes from self-employment were subject to a social security tax and a two-tier surtax, with rates of 9.5 percent commencing at NOK200,000 and 13 percent tax starting at NOK225,000. The surtax is, of course, a key progressivity-maintaining feature of the tax system. This part of the tax system has undergone substantial changes over the subsequent period. By 2004, the first rate had increased to 13.5 percent and kicked in at NOK354,300, while the second rate had risen to 19.5 percent, starting at NOK906,900. The combination of exemption and rate increases makes the overall effect difficult to gauge by only considering statutory tax-law regulations. Such changes reinforce the demand for micro data analysis.

Dividends, mainly an income source for high-income earners in Norway, were taxed at the individual level in 2001, prior to which they had been taxed at the corporate level only. This tax meant less profits transferred to individuals and more corporate savings.¹⁶ It was widely believed that the individual tax would be temporary, which was shown to be true.

Several characteristics of the Norwegian tax system represent potential sources of horizontal inequity. Obviously, the dual income tax system implies that taxpayers may face different tax burdens, dependent on income composition, for the same size of income. There are also special tax breaks for individuals living in the northernmost parts of the country, for charitable giving and for mortgage interest, along with a joint filing scheme for married couples. Obviously, these tax breaks may serve respectable social goals, such as maintaining population figures in the north, promoting home ownership, and so on. As noted in Section 2, these special treatments are assessed here in terms of costs of foregone redistribution.¹⁷

¹⁶ This finding also draws attention to measurement problems, created by difficulties of assigning corporate sector incomes to individuals.

¹⁷ See also Aarbu and Thoresen (2001) and Thoresen (2004) for further descriptions of the Norwegian tax system.

The Kasten-Sammartino-Toder procedure

Kasten, Sammartino and Toder (1994), henceforth KST, isolate effects of tax policy changes on progressivity through a fixed-income approach, which means holding the pre-tax income distribution fixed and letting it be exposed to taxation as per the tax schemes of the period.¹⁸ This method might provide robust measures of tax-policy effects over time: for example, one particular tax scheme might be seen to dominate all others with respect to distributional effects. However, KST reported that their results were not independent of which base year was chosen for the tax law comparisons.

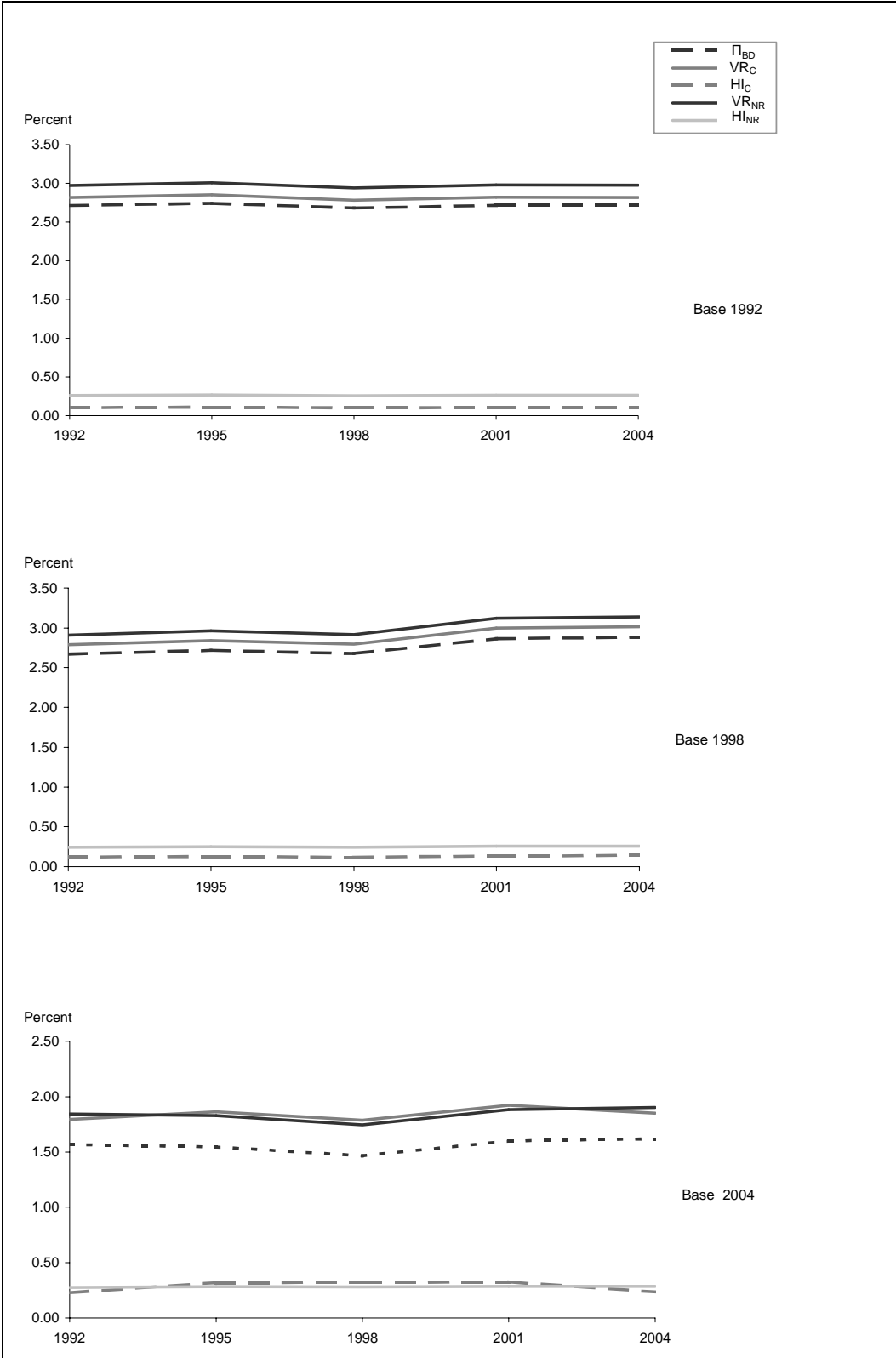
KST based their evaluations on measures of effective tax rates by quintiles. In our application, we use the summary measures presented in Section 2. In accordance with KST, we present results for several pre-tax income distribution bases; we choose those of 1992, 1998, and 2004. For each of these data sets, we let individuals be exposed to taxation as per the tax laws of 1992, 1995, 1998, 2001 and 2004. When tax regulations diverge from the point of time of the pre-tax income distribution, they are inflated or deflated to match the pre-tax income distribution, by a factor based on developments in wage per normal man-year. This means that all thresholds, including income deduction thresholds and tax deduction thresholds, are adjusted by the wage growth.

Under this method, special care must be executed with respect to new taxes and taxes that disappear during the period. The home care allowance, which was introduced in 1998, serves as an example. The home care allowance gives parents of preschool children aged 1–2 a transfer in cash that depends on utilization of public or private day care centers. After 1998, the data includes information on home care allowance eligibility, while older data sets do not contain information that permit such simulations. Therefore, when we apply the tax scheme of 2004 and 2001 to the datasets of 1998, 1995 and 1992, an equal size transfer to all families with children aged 1–2 is our only way of representing the home care allowance.

Figure 2 shows how tax policies of the period 1992–2004 are evaluated in progressivity terms under the KST approach, dependent on base year (*cf.* Table A2 in Appendix A for more detailed information). Note that results are shown for $e = 0.75$, only. The most striking feature of Figure 2 is the lack of variation over time in progressivity and also in horizontal and vertical effects according to this method. For instance, the upper panel (base 1992) shows hardly any variation at all. The problem of base-dependent results, as reported by KST, does also to a limited extent apply to the results of Figure 2: A small increase in tax progressivity from 1992 to 2004 is seen when pre-tax income distributions of 1998 and 2004 are employed as base years, while no significant results can be derived when data of 1992 are utilized.

¹⁸ KST carried out a comprehensive evaluation of U.S. federal tax burdens in the period 1980–93, including effects of income taxes, excise taxes, payroll-taxes and corporate taxes. See Decoster and Van Camp (2001) for analysis in the same vein.

Figure 2. Tax progressivity evaluations of tax policies 1992–2004. The Kasten-Sammartino-Toder procedure for different pre-tax income bases: 1992, 1998 and 2004



Let us point out some significant problems for practitioners that this application of the KST methodology has brought up. Firstly, we find that results are sensitive to the choice of factors used to inflate or deflate tax systems to the base year. For instance, we find that the 1992 tax-law wage-inflated to 2004 generates less tax revenue when applied to the 2004 pre-tax income distribution than the tax system of 2004 applied to the same 2004 pre-tax income distribution. If we adjust post-tax income in 2004 for these differences in tax revenues, the overall tax progressivity of 2004 falls below the estimate according to the 1992 tax-law. This problem, *i.e.* that results are sensitive to tax revenue effects, did not apply to the analysis in KST's paper, as they employed measures that are tax scale invariant (average tax burdens over quintiles).

Secondly, results are necessarily sensitive to how we define pre-tax income. In this study we employ a rather broad measure, which includes transfers. These transfers themselves may be subject to regulation over time, and thereby affect the pre-tax income distribution as well as the post-tax distribution. For example, when we exposed the 2004 base distribution to previous changes in the child benefit, we found that by making a concomitant change in the base (pre-tax) distribution itself, we got significantly different results from those obtained by ignoring this adjustment.

The Dardanoni-Lambert approach

Dardanoni and Lambert (2002), henceforth DL, suggest another method of transplanting one tax system into another, in order to undertake tax progressivity comparisons over time, based on the same motivational underpinning that applies to the approach of KST. The basic idea is that one schedule be imported into the income distribution of the other, controlling for inequality differences between the two regimes (or from both of them into a common regime). Distributional comparisons after such importation are guaranteed invariant to the choice of baseline if candidate distributions are isoelastic transformations of one another. An isoelastic function takes the form

$$(11) \quad g(x) = Ax^b \quad A > 0, b > 0$$

Making an isoelastic transformation means that pre-tax income distributions, in logarithms, differ only by location and scale, or in terms of the log of equation (11), differ only by the intercept $\ln A$ and the slope parameter b .

More formally, if we think of an income schedule N as mapping pre-tax income, x , into post-tax income, y , the DL procedure is based on finding a deformation function, $g(x)$ that defines a modified post-tax income schedule N^g , induced by N on deformed incomes. If the pre-tax income distribution is F , the definition of the transformed regime, $\langle N, F \rangle^g$, is such that N^g in fact operates on the distribution $F \circ g^{-1}$ and $g(x)$ can be chosen to render $F \circ g^{-1}$ into any desired other distribution.

A key result of the DL procedure is that base-independent results are guaranteed when the deformation function, $g(x)$, is isoelastic (and only then). In more simple terms; base-independency is ensured if the pre-tax income distributions only differ in logarithms by location point and spread (thus, not admitting shape dissimilarities: differences in scale involve expansions or contractions of distributions without altering their basic form). A way to approach this requirement is to assess goodness of fit with respect to a location-scale-invariant family of distributions, examples of which are the lognormal, Pareto and Singh-Maddala families of distributions. According to DL's Theorem 2, part (a), if the fit is good, the estimated parameters can be applied in order to transform distributions into a chosen one, for example the standard lognormal distribution based upon $N(0,1)$. We may find that traditional goodness-of-fit tests based on empirical distribution functions, such as the test statistics of Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling, lead to rejection of such models. However, one might question the adequacy of relying on statistical tools only in this case, as large samples might reject the null hypothesis even if the approximation is appropriate. This issue is discussed by *e.g.*, McCloskey (1985) and Hendry (1995) from a more general perspective.

Instead of relying on one specific parametric model of size distributions, we may utilize another result of DL, in part (b) of their Theorem 2, which vouches that comparisons can be carried out without involving any specific model distribution, whenever distributions differ in logarithms only by location and scale. From a practical point of view, this means that we should be able to reproduce the income distribution of year 1 on the basis of year 2, if we adjust year 2 by measures that represent differences of location and scale between the two, represented by the A and b of equation (11) above. In this case, comparisons can be undertaken by "*importing*" the schedule of year 1 into year 2.

To make the procedure more clear, let us say that we would like to compare the tax progressivity of 1992 with the tax progressivity of 2004, employing the same data sources as above. If we transform incomes into logarithmic form,¹⁹ the comparison will produce base-independent results if the vector $\ln x^{1992}$ is sufficiently close to the vector $a + b \ln x^{2004}$, for some a and $b > 0$, where the latter vector follows from taking logarithms of equation (11) ($\ln A = a$). Thus, the method implies finding estimates of a and b that minimize the differences between the two distributions in terms of location and scale. This corresponds to finding the intercept and slope in a traditional OLS regression, with pre-tax income in 1992 at each quantile as the dependent variable, and pre-tax income of 2004 at the same quantile as the explanatory variable (both in logarithms). The R^2 statistic becomes the relevant measure of goodness-of-fit. Before undertaking the regression, we need to transform the

¹⁹ A variable s belongs to a log-location-scale family if $t=\log(s)$ is a member of the location-scale family, see Meeker and Escobar (1998) for more on this.

larger sample, 2004, to the same size as the smaller, 1992, see footnote 23 in DL. Then we regress the pre-tax vector for 1992 on that for 2004:

$$(12) \quad \ln x_i^{1992} = a + b \ln x_i^{2004} + \varepsilon_i,$$

where the ε_i are random errors (which are assumed independently and normally distributed with zero mean and common variance), and subscript i refer to the rank of observations in the two equal-sized samples. If the fit is good, we can transform 2004 post-tax incomes, which reflect the tax schedule of 2004, into 1992-adjusted 2004 values of post-tax income, $y_i^{\hat{g}}$, where the topscript indicates that the post-tax incomes have been adjusted by a fitted deformation function. The comparison-relevant post-tax income distribution is now derived by employing estimates of location and scale from equation (12). Although the normality assumption for ε_i is *not* invoked by DL, results presented in Table C1 in Appendix C clearly show that incorporating a measure for the estimated residual in (12) is advantageous, in order to reproduce pre-tax income inequalities. Thus, we used:

$$(13) \quad y_i^{\hat{g}} = e^{\hat{a} + \hat{b} \ln y_i^{2004} + \hat{\varepsilon}_i}$$

to estimate the post-tax income distribution, where the random component is derived by making independent draws from a normal distribution with zero mean, and with standard deviation equal to the root mean-square error. The transplanted distribution of pre-tax income is derived similarly (though it could be argued that one might employ the pre-tax income distribution of 1992, F_{1992} , instead of the distribution function $F_{2004} \circ \hat{g}^{-1}$, as the latter is meant to approach the former, an issue to which we return shortly). The relevant regimes for comparing tax progressivity of 1992 and 2004 now become the pairs $\langle N_{1992}, F_{1992} \rangle$ and $\langle N_{2004}, F_{2004} \rangle^{\hat{g}}$, where \hat{g} is defined by equations (11)–(12).

If relative pre-tax income differentials are smaller in 1992 than in 2004, which is signified by estimates of $b < 1$, the 1992-transformed post-tax income distribution of 2004 is less dispersed than in 2004. The fundamental question, which will be discussed with respect to the Norwegian case below, is to what extent this control for pre-tax income inequality differences is able to eliminate, or even reverse, differences between post-tax income distributions of 1992 and 2004 that follow from differences in pre-tax income distributions. Hence, if the regime $\langle N_{2004}, F_{2004} \rangle^{\hat{g}}$ holds more progressivity than $\langle N_{1992}, F_{1992} \rangle$, we record a progressivity-enhancing policy change in the period. This method holds the promise of providing base-independent results. We examine this issue by providing empirical results with respect to various base years in the following.

The results of the OLS regressions are presented in Table 2. The log of gross income in each of the base years 1992, 1998 and 2004 is regressed against the log of gross income in each of the five years: 1992, 1995, 1998, 2001 and 2004.

Table 2. Ordinary least square regression results; log of gross income in 1992, 1998 and 2004 regressed against log of gross income 1992, 1995, 1998, 2001 and 2004

Year of data, regressors	Dependent variable			
	Log of gross income 1992	Log of gross income 1998	Log of gross income 2004	
1992	Slope, b	-	1.071	1.217
	Intercept, a	-	-0.607	-2.115
	R^2	-	0.973	0.917
	Root mean-square error	-	0.092	0.190
1995	Slope, b	0.909	0.993	1.149
	Intercept, a	1.040	0.270	-1.366
	R^2	0.977	0.993	0.961
	Root mean-square error	0.078	0.035	0.131
1998	Slope, b	0.909	-	1.152
	Intercept, a	0.877	-	-1.616
	R^2	0.973	-	0.967
	Root mean-square error	0.085	-	0.117
2001	Slope, b	0.879	0.966	1.119
	Intercept, a	1.111	0.267	-1.383
	R^2	0.953	0.987	0.967
	Root mean-square error	0.112	0.062	0.059
2004	Slope, b	0.754	0.839	-
	Intercept, a	2.609	1.783	-
	R^2	0.917	0.967	-
	Root mean-square error	0.149	0.099	-

The slope parameters in Table 2 reflect differences in scale or spread between the two distributions at hand. For instance, the slope parameter of the example we discussed above, in which pre-tax log income of 1992 is regressed on pre-tax log income of 2004, is 0.754. This signifies that the pre-tax income distribution of 1992 is significantly less dispersed than the 2004 distribution. As noted above, the goodness-of-fit measure, R^2 , is the key indicator of the isoelastic link, which is essential within this approach. The measures of R^2 in Table 2 are generally high; never below 0.9 and mainly in the range 0.95–0.99. However, it remains to be inferred at what level of goodness-of-fit this

procedure can be safely undertaken, or in other words: what does it mean to say that "the isoelastic form is satisfied"? We suggest discussing this issue in terms of how well the fitted distributions are able to reproduce the pre-tax income distributions they are intended to represent. These types of model evaluations are shown in Appendix C. There we also show that results to some extent depend on whether we let the dependent variable represent pre-tax income or employ the fitted distributions.

Next, estimates from Table 2 are employed to evaluate progressivity trends from 1992 to 2004. Figures 3, 4 and 5 show results with respect to the three selected pre-tax income distribution bases, and for the same three combinations of inequality and horizontal inequity aversion as utilized above: ($e = 0.25, f = 0.25, a = 0.025$), ($e = 0.75, f = 0.50, a = 0.05$) and ($e = 1.25, f = 0.75, a = 0.05$). See also Tables A3, A4 and A5 in Appendix A.

Let us first consider what Figures 3 to 5 tell us about base independence. All diagrams tell the same story: the overall distributive performance of the Norwegian tax system is greater in 1992 than in 2004. Moreover, orderings of vertical redistributive effects of tax schemes of 1992, 1995, 1998, 2001 and 2004 are identical across income bases, which means that the DL approach provides evaluations of the trend for Norway 1992–2004 which may safely be regarded as base-independent, unlike those obtained using the KST approach. According to Table 2, R^2 falls toward 0.9 when regressions involve incomes of 1992 and 2004. It is therefore encouraging to see that the results when applying the 1998 pre-tax income distribution, which are based on regressions where all measures of R^2 are above 0.95, are very similar to results based on incomes of 1992 and 2004.

What do Figures 3-5 tell us about Norwegian tax policy over time? A very distinctive feature is the reduction that has been caused in tax progressivity in 2004. As noted above, the period from 2001 to 2004 is characterized by high degree of permanence of tax schemes, except that the temporary individual tax on dividends was removed from 2002 onwards. As dividends increased from NOK13 billion in 2001, to NOK42 billion in 2002, NOK55 billion in 2003 and NOK65 billion in 2004²⁰, and as about 90 percent of this income component is transferred to tenth decile persons (in terms of post-tax income), there are reasons to relate the reduction in tax progressivity to the termination of the dividend tax. The KST approach did not expose this effect.

²⁰ Note that the 2004 income distribution will reflect these income changes, as projection methods utilize updated information on dividends.

Figure 3. Tax progressivity evaluations of tax policies 1992–2004. The Dardanoni-Lambert procedure for the 1992 pre-tax income distribution

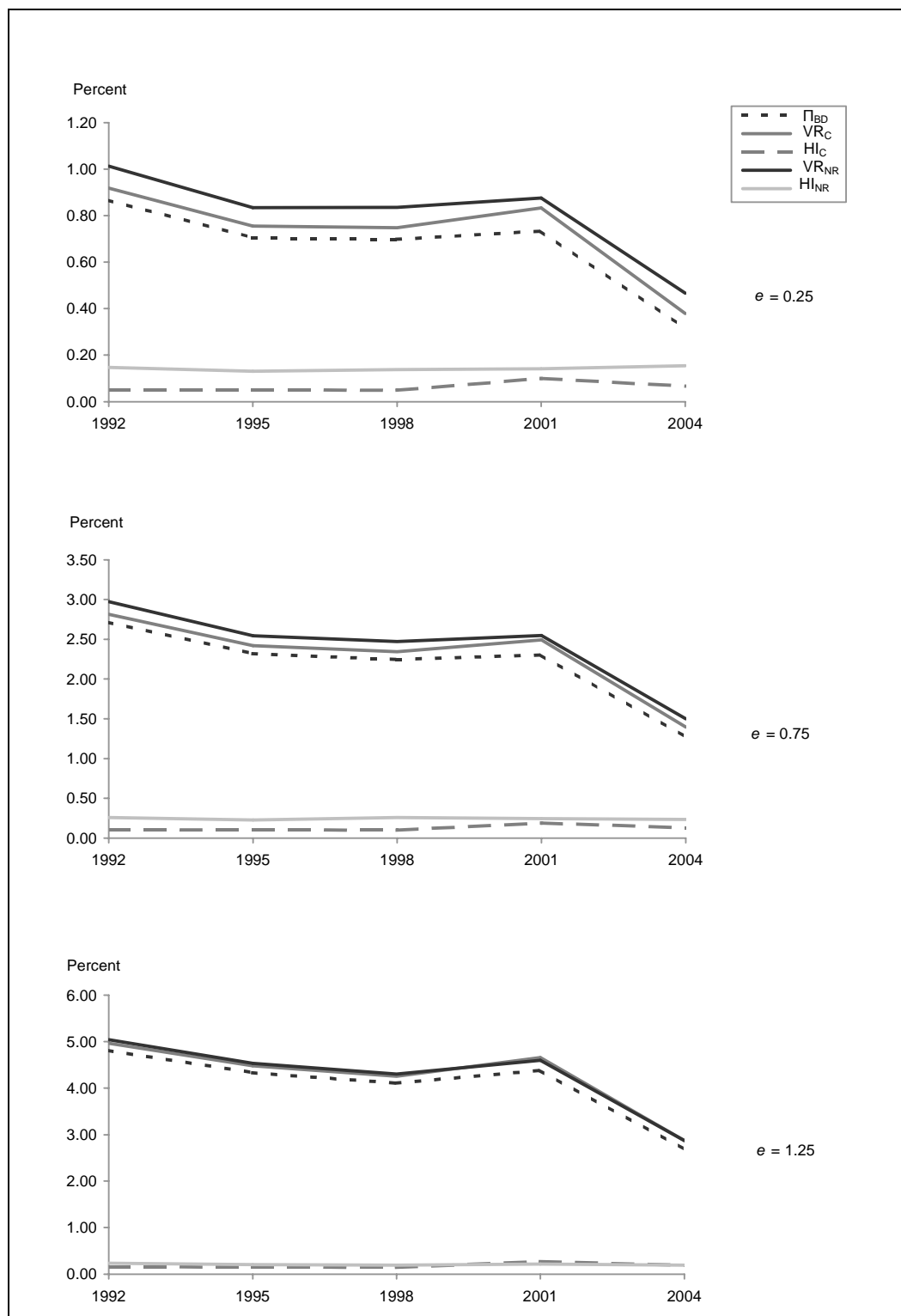


Figure 4. Tax progressivity evaluations of tax policies 1992–2004. The Dardanoni-Lambert procedure for the 1998 pre-tax income distribution

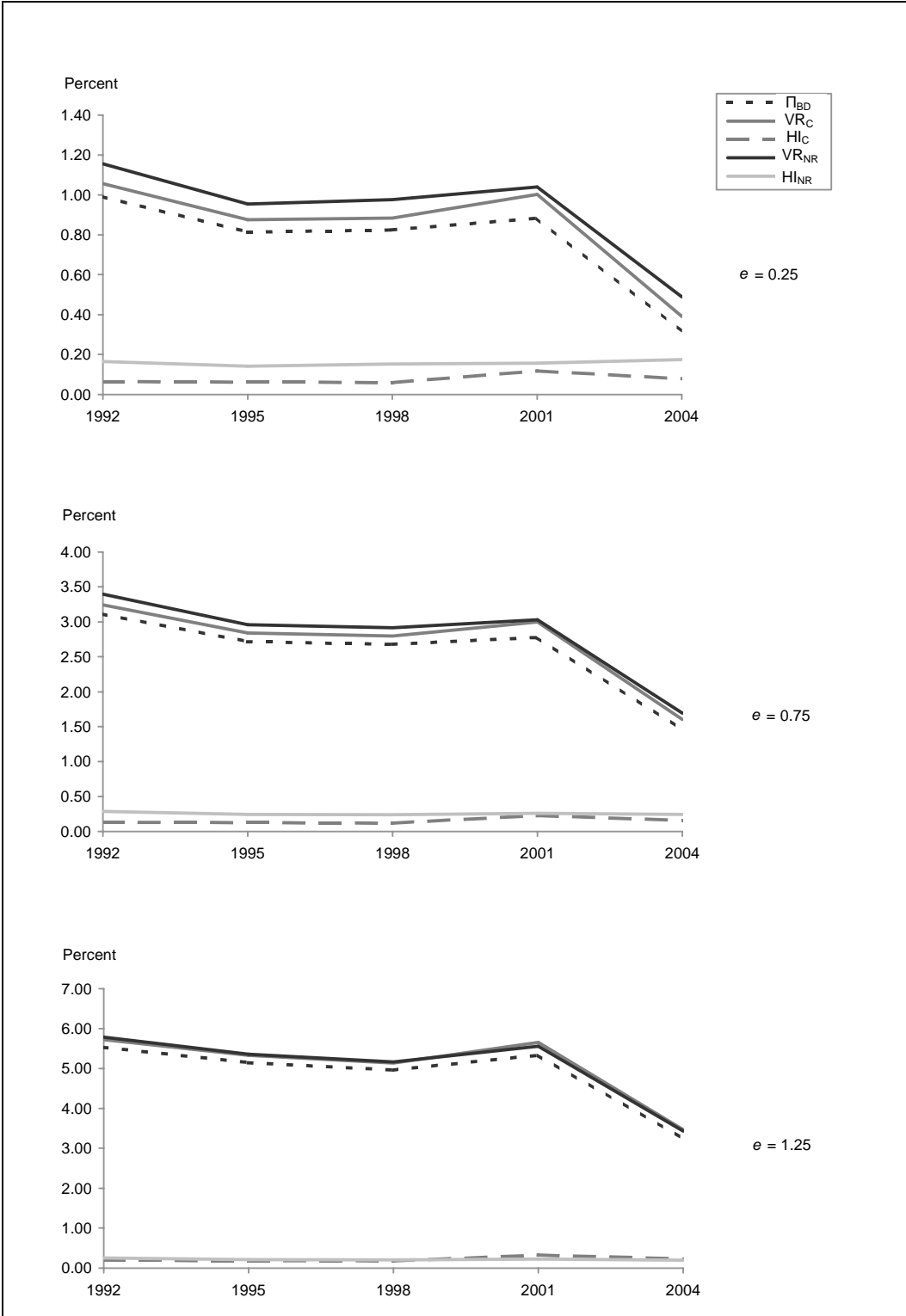
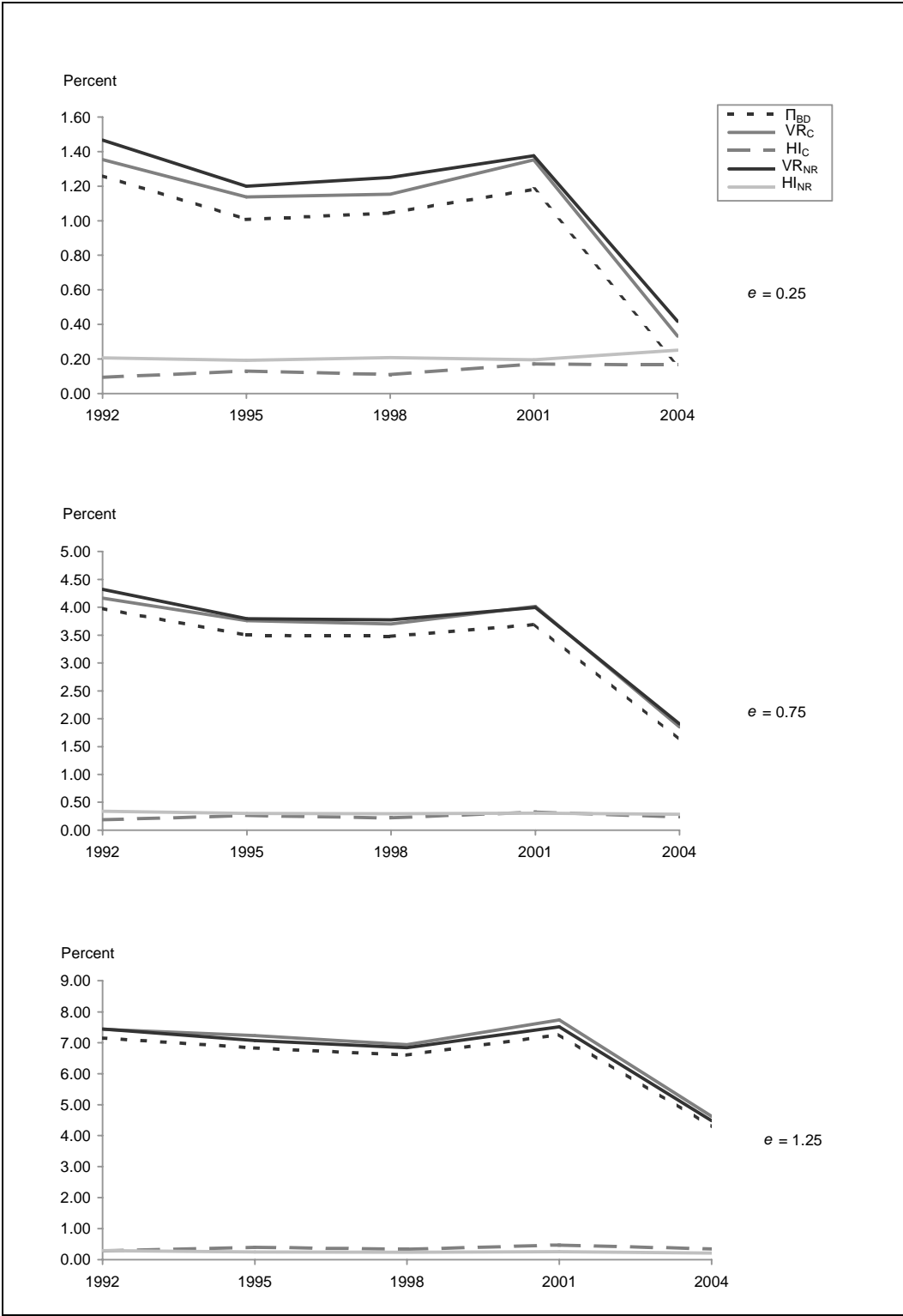


Figure 5. Tax progressivity evaluations of tax policies 1992–2004. The Dardanoni-Lambert procedure for the 2004 pre-tax income distribution



Now let us turn to horizontal inequity. The classical and no reranking measures evaluate the period somewhat differently: the degree of classical HI is higher in 2004 than in 1992, peaking in 2001 for all base years and for all combinations of horizontal inequity and inequality aversions, while results according to the no reranking measure depend on the level of inequality aversion, as also seen in Section 3. However, the overall picture, also signified by Figures 3–5, is that the degree of horizontal inequity has not changed very much over the period.²¹

The estimates for classical and no reranking vertical redistribution in Figures 3-5 show vertical redistribution to have been highest for the 1992 tax law. The DL procedure has allowed us to filter out other factors, such as demographic and cyclical influences on pre-tax distribution, and reveals that, if not for the blip in 2001 (the year of the temporary dividend tax), the vertical stance of the tax schedule *per se* would have been in more or less continuous decline over the period.

5. Conclusion

Over time descriptions of tax progressivity are important components of tax policy evaluations. The twofold informational content of tax progressivity measures raises an important possibility to isolate effects of tax policy changes in order to improve on the understanding of driving forces behind a particular observed pattern. The current analysis has employed the only two methods yet put forward in the literature; the fixed-income and the transplant-and-compare approaches. The first method keeps incomes fixed and makes tax schemes comparable by inflating or deflating different tax laws to a common base year, while the second, the transplant-and-compare procedure, adjusts schedules on the basis of changes that have occurred in pre-tax distributions. We find that only the latter procedure is able to produce robust results, i.e., results that are independent of base year when ordering progressivity effects of tax schemes.

The results of the fixed-income procedure did not yield any illuminating insights, and were also found to be sensitive to income definitions and to the choice of factors used to inflate or deflate tax systems to base years. Employing a tax scale invariant measure, such as the Kakwani measure, might counteract the latter problem: we are not convinced that the combination of the Blackorby-Donaldson measure and the fixed-income approach to identify tax policy effects can be recommended. One should also be aware that the fixed-income approach might be technically more

²¹ As noted in footnote 6 above, we have deleted an observation that received a dividend payment of NOK70 mill. in 1998. In the original data set this observation has been given an infinitesimally small weight due to its extreme character. However, when "enlarging" inequality of 1998 to the 2004 level under the DL approach, this observation was seen to have an unreasonable strong influence on the estimate of classical horizontal inequity. Thus, we decided to remove this observation from the analysis.

demanding than the transplant-and-compare procedure, as it requires some sort of tax-benefit modelling technology.

Given that the transplant-and-compare procedure appears to yield both sensible and robust results, it is worth emphasizing that data are derived from a fairly "stable society": even if the time period is long, incomes and income distributions develop gradually. It would be interesting to see if the isoelastic form provides a reasonable fit to data of other countries, as well.

This analysis also shows clearly the potential of the normative approach to tax progressivity measurement of Blackorby and Donaldson (1984), when supplemented by commensurate indices of vertical redistribution and horizontal inequity. Because it is based on a social welfare function, this approach yields estimates of the monetary cost of removing tax inequities with no welfare loss, which will have natural appeal to policy makers as well as being in the general tradition of measurement in other areas of public economics. The use of the BD and associated indices has shown, though, that results with respect to intertemporal horizontal inequity are to some extent sensitive to which approach is followed, the classical or the no reranking, and to the assumptions required for the procedures (*e.g.* parameter values for inequality and horizontal inequity aversion). Our recommendation is that one should be aware of such sensitivity, but not let it get in the way, as valuable information on tax schemes can be derived through such applications.

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Appendix A. Table Appendix

Table A1. Year-specific measures of redistributive, vertical and horizontal inequity effects, 1992–2004

	$e = 0.25$					$e = 0.75$					$e = 1.25$				
	Classical			No rerank.		Classical			No rerank.		Classical			No rerank.	
	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}
1992	0.866	0.918	0.051	1.013	0.147	2.714	2.817	0.103	2.974	0.260	4.808	4.962	0.154	5.043	0.236
1993	0.852	0.966	0.114	0.996	0.144	2.748	2.977	0.230	2.998	0.250	5.023	5.371	0.348	5.246	0.223
1994	1.156	1.225	0.070	1.310	0.154	3.528	3.668	0.140	3.796	0.268	6.249	6.459	0.210	6.491	0.242
1995	0.822	0.881	0.059	0.965	0.142	2.750	2.869	0.119	2.991	0.241	5.217	5.395	0.178	5.429	0.212
1996	0.899	0.987	0.088	1.066	0.166	2.875	3.053	0.178	3.122	0.247	5.355	5.625	0.270	5.566	0.210
1997	0.893	0.961	0.068	1.046	0.153	2.907	3.044	0.137	3.146	0.240	5.450	5.656	0.206	5.655	0.205
1998	0.824	0.883	0.059	0.976	0.152	2.678	2.795	0.118	2.917	0.240	4.960	5.137	0.177	5.164	0.204
1999	0.784	0.846	0.062	0.949	0.165	2.596	2.717	0.122	2.845	0.249	5.478	5.658	0.180	5.586	0.208
2000	0.866	0.918	0.051	1.013	0.147	2.714	2.817	0.103	2.974	0.261	4.857	5.020	0.163	5.098	0.241
2001	0.946	1.070	0.125	1.108	0.162	2.962	3.200	0.237	3.228	0.266	5.731	6.070	0.339	5.958	0.227
2002	0.426	0.589	0.162	0.634	0.208	1.997	2.302	0.306	2.288	0.292	4.441	4.877	0.437	4.673	0.232
2003	0.491	0.634	0.144	0.702	0.212	2.118	2.338	0.220	2.396	0.278	4.610	5.033	0.423	4.828	0.218
2004	0.166	0.331	0.164	0.417	0.251	1.618	1.851	0.233	1.903	0.284	4.278	4.623	0.345	4.488	0.211

Table A2. Measures of redistributive, vertical and horizontal inequity effects, 1992–2004. The Kasten-Sammartino-Toder approach for different pre-tax income bases, for $e = 0.75$

	Base 1992					Base 1998					Base 2004				
	Classical			No rerank.		Classical			No rerank.		Classical			No rerank.	
	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}
1992	2.714	2.817	0.103	2.974	0.260	2.669	2.790	0.121	2.910	0.241	1.567	1.793	0.226	1.843	0.276
1995	2.743	2.851	0.108	3.009	0.266	2.717	2.841	0.125	2.964	0.247	1.546	1.865	0.318	1.828	0.282
1998	2.681	2.783	0.102	2.940	0.258	2.677	2.795	0.118	2.917	0.240	1.464	1.786	0.321	1.745	0.280
2001	2.716	2.820	0.104	2.978	0.263	2.864	2.998	0.133	3.119	0.256	1.599	1.924	0.326	1.883	0.284
2004	2.715	2.818	0.103	2.975	0.261	2.881	3.015	0.139	3.138	0.256	1.618	1.851	0.233	1.903	0.284

Table A3. Measures of redistributive, vertical and horizontal inequity effects, 1992–2004. The Dardanoni-Lambert approach for the 1992 pre-tax income distribution

	$e = 0.25$					$e = 0.75$					$e = 1.25$				
	Classical			No rerank.		Classical			No rerank.		Classical			No rerank.	
	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}
1992	0.866	0.918	0.051	1.013	0.147	2.714	2.817	0.103	2.974	0.260	4.808	4.962	0.154	5.043	0.236
1995	0.704	0.755	0.051	0.834	0.130	2.319	2.420	0.101	2.545	0.226	4.330	4.482	0.152	4.533	0.203
1998	0.698	0.748	0.049	0.835	0.137	2.245	2.344	0.099	2.471	0.256	4.107	4.256	0.148	4.305	0.197
2001	0.734	0.833	0.100	0.876	0.142	2.303	2.493	0.190	2.548	0.244	4.385	4.657	0.272	4.601	0.216
2004	0.313	0.379	0.066	0.467	0.154	1.267	1.397	0.130	1.500	0.233	2.678	2.869	0.191	2.874	0.197

Table A4. Measures of redistributive, vertical and horizontal inequity effects, 1992–2004. The Dardanoni-Lambert approach for the 1998 pre-tax income distribution

	$e = 0.25$					$e = 0.75$					$e = 1.25$				
	Classical			No rerank.		Classical			No rerank.		Classical			No rerank.	
	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}
1992	0.991	1.056	0.064	1.156	0.165	3.112	3.241	0.129	3.398	0.286	5.533	5.726	0.193	5.787	0.254
1995	0.813	0.875	0.062	0.955	0.142	2.717	2.841	0.124	2.958	0.241	5.147	5.333	0.186	5.359	0.212
1998	0.824	0.883	0.059	0.976	0.152	2.678	2.795	0.118	2.917	0.240	4.960	5.137	0.177	5.164	0.204
2001	0.883	1.002	0.118	1.039	0.156	2.777	3.000	0.226	3.029	0.259	5.334	5.658	0.324	5.558	0.224
2004	0.314	0.392	0.079	0.489	0.175	1.448	1.603	0.155	1.691	0.243	3.236	3.464	0.228	3.433	0.197

Table A5. Measures of redistributive, vertical and horizontal inequity effects, 1992–2004. The Dardanoni-Lambert approach for the 2004 pre-tax income distribution

	$e = 0.25$					$e = 0.75$					$e = 1.25$				
	Classical			No rerank.		Classical			No rerank.		Classical			No rerank.	
	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}	Π_{BD}	VR_C	HI_C	VR_{NR}	HI_{NR}
1992	1.261	1.354	0.093	1.467	0.206	3.977	4.163	0.186	4.318	0.341	7.154	7.432	0.278	7.445	0.291
1995	1.007	1.137	0.130	1.199	0.192	3.496	3.758	0.262	3.793	0.298	6.829	7.226	0.397	7.077	0.248
1998	1.045	1.154	0.109	1.251	0.207	3.477	3.697	0.219	3.774	0.297	6.606	6.937	0.331	6.845	0.238
2001	1.182	1.352	0.170	1.376	0.194	3.690	4.015	0.324	3.993	0.303	7.266	7.734	0.468	7.516	0.250
2004	0.166	0.331	0.164	0.417	0.251	1.618	1.851	0.233	1.903	0.284	4.278	4.623	0.345	4.488	0.211

Appendix B. Sensitivity with respect to horizontal inequity measurement assumptions

Table B1 presents results of some sensitivity tests that have been carried out with respect to assumptions used in deriving estimates of the degree of horizontal inequity in Section 3. To simplify we show results for five years only; 1992, 1995, 1998, 2001, and 2004, and for $e = 0.75$. The values in bold refer to the horizontal inequality aversion parameters and number of groups for the kernel procedure of the classical approach that are utilized in the analyses. In each line of the table we show the ordering when one of these options is altered. The table shows that the orderings (the \rightarrow indicates an increase in horizontal inequity) are unaffected by altered horizontal inequality aversion under the no reranking measure, while orderings of 2001 and 2004 according to the classical measure are sensitive both to the number of equal groups and to the HI aversion assumption. However, we see that under all assumptions classical HI is lower in 1992 than in 2004, which signifies that meaningful results can be derived even if some sensitivity is observed.

Table B1. Sensitivity tests for the classical and the no reranking measures of horizontal inequity, $e = 0.75$

	Assumptions	Horizontal inequity orderings
HI_C	$f = 0.50, ng^* = 500$	1992 \rightarrow 1998 \rightarrow 1995 \rightarrow 2004 \rightarrow 2001
	$f = 0.25$	1992 \rightarrow 1998 \rightarrow 1995 \rightarrow 2004 \rightarrow 2001
	$f = 0.75$	1992 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 2004
	$ng^* = 400$	1992 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 2004
	$ng^* = 600$	1992 \rightarrow 1998 \rightarrow 1995 \rightarrow 2004 \rightarrow 2001
HI_{NR}	$a = 0.05$	1998 \rightarrow 1995 \rightarrow 1992 \rightarrow 2001 \rightarrow 2004
	$a = 0.025$	1998 \rightarrow 1995 \rightarrow 1992 \rightarrow 2001 \rightarrow 2004
	$a = 0.075$	1998 \rightarrow 1995 \rightarrow 1992 \rightarrow 2001 \rightarrow 2004

*Number of groups under the kernel procedure of the classical approach

Appendix C. Reproduction of pre-tax income distributions

To gain further insight into the working of the DL procedure with respect to the data utilized in this analysis, we assess to what extent the estimated pre-tax income distributions are able to reproduce the distributions they are intended to replicate. This means, for instance, that we compare the pre-tax income distribution of 1992 with the 1992-fitted pre-tax income distributions of 1995, 1998, 2001 and 2004. Further, it is also illuminating to see if the intertemporal characterizations are sensitive to whether fitted or actual pre-tax income distributions are employed.

Let us first consider the degree of correspondence between fitted pre-tax income distributions and the observed income distributions they are fitted to. Note that the fitted distributions are derived by employing estimates of intercept and slope together with random draws from a normal distribution with zero mean, and estimates of the root square-mean error as measures of the standard deviation, as in equation (13) above, but with x_i replacing y_i . The estimates in parentheses in Table C1 are summary measures of pre-tax income inequality for the three pre-tax income distributions that are used as dependent variables under the DL approach in Section 4, and these estimates are contrasted to measures of inequality that follow from the fitted distributions. Table C1 also presents estimates of pre-tax inequality when the error term component is omitted from the reproduction equation. Undoubtedly, the fit is poorer under this alternative.

Table C1 reveals that deviations between inequalities of fitted distributions and of the actual distributions they intend to reproduce do not exceed 12 percent. The main impression is that the transplantation method underestimates pre-tax inequality, which indicates that even if the fit is good and parameter estimates are supplemented by a stochastic element through random draws, the full extent of data variance might be difficult to replicate. However, results are not entirely consistent in that respect; pre-tax inequality in 2004 is overrated by the 2004-fitted pre-tax income distribution of 1992.

Another notable finding is that transplants based on the pre-tax distribution of 2001 deliver the poorest results according to all the three cases of Table C1. As discussed in Section 3, the temporary tax on dividends in 2001 is expected to have had a major effect on incomes of high-income earners. Has this shifted the basic form of the income distribution in that year, which makes it less suitable for this type of location-scale transplant procedure? The measures of R^2 in Table 2 do not suggest any inferiority of 2001 incomes in that respect.

Table C1. Pre-tax inequality in 1992, 1998 and 2004 compared to estimates based on transplanted distributions, $e = 0.75$

		Measures of pre-tax income inequality based on transplant procedures [actual values on diagonal in parentheses]				
		1992	1995	1998	2001	2004
With stochastic element	1992	[9.575]	9.182	9.267	8.759	8.930
	1998	11.200	10.648	[10.835]	10.105	10.558
	2004	15.011	14.585	14.820	13.328	[14.998]
Without stochastic element	1992	[9.575]	8.969	8.999	8.340	8.200
	1998	10.869	10.603	[10.835]	9.984	10.252
	2004	13.738	14.000	14.335	13.213	[14.998]

The results of the transplant-and-compare procedure in Section 4 are produced using estimates from the OLS regressions with respect to both pre-tax and post-tax income distributions. To get further information on the significance of any lack of fit, we examine if intertemporal tax policy orderings are affected if we let pre-tax distributions be represented by actual distributions instead of fitted ones. In Table C2 we compare orderings, both according to classical and no reranking vertical redistribution, when measures of pre-tax inequality are derived from fitted distributions (the approach in Section 4) and from actual distributions.

Table C2 shows that results to some extent depend on this choice. When actual pre-tax distributions are employed, the tax scheme of 2001 becomes the one with most vertical redistribution (the \rightarrow indicates an increase) according to 1998 and 2004 pre-tax income bases. As seen in Table C1, the estimates of inequality in 2001 based on transplants tend to understate actual pre-tax inequality. Furthermore, results are no longer independent of base.

Table C2. Vertical redistribution orderings when measures of pre-tax income inequality are based on fitted distributions and actual distributions, $e = 0.75$

		Fitted pre-tax income distributions	Actual pre-tax income distributions
1992	VR_C :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 2001 \rightarrow 1995 \rightarrow 1992
	VR_{NR} :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 2001 \rightarrow 1995 \rightarrow 1992
1998	VR_C :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 1992 \rightarrow 1995 \rightarrow 2001
	VR_{NR} :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 1992 \rightarrow 1995 \rightarrow 2001
2004	VR_C :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 1992 \rightarrow 2001
	VR_{NR} :	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 2001 \rightarrow 1992	2004 \rightarrow 1998 \rightarrow 1995 \rightarrow 1992 \rightarrow 2001