

4J SCHOOL DISTRICT: A CASE STUDY CORRELATING
CONTENT STANDARDS TO TEACHER PRACTICE

by

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In light of new legislation defining society's newest standards for math learning, my research aims to observe how teachers are adapting to put these mandates into practice. Through a case study of one high school Algebra 1 teacher, I analyzed how differences in pedagogical practices affected student learning outcomes. In observing the shifts in teacher practices in the facilitation of math discourse and the building of procedural fluency from conceptual understanding, I have found a strong correlation between the change in teaching practices and the shifts from the McDougal & Littell (M&L) and College Preparatory Mathematics (CPM) textbooks. In this study, "Conceptual Understanding" is defined as the ability for a student to "understand why a mathematical ideal is important and the contexts for which it is useful", and "Procedural Fluency" means that "students understand when to use certain procedures and how to perform them with both flexibility and precision." (National Research Council, 2001, p. 118) By creating a more encouraging environment where students are unafraid to ask for help, and providing more opportunities for students to justify their reasoning, the changes in Cornelia's teaching practices are a positive adaption to meet the Common Core State Standards for Mathematics (CCSSM) standards, and highly aligned to the

shift in textbooks. As the CPM lesson specifically dictates that students work in groups, there is an explicit emphasis on student communication as members must check-in with each other to verify their solutions. Additionally, CPM provides a higher percentage of problems that do not have solutions to reinforce the idea that students must justify when they can use a procedure. Overall, the shifts between Chapter ten of McDougal & Littell textbook and Chapter eight of College Preparatory Mathematics textbook are moderately aligned to the change in content standards. While CPM presents students with more opportunities to justify their understanding in writing and via peer communication, many improvements to Chapter 8 of the CPM text can be made to fully align the text to the CCSSM standards regarding quadratic equations. These changes include limiting the use of Learning Logs, (notebooks where students explain their conceptual understanding), until students can fully prove a hypothesis, including more sections that begin with contextual problems like Sections 8.2.1 and 8.2.4, and better connecting the 8 Standards of Mathematical Practice (Practices students should use in the math classroom) to each lesson. Because the curriculum shifts are moderately aligned to the changes in content standards, we can conclude that the changes in student standards have made a moderate impact on teacher practices.

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Thesis Questions, Relevancy of Research

In the last 3 years, Oregon made a momentous shift in K-12 math education by switching to the Common Core. While students will be faced with the challenge of learning new content and providing explanation for their thinking processes, educators are faced with the challenge of adapting their methods to accommodate the new expectations for student achievement, or else face the same consequences of mile wide, inch deep curricula that we face today. As the Common Core State Standards for Mathematics (CCSSM) is in its first year of implementation in Oregon, it is vital to determine if educators are changing their teaching practices to produce new results in their students. For my thesis I plan to analyze one high school Algebra 1 teacher, and determine what adjustments Cornelia has made in the last three years in her facilitation of mathematical discourse and the building of procedural fluency from conceptual understanding. Secondly, I plan to determine the reasoning behind why such changes were made, and if such changes have resulted in different student performance.

Introduction of Current and Former Math Standards

In order to understand if the shifts in teacher practice have been affected by the change in content standards, it is first necessary to review both content standards and the differences between them. As my thesis focuses specifically high school algebra math, I will provide a general comparison of Algebra standards, as well as an in-depth analysis in their expectations for quadratic equations.

Abstract Summary of Shifts between both Math Standards

My analysis of the 2009 algebra high school math content standards and algebra, and functions high school CCSSM standards were categorized into two sections, organization and content. Additionally, each set of standards was also divided into content, and student practice. For brevity, any mention of the 2009 high school algebra standards henceforth will be referred to as 2009 standards, and any mention of standards in the CCSSM algebra and function domains will be referred to as the CCSSM standards

In terms of organization, the main shifts between the 2009 and CCSSM standards are that the CCSSM are not partitioned by function type, but by general equations. Therefore, students are expected to apply concepts to linear, exponential, and quadratic equations regardless of the initial function that was used to teach the property. In terms of content, the CCSSM require more conceptual knowledge and procedural application than the 2009 standards, as shown in the Oregon Crosswalk example in later sections. Regarding quadratic equations in particular, which was the topic of my classroom observation, students are expected to meet all 2009 standards, and also

extend their knowledge of factoring to apply to quadratics with complex roots as well as leading coefficients greater than one. This, by extension, also requires students to show the roots, vertex, and line of symmetry of quadratics that give complex solutions, and not merely quadratics with integer roots.

In looking at the National Council of Teachers of Mathematics (NCTM) Process Standards and 8 *Standards for Mathematical Practice* (abbreviated as SMP's) from a student and teacher perspective, the SMP's provide a better basis for teachers to recognize the use of these practices in the classroom. In terms of student expectations, the SMP's are considered a more in-depth version of the Process Standards, and thus require more complex levels of reasoning, justification, and communication.

Background information on 2009 math standards

The 2009 Math Content Standards were drafted in October 2008 by the Oregon Department of Education (ODE). The standards were divided into two categories, “Standards for All Students” and “The Advanced Mathematics Standards”. The 3 primary goals of the ODE when drafting the Core standards were to:

- 1) Ensure a smooth transition from the 2007 K-8 Math standards,
- 2) Ensure alignment with the 3rd Oregon Essential Skill, “Apply Math to a variety of settings”, and
- 3) Refine the 2002 high school math standards so that they were more focused, coherent, and supported deeper understanding.

(Essential skills are requirements that students must show competency in to graduate high school. Most students achieve these through passing various standardized tests.)

To ensure alignment with Oregon's 3rd Essential Skill, the ODE added the NCTM

Process Standards to the new high school standards to “extend the same “coherent” and “connected” K-8 Content Standards through high school”(ODE, New High School Mathematics Content Standards Information, 2009). These NCTM Process standards help create a deeper understanding of math by changing students responsibilities from merely “doing” to “thinking about and applying math” (ODE, 2009).

Refining the 2002 High School Standards

The 2009 standards were an improvement on the 2002 high school math standards by limiting the number of focuses so as to prevent a mile wide, inch deep curricula. By limiting the number of standards from 79 to 40, the 2009 standards were organized into a Core Standard structure which created two or three Core Standards for the Algebra domain, with 3-8 supporting standards for each Core goal. In conjunction with the Core Standard structure, the standards were rewritten to help foster a deeper understanding of core content. For example, instead of recognizing or merely “Identify[ing]” the existence of independent and dependent variables in the 2002 standards, the 2009 version was revised to “Identify and analyze the relationship between” (ODE 2009) independent and dependent variables. This change moves beyond sheer recognition to looking at the interplay that set one apart from the other.

What Students were expected to Learn in 2009

The 2009 Math standards want students to demonstrate a deep understanding of Algebra by determining useful information, identifying equivalent expressions, fluently converting between mathematical representations, and distinguishing relationships among functions of different families.

Broken into 3 categories; Real Number Numeracy, Fluency with Linear models and their representations, and Quadratics and Exponential equations, there is an emphasis on using a particular format of expression and determining other useful information from an equivalent form. An example is H.2A.2, which states “Given a rule, a context, two points, a table of values, a graph, or a linear equation in either slope intercept or standard form, identify the slope of the line, determine the x and/or y intercept(s), and interpret the meaning of each.”(ODE 2009).¹

There is also an emphasis on fluently converting between various representations of linear equations. In H.2A.4, students are expected to “fluently convert” among the same representations listed in H.2A.2 above.

Lastly, students are expected to extend their fluency of converting representations to two functions of different families (H.2A.7). This includes the ability to distinguish among linear, exponential, and quadratic functions that are expressed in various forms (H.3A.3). This emphasis on analyzing the relationship between two sets, whether it is between different representations of the same function (H.2A.4), between the independent and dependent variables (H.2A.5), or between two functions of different families (H.3A.3) is a dominating characteristic of these standards.

As my case study deals specifically with quadratic functions, I have summarized the requirements regarding quadratic equations below.

¹ Notation for H.2A.1: H refers to grade level (high school), 2A refers to the 2nd core goal within the Algebra band, and 1 refers to the first standard within the 2nd core goal.

The 2009 algebra standards are divided between linear functions (H.2A) and quadratic and exponential equations (H.3A.1-5). By the end of high school, a student needed to be able to:

Expectations regarding Quadratic Equations (2009)

1. Factor quadratic equations [limited to leading coefficients =1]
2. Distinguish between linear, quadratic, and exponential functions presented in any form.
3. Convert between a quadratic equation and table and graph
4. Given a table or graph, extend the pattern to make predictions of other outputs.
5. Evaluate a function for specific values of the domain.
6. Determine and interpret the roots, vertex, and line of symmetry of a parabola (given integer roots) both graphically and algebraically.

Student Expectations regarding Quadratic Equations

One difference between the 2009 and CCSSM standards is that students are only expected to factor quadratics with a leading coefficient equal to 1, which does not include equations with complex roots. Additionally, while students must be able to interpret the roots, vertex, and line of symmetry of a parabola algebraically, it is not required for students to learn the “Complete the Square” method to convert between graphing and standard form. As we will see in the next chapter, a typical unit on quadratics in high school algebra 2 classrooms only focuses on converting between the standard and factored form of a quadratic equation.

The NCTM Process Standards

The NCTM Process Standards are thinking practices for students created to “highlight the mathematical processes that students draw on to acquire and use their content knowledge” (ODE, Adopted 2009 High School Math Standards, 2009). These processes are abilities that all students should be able to do at each grade level, regardless of their decision of where to go after high school. The five NCTM Process Standards include Communication, Connections, Representation, Problem Solving, and Reasoning and Proof. In addition, the ODE added a sixth process standard requiring students to reflect on one’s solution. The general progression of each Process Standard includes recognizing each standard’s importance to problem solving, developing said skill, and then extending one’s to include different strategies or modeling the standard in contextual situations. Using the standard for Reasoning and Proof as an example, students must recognize reasoning as a fundamental aspect of math, make conjectures, and then be able to “Select and use various types of reasoning and methods of proof” (ODE, 2009). While this standard has a clear definition of various levels of math reasoning, the definition lacks examples to help teachers and students alike understand what this practice would look like in the classroom.

Aligning to the 2007 K-8 Standards via the NCTM Process Standards

As the 2007 K-8 Content Standards used the NCTM Process Standards to align to the Oregon Essential Skills, the ODE included them in the high school math content standards to simultaneously create a smooth transition from the K-8 Standards as well as ensure alignment with the 3rd Essential Skill. These process standards are an

important aspect to the standards, as reflected by writing this statement on all pages of the high school standards:

“It is essential that the high school mathematics content standards be addressed in instructional contexts that promote problem solving, reasoning and proof, communication, making connections, designing and analyzing representations, and reflecting on solutions. Every student should understand and apply all mathematical concepts and skills from previous grade levels to these standards.” (ODE, 2009 Adopted High School Math Standards, 2009)

By listing the importance for students to practice these process standards on the front page of every subject, the ODE sent a clear message that concepts should come before procedures.

Challenges to these standards

Some challenges to the 2009 standards are that each standard is not of equal weight or difficulty. As shown in the below example, some standards require much more combinations to prove mastery than others.

As an example, H.2A.1, “Identify, construct, extend, and analyze linear patterns and functional relationships that are expressed contextually, numerically, algebraically, graphically, in tables, or using geometric figures” lists a stream of actions and formats that amount to 48 different types of analysis. Although the 2009 standards make it clear to student that they are expected to switch between all types of representations, the succinctness of these standards may make them difficult for teachers to apply to their lesson planning. For example, H.2A.1 could be rewritten as “Given a linear function, fluently convert between all aspects of the linear web as well as geometric figures”, with a linear web referring to all of the conversions between graphs, tables, equations, and contextual situations as they apply to linear equations.

Background Information on the Common Core State Standards of Mathematics

The CCSSM is a set of academic standards that provides outlines of skillsets for each individual grade level. It was developed in conjunction by the National Governors Association Center for Best Practices (NGA) and Council of Chief State School Officers (CCSSO) with the aim to develop students that are “College and Career ready”(Achieve, 2015). The definition of a College and Career ready student, as defined by the authors of the Common Core, is a high school graduate [that] has the knowledge and skills necessary to qualify for and succeed in:

1. Entry-level, credit-bearing college courses without the need for remedial coursework.
2. The postsecondary job training and/or education necessary for their chosen career. A career is defined as employment that provides a family-sustaining wage and pathways to advancement and requires postsecondary training or education.(National High School Center, 2012)

A Brief Summary on Math Education in the United States

The desire for US students to be “College and Career ready” is a goal that evolved over time. Since the 1980’s, the United States’ national math standards have consistently emphasized mastery of basic skills to dilute “employers' complaints about the costs of teaching basic skills to entry level workers”(Klein, 2003). A second influencing organization is the United States military, which throughout times of warfare complained of the millions of dollars spent on “costly remedial education and training programs in such basic skills as reading, writing, spelling, and computation(Klein, 2003). The third and most distinguishable influence stems from

international educational achievement comparisons. In a 2003 study called the Third International Math and Science Study (TIMSS), researchers collected data from over 42 countries from three different age groups; grades third and fourth, grades seventh and eighth, and 12th graders. Through a mixture of student exam scores, examinations of national curricula and textbooks, and in some instances, classroom observations, “evidence proved that American students and teachers are greatly disadvantaged by our country’s lack of a common, coherent curriculum and the texts, materials, and training that match it.”(Schmidt, Houang & Cogan, 2002) In order to articulate the potential differences between the highest and lowest achieving countries, the researchers from TIMSS compiled a set of core topics that at least two thirds of the highest achieving countries believed to be essential. This composite is the basis for the CCSSM. Since its creation in 2009; forty-three states as well as other US territories have adopted the CCSSM as their own curricula (NGA & CCSSO, 2015).

Improving thinking to enhance skills, versus learning skills to enhance thinking

In order to make students from the United States competitive on an international scale, the CCSSM standards were developed from a composite of essential topics in high achieving countries. This composite of standards is considered more rigorous than previous state standards because of its equal emphasis on developing student reasoning, and extending previous knowledge to future applications. As a way to help students achieve these more rigorous standards, the CCSSM also created the 8 Standards for Mathematical Practice (SMP’s), which changes the focus from exercising skills to developing student reasoning.

ODE Crosswalk tables

One way to exemplify the change in rigor between the two standards is with the ODE crosswalks. These crosswalks were made by the ODE as a tool to help transition educators to the CCSSM is by matching corresponding standards, estimate their alignment on a 0-3 scale, and provide short comments on which standards have more content or performance expectation, and why.

Out of the 18 Algebra benchmarks in the 2009 Math standards, there are 51 corresponding CC standards that at least partially incorporate the content from the 2009 standards (ODE, Oregon Math Crosswalks, 2004). Out of those 51 standards, 40% of them were at the middle school grade level. Of the remaining, none of the 2009 standards were considered equivalent to a CCSSM standard of a higher grade level, indicating that the CCSSM standards have higher expectations for students than the 2009 standards. Below is an example of a corresponding standard regarding factoring equations, which was the focus of my classroom observation:

Oregon Crosswalk Table; Factoring Quadratics

2009 Standard	CCSSM equivalent	Grade Change	Alignment	Partial Type	Comment
Factor quadratic expressions limited to factoring common monomial terms, perfect-square trinomials, differences of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers.	A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives	0	0	2	Match on factoring quadratics. OR standard is specific to quadratics with a leading coefficient of 1 (e.g. $a = 1$). CCSS does not limit the types of quadratics used. CCSS specifically identifies completing the square and quadratic formula, and

	complex solutions and write them as $a \pm bi$ for real numbers a and b .				includes complex solutions. OR includes solving by factoring in 12.3A.5
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As we can see, while both standards focus on procedural fluency, A.REI.4b emphasizes strategies that convert quadratics to equivalent forms (such as between graphing, standard, and factored form), and not just recognize shortcuts within standard and factored forms.² Additionally, A.REI.4b does not limit knowledge of quadratic equations to those that factor over the integers but within the complex number system, by requiring students to recognize when quadratics do not have roots. In requiring students to learn the complete the square method, students recognize that a quadratic written in graphing form is a quadratic of a perfect square trinomial pattern $(ax - b)^2$, with its vertex shifted up or down. Therefore, it is still possible to graph a quadratic equation without real roots by translation its parent graph to a different vertex. What's more, this perspective redefines students' notions of a factorable quadratic to mean any quadratic with real or complex solutions, instead of equations that "factor over the integers".

Shifts in Standards for Quadratic Equations

In the CCSSM standards, high school content is divided by subject and not grade level. Therefore Algebra 2 students may be taught content from both the Functions and Algebra domains. As it pertains to quadratic functions, by the end of high school, students should be able to:

² CCSSM standards are organized first by domain (Algebra), then cluster (Reasoning with Equations and Inequalities), and then standard within that cluster (4b).

Expectations regarding Quadratic Equations (CCSSM)

CCSS.MATH.CONTENT.HSA.SSE.B.3.A

Factor a quadratic expression to reveal the zeros of the function it defines. (no limitations on leading coefficient)

CCSS.MATH.CONTENT.HSA.SSE.B.3.B

Complete the square in a quadratic expression to reveal the vertex.

CCSS.MATH.CONTENT.HSA.REI.B.4.A

Use the method of completing the square to convert quadratics into graphing form $y = (x-p)^2 + q$.

CCSS.MATH.CONTENT.HSA.REI.B.4.B

Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

CCSS.MATH.CONTENT.HSF.IF.C.8.A

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

CCSS.MATH.CONTENT.HSA.REI.B.4

Solve quadratic equations in one variable.

Within these two domains, students' knowledge of factoring is extended to apply to quadratics with complex roots as well as leading coefficients greater than one. Therefore, the ability to show the roots, vertex, and line of symmetry also extend to these quadratics, not merely ones with integer roots. Students have to interpret key parts of a quadratic in terms of a context. In addition, there is explicit mention of using the complete the square method, which is not mentioned at all in the 2009 standards.

8 standards of Mathematical Practice

Similar to the NCTM Process Standards, the authors of the CCSSM created a list of mathematical practices that students of all grades should exemplify in the classroom. Indeed, the Process Standards and 8 SMP's share many similarities as the Process Standards make up the first string of the 8 math practices, with the second strand citing practices from the "National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding , procedural fluency and productive disposition" (NGA & CCSSO, 2015). In general, both the process standards and 8 SMP's are considered essential to the implementation of their respective math content standards. While both aim to develop high levels of problem solving and reasoning in students, the SMP's can be considered a more in-depth version of the Process Standards, as they are more descriptive, and provide a better basis of student expectations in class. The 8 Standards for Mathematical Practice are:

8 Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Content retrieved from the National Governors Association Center for Best Practices (NGA) & Council of Chief State School Officers (CCSSO), 2010.

The Differences in Development between the 2009 and CCSSM Standards

The biggest differences between the Common Core and other state standards are its organization and development process. After determining the essential body of knowledge needed for a "College and Career ready" student, the CCSSM standards for

each grade level were developed backward, starting at the high school level and working backward until they arrived at kindergarten. Therefore, as children get older, all new concepts are built upon a foundational knowledge from previous grades. This development, of creating all K-12 standards at the same time, is something that distinguishes the CCSSM from the Oregon 2009 and other math content standards, which were developed in separate grade bands of high school and K-8th grade.

National Standards versus State Standards

Another distinguishing factor between the standards is their similarities to other states' standards. Now adopted as the state standards in 43 total states, the CCSSM represents the first semblance of national curricula standards since the NCTM standards of 1989. The fact that so many states have adopted a version of the Common Core implies that teachers will have greater opportunities to materials as resources can be shared across states. What's more, textbook companies will have newfound motivations to publish textbooks with a strong alignment to the CCSSM in order to distinguish themselves from other companies.

NCTM Process Standards versus 8 Standards for Mathematical Practice

One shift between the Process Standards and the SMP's is that the SMP's specify how to gauge these practices by breaking the process standards into multiple categories. For example, the process standard "Communication", which requires students to consolidate thoughts, evaluate the arguments of others, and use math language to express precision, is broken up into 2 corresponding Math Standards: Construct viable arguments and critique the reasoning of others, and Attend to precision. In other examples, a process standard may correspond to a single SMP, but

require more levels of development to demonstrate complete performance. For example, in the math practice corresponding to Reasoning and Proof, reasoning is broken down into two categories (abstract and quantitative) and then further defined through that ability to “*decontextualize*—to abstract a given situation and represent it symbolically ... without necessarily attending to their referents—and *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.” (Standard for Math Practice #2) These added levels, while more challenging for students to achieve, provide a better framework for students to demonstrate and for teachers to recognize these practices.

Another shift is that the SMP’s are more specific and articulate. Although the SMP’s apply to a larger grade band (K-12) compared to the Process Standards (high school), each SMP provides examples of what the use of this practice looks like at different grade levels. These examples are extremely helpful for readers (parents, or teachers alike) who are learning how to gauge their students’ level of practice engagement in class. For example, in Modeling with Mathematics, 3 examples are provided that describe this practice in use across different grade bands:

In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. (SMP #4)

While it is assumed readers cannot rely on these examples as the only indication of Modeling with mathematics, their presence provides a better foundation for teachers to understand how to quantify these traits in their students.

Conclusions of Differences between CCSSM and 2009 Standards

In summary, the differences between both standards are that the CCSSM standards are better organized than the 2009 math standards, which thus aid teachers in aligning standards to their lesson plans. From the student perspective, students must have a stronger foundation of conceptual understanding, and expand their applications of procedures. Regarding student practices, the SMP's provide more examples for teachers to recognize student demonstrations of such practices in their classrooms. Although the SMP's require more complex levels of thinking from students, they also provide more support by providing examples for how to gauge student practice of these qualities. In my next chapter, I will analyze whether or not these same differences align to the shifts found in my comparison of a past and current chapter on quadratic equations.

Introduction of Case Study

To gauge if teachers are teaching differently because of the implementation of the CCSSM, I completed a case study with one high school Algebra 1 math teacher. My goals were to develop a comparison of how she taught Algebra 1 with the 2009 math content standards as the benchmark versus the CCSSM³. Setting the comparison at 3 years ago, I broke my research into four different sections: Past teaching vs present teaching and content vs pedagogy. With each section, I attempt to determine how the math content standards affected her textbooks and teaching style, and perhaps correlate changes in teacher practice to the change in math standards.

Modeling with the 8 Principles to Action

In comparing the shifts in teacher practice, I decided to model my observations off of the NCTM's 8 teaching practices from the book *8 Principles to Action*. This model describes 8 qualities that effective math teachers practice and is currently used as a model in Education Master's programs. However, it is important to note that while 8 Principles to Action provide one model to gauge effective math teaching, NCTM is a separate organization that through its own research came up with a list of qualities math teacher should have, and is by no means the only model. Thus, while I will be using a portion of this model to gauge changes in teacher practice, my focus is not on comparing how well Cornelia aligned to this model, but in finding which practices led students to better mastery of the CCSSM content and student practices. The eight teaching practices are:

³ Names have been altered to protect the privacy of teachers

8 teaching practices from *Principles to Action*

1. Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

During my classroom observation I chose to focus my research and observations on 2 of the 8 practices; Facilitating Mathematical Discourse, and Building Procedural Fluency from Conceptual Understanding, and created a scoring rubric to rate Cornelia each day on how she exhibits these principles.

The three things I looked for when evaluating Cornelia regarding Facilitating Discussion was creating an encouraging environment, basing discussion off of student work, and providing varying arguments that build off of one another.

Likewise, in evaluating Building Procedural Fluency from Conceptual Understanding, the three things I looked for was the level of student centered investigation, and at what point and how often did teachers “ask students to discuss and explain why the procedures they were using solved particular problems?” (NCTM, 8 Principles to Action, 45)

The first part of my research develops a complete picture of how Cornelia taught Algebra 1 three years ago. Before I describe my methodology, there are a few restrictions that should be discussed to limit the scope of my data.

Limitations to data

Single case study

Due to time restraints, the scope of this project was limited to observing one math teacher and gauging how that teacher has changed in the past 3 years. Any correlations that I find between the shifts in standards and pedagogical practices will not necessarily apply to other Algebra 1 teachers.

Limitations on textbook:

My initial reasoning for comparing teacher practices from 3 years ago was because 2011-12 was the last year the McDougal & Littell textbook (M&L) was used before the switch to College Preparatory Mathematics textbook (CPM). As one of the reasons CPM was introduced to classrooms was to help teachers transition to the

CCSSM, I had assumed that the M&L text was introduced for a similar reason.

However, I later found out that the M&L textbook has been used since 2002. Although my findings suggest that that the M&L textbook has a weak alignment to the 2009 content standards, it is important to note that the 6A district still used the M&L textbook until 3 years ago, and that there were still advantages that the textbook held for students.

Methodology for Curriculum Analysis

To see how these standards may be affecting the curricula, I analyzed two textbooks, one that was used during the era of the 2009 content standards from McDougal and Littell Company (M&L), and another that is currently used during the first year of CCSSM implementation called College Preparatory Mathematics (CPM). As the content that teachers use can have a significant impact on what students learn, I wanted to analyze at what levels each textbook aligned with the content standard of its time. My second intention was to compare whether the change between textbooks aligned to the changes in the content standards.

Model: EQuIP Rubric

Currently, the ODE uses Oregon Instructional Materials Evaluation Toolkit (OR-IMET) to “evaluate alignment of instructional materials to the Shifts and the major features of the CCSSM.” (ODE, Instructional Materials, 2015)

This rubric looks at the Alignment to the CCSSM content as well as the SMP’s. As my project compared two units from different textbooks and not the textbook as a whole, I used the Educators Evaluating Quality Instructional Products (EQuIP) rubric, which was created by Achieve the Core for unit lesson plans. This rubric is written in the same format as the IMET except on a smaller scale. Therefore, as the EQuIP rubric is the equivalent to the OR-IMET in terms of evaluating a unit of lessons, it is the assessment tool that I chose to use.

Because the 2009 content standards were designed to be more focused, coherent, and support a deeper understanding of the 2002 models, I analyzed the M&L textbook using the same rubric, with a few adjustments. For example, while the NCTM Process

standards clearly make up the first string of the 8 Standards for Mathematical Practice, I evaluated the M&L text based on the NCTM process standards and not on the SMP's.

Abstract

Overall, the shifts between Chapter 10 of M&L and Chapter 8 of CPM are moderately aligned to the change in content standards. CPM creates a better foundation of conceptual understanding than M&L by sequencing conceptual questions at the beginning of a chapter instead of the end, and providing exercises such as Learning Logs that prompt students to generalize patterns they learn in class. By presenting procedures within contextual problems, like the Water Balloon Problem and advice columns in Sections 8.2.3 and 8.2.5, students are not only prompted to investigate patterns, but also learn of the advantages and disadvantages of the procedures. However, while CPM is more closely aligned to the CCSSM than M&L is to the 2009 standards, CPM has much room for improvement. Such improvement includes explicitly connecting the 8 SMP's to the lessons, and challenging students to apply their knowledge to model real life situations.

Curriculum Analysis

Evaluation of McDougal and Littell Textbook, Chapter 10

I begin my curriculum analysis with an evaluation of the alignment between Chapter 10 of the McDougal & Littell textbook and the 2009 content standards. Overall, M&L includes too little opportunities for student investigation for a proper foundation of conceptual understanding. Additionally, concept reviews, when included, are not presented early enough in the lesson. Thus, any hope of procedural fluency is moot, because students are not challenged to understand the advantages and limitations of a procedure. Overall, procedures are practiced with such frequency that they become memorized. This, in conjunction with a lack of student justification of why a procedure works, provides an imbalance between conceptual understanding and procedural fluency.

Chapter 10 EQUIP Evaluation (M&L)



EQUIP Quality Review Process
 EQUIP Quality Review Rubric for Lessons & Units: Mathematics

Reviewer Name or ID: Greta Gleason

Grade: Algebra 1, M&L Mathematics Lesson/Unit Title: Polynomials and Factoring

I. Alignment to the Depth of the 2009 Math Content Standards

The lesson/unit aligns with the letter and spirit of the 2009 Math content standards:

- Targets a set of grade-level 2009 mathematics standard(s) to the full depth of the standards for teaching and learning.
- NCTM Process Standards that are central to the lesson are identified, handled in a grade-appropriate way and well connected to the content being addressed.
- Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the 2009 standards.

Summary of Observations and Suggestions for Improvement:

Chapter 10 of M&L covers all of the requirements listed in H.1A.5, "Factor quadratic expressions limited to factoring common monomial terms, perfect-square trinomials, differences of squares, and quadratics of the form x^2+bx+c that factor over the integers." While M&L extends factoring to include quadratics with leading coefficients < 1 , Chapter 10 only focuses on converting between standard and factored form.

The NCTM Process Standards are not identified in the lesson, and are weakly aligned to the content in Chapter 10. Overall, the unit could be strengthened by providing more open ended questions that can be solved with a variety of strategies, encouraging more communication with peers, and emphasize more student led investigation.

M&L includes too little student investigation for a proper foundation of conceptual understanding. Additionally, concept reviews, when included, are not presented early enough in lesson. Thus, any hope of developing flexibility with procedures is moot. In terms of balancing conceptual understanding and procedural fluency, procedures are heavily emphasized.

Rating for Dimension I: Alignment is non-negotiable and requires a rating of 2 or 3. If rating is 0 or 1 then the review does not continue.

Rating: 3 2 1 0

Rating Scale for Dimensions I, II, III, IV:

- 3: Meets most to all of the criteria in the dimension
- 2: Meets many of the criteria in the dimension
- 1: Meets some of the criteria in the dimension
- 0: Does not meet the criteria in the dimension

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EQUIP Quality Review Process

EQUIP Quality Review Rubric for Lessons & Units: Mathematics

Reviewer Name or ID: Greta Gleason

Grade: Algebra 1, M&L

Mathematics Lesson/Unit Title: Polynomials and Factoring

II. Key Shifts in the 2009 Math Content Standards

The lesson/unit reflects evidence of key shifts that are reflected in the 2009 math content standards:

- Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
 - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
 - **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports, and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Summary of Observations and Suggestions for Improvement:

Chapter 10 of M&L goes beyond the expectations of H.1A.5 by teaching students how to convert from standard to factored form with leading coefficients greater than 1.

Each Section of Chapter 10 uses knowledge from the previous sections to use as a basis for previous understandings. However, the order of sections could be improved to include a more developed progression. For example, instead of presenting students with an algebraic formula to find the roots and vertex of a quadratic in each different form, the text can sequence the lesson such that students practice switching between forms to easily retrieve the same information.

M&L does not provide a sufficient conceptual foundation for students to understand its material. While the book provides verification of concepts presented to students, there are not enough exercises where students verify their understanding by themselves. In terms of balancing conceptual understanding and procedural fluency, procedures are heavily emphasized. The text's emphasis on procedural fluency parallels the Algebra Conceptualized tests, (a series of 16 tests that must all be passed in order for a student to graduate), which are composed of 90% procedural questions.

Rating: 3 2 1 0

Rating Scale for Dimensions I, II, III, IV:

3: Meets most to all of the criteria in the dimension

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Close Analysis of M&L Rubric Score

Chapter 10 targets all of the requirements listed in H.1A.5: “Factor quadratic expressions limited to factoring common monomial terms, perfect-square trinomials, differences of squares, and quadratics of the form $x^2 + bx + c$ that factor over the integers.” (ODE 2009 Adopted High School Math Standard, 2009)

Summary

By the end of chapter 10 students should be able to add, subtract, and multiply polynomials, factor polynomials (including leading coefficients >1) and solve polynomial equations by factoring.

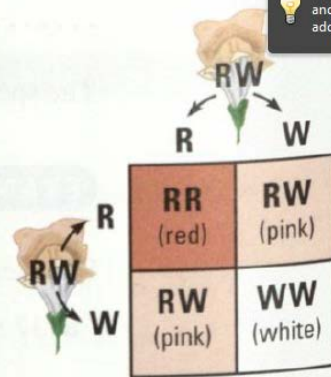
Students begin by learning how to add, subtract, and multiply polynomials [10.1]. In section 10.2 students learn how to multiply polynomials by modeling a generic rectangle and by using the FOIL method. After practicing how to use a generic rectangle for all quadratics in factored form, they progress to identifying shortcuts for quadratics in factored form. In 10.7 the process is reversed, and students learn shortcuts for factoring from standard form to factored form. In 10.4, 10.5, and 10.6, students learn about the zero product property and how to solve for all quadratics in standard form, including when the leading coefficient is greater than one. Lastly, section 10.8 uses the distributive property to find the GCF and “completely factor” a quadratic expression.

Minimum connection to Process Standards, Lack of challenging homework:

In general, the M&L text misses opportunities for students to demonstrate the Process Standards by 1) providing examples and solutions in every lesson and 2) basing homework off of the same models. Following the general outline of each lesson,

students are always given 4-5 examples with solutions directly after receiving formal definitions of that day's topic. Afterwards, homework questions almost always dictate which procedure to use, thereby limiting the number of opportunities for students to "choose a variety of appropriate strategies to solve problems (NCTM Process Standard #1). While every section includes real-world problems for student to apply that day's lesson, the majority of homework questions have scaffolding to such a degree that students hardly forge new connections. For example, in section 10.3, students are presented with Punnet squares describing the genetic makeup of a snapdragon, which acts as an interesting way to apply the shortcut of a perfect square trinomial pattern $[(a+b)(a+b)]$ in a new way.

SCIENCE CONNECTION The Punnett square at the right is an area model that shows the possible results of crossing two pink snapdragons, each with one red gene R and one white gene W . Each parent snapdragon passes along only one gene for color to its offspring. Show how the square of a binomial can be used to model the Punnett square.



SOLUTION

Each parent snapdragon has half red genes and half white genes. You can model the genetic makeup of each parent as $0.5R + 0.5W$. The genetic makeup of the offspring can be modeled by the product $(0.5R + 0.5W)^2$.

$$\begin{aligned}
 (0.5R + 0.5W)^2 &= (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2 \\
 &= 0.25R^2 + 0.5RW + 0.25W^2
 \end{aligned}$$

↑ Red
 ↑ Pink
 ↑ White

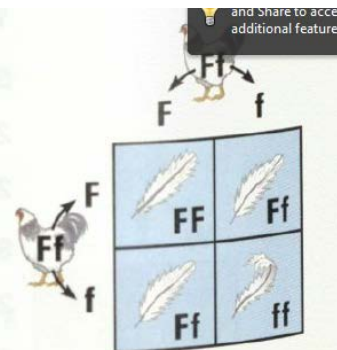
▶ 25% of the offspring will be red, 50% will be pink, and 25% will be white.

Section 10.3, page 590, M&L

While the example does provide an interesting connection between math and science, the following homework problem merely substitutes chickens in for snapdragons.

51. The Punnett square at the right shows the possible results of crossing two chickens with mildly frizzled feathers. Find a model that can be used to represent the Punnett square and write the model as a polynomial.

52. What percent of the offspring will have normal feathers? What percent will have mildly frizzled feathers? What percent will have extremely frizzled feathers?



Such plug and chug methods allow no opportunities or students to “Apply and adapt a variety of strategies to solve problems (Process Standard #1).

Conceptual Understanding versus Procedural Fluency


M&L does not provide a sufficient conceptual foundation for students to understand its material. While the book provides verification of concepts/definitions presented to students, there are not enough math problems that quiz students to verify their understanding by themselves. The only examples of student led verification are 3 concept questions in the “Guided Practice” section of every lesson, which would theoretically be presented in the last 20 minutes of a class. In general, roughly two thirds of these concept questions require students to expand on procedures introduced in the section. A good example of this type of concept question is in section 10.5, which asks “When testing possible factorizations for $x^2 + 2x - 3$, why is it unnecessary to test $(x-1)(x-3)$ and $(x+1)(x+3)$?” (M&L, Section 10.5, p. 607) By asking this question, students must recognize how unnecessary it is to test factor pairs that give a positive constant value, as the only way to get a negative constant value of -3 is by a product of a positive and negative integer. However, other concept questions ask students to “Tell whether the statement is true or false”, but do not require them to explain why. (M&L, Sections 10.4 & 10.8, p. 593 & p. 629)

Conclusion

Overall, M&L includes too little student investigation for a proper foundation of conceptual understanding. Additionally, concept reviews, when included, are not presented early enough in lesson. Thus, any hope of developing a sense of procedural fluency based on conceptual understanding is moot. In terms of balance between the two, procedures are heavily emphasized. The text’s emphasis on procedural fluency parallels the “Algebra Conceptualized” tests, which were a series of 16 tests that

students in this particular school district had to pass in order to receive school credit for Algebra. As 90% of each test was composed of purely procedural questions, and passing all 16 tests was a requirement for students, the Algebra Conceptualized tests helped perpetuate an emphasis of procedural fluency and lack of conceptual understanding.

Evaluation of CPM textbook, Chapter 8

	EQuIP Quality Review Process <i>EQuIP Quality Review Rubric for Lessons & Units: Mathematics</i>
Reviewer Name or ID: Greta Gleason	
Grade: Algebra 1, CPM	Mathematics Lesson/Unit Title: Quadratic Functions
I. Alignment to the Depth of the CCSS	
<i>The lesson/unit aligns with the letter and spirit of the CCSS:</i>	
<input checked="" type="checkbox"/> Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.	
<input type="checkbox"/> Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.	
<input checked="" type="checkbox"/> Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	
Summary of Observations and Suggestions for Improvement:	
<p>The major learning targets of chapter 8 correlate to three CCSSM standards (HSA.SSE.A.2, HSA.SSE.B.3, HSF.IF.C.7.A). By the end of the unit, students are able to convert between graphing form, standard form, and factored form of a quadratic equation, and discover the advantages each format provides. Additionally, students also learn how to convert a quadratic equation to a table, graph, or situation, and vice versa. While these standards do not specifically target quadratic equations, it is known that the same applications to linear and exponential functions are the focus of other chapters.</p> <p>The 8 standards of Mathematical Practice are not explicitly identified to each lesson, and therefore are in no way connected to the content being addressed.</p> <p>Chapter 8 provides ample opportunities to practice both mathematical procedures and demonstrate conceptual understanding. Regarding Procedural Skill and Fluency, students are given ample opportunities to practice their various factorization strategies (8.1.1-2), convert between quadratic forms (8.1.4, 8.2.4-5), as well as switch between representations (8.2.1, 8.2.2). However, when placed in a timed lesson plan, the emphasis of procedural mastery may diminish conceptual understanding among students. For example, while Learning Logs act as a great resource to record and justify conceptual understanding, students may write procedures in their Logs before fully understanding the process (Casey's theorem, 8-4, 3 points to sketch a parabola, 8-64)</p>	
<i>Rating for Dimension I: Alignment is non-negotiable and requires a rating of 2 or 3. If rating is 0 or 1 then the review does not continue.</i>	
Rating: <input type="radio"/> 3 <input checked="" type="radio"/> 2 <input type="radio"/> 1 <input type="radio"/> 0	
Rating Scale for Dimensions I, II, III, IV:	
3: Meets most to all of the criteria in the dimension	
2: Meets many of the criteria in the dimension	
1: Meets some of the criteria in the dimension	
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Chapter 8 EQuIP Evaluation (CPM)



EQuIP Quality Review Process

EQuIP Quality Review Rubric for Lessons & Units: Mathematics

Reviewer Name or ID: Greta Gleason

Grade: Algebra 1, CPM

Mathematics Lesson/Unit Title: Quadratic Functions

II. Key Shifts in the CCSS

The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:

- **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades.
- **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions.
- **Rigor:** Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:
 - **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations.
 - **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding.
 - **Procedural Skill and Fluency:** Expects, supports, and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.

Summary of Observations and Suggestions for Improvement:

While each lesson develops on the basis of previous knowledge, the unit could be strengthened by incorporating identification of a common factor into the general process of factoring using a generic rectangle. In particular, section 8.1.4 summarizes the process for "Factoring Quadratic Equations" directly after teaching students how to "completely factor", yet does not incorporate common factors into its generalization.

Application: Sections 8.2.2 and 8.2.4 are particularly strong in their use of real-world problems. In these lessons, students convert between all aspects of the quadratic web in the context of a Water Balloon toss, and create proper parabolic expressions to angry customers as Customer Service employees at a Quadratic Factory. However, the chapter could improve by including contextual problems in the other 8 sections.

Chapter 8 challenges students' conceptual understanding of quadratic equations by translating expressions into tables, graphs, situations, and vice versa. In addition to daily verbal justification, students are given 6 Learning Log assignments in the course of the unit to write down generalizations they make in class. Suggestions to strengthen the unit would include pushing the Learning Log assignment in 8.1.2 to a later day to prevent generalizing "Casey's pattern" without proper justification. Regarding Procedural Skill and Fluency, students are given ample opportunities to practice their various factorization strategies (8.1.1-2), convert between quadratic forms (8.1.4, 8.2.4-5), as well as switch between representations (8.2.1, 8.2.2).

Rating: 3 2 1 0

Rating Scale for Dimensions I, II, III, IV:

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Close Analysis of CPM Rubric Grade

Principal Standards that Chapter 8 targets:

CCSS.MATH.CONTENT.HSA.SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* (8-49)

CCSS.MATH.CONTENT.HSA.SSE.B.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

CCSS.MATH.CONTENT.HSA.SSE.B.3.A

Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.MATH.CONTENT.HSA.SSE.B.3.B

Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

CCSS.MATH.CONTENT.HSF.IF.C.7.A

Graph linear and quadratic functions and show intercepts, maxima, and minima.

Retrieved from CCSSM website (NGA & CCSSO, 2010)

Summary of content standard alignment

SSE.B.3 Standard form to factored form

One way students learn to produce equivalent forms of a quadratic equation is by switching between standard and factored form using an area model. After modeling with algebra tiles, students will develop a process to factor quadratics in a generic rectangle that does not involve a physical model. The book explains that using a generic

rectangle is helpful “when algebra tiles are not available or when the number of necessary tiles becomes too large to manage.” (CPM, Chapter 8, Problem 8-18) This process can be used to even when terms are missing or not in standard order.

Graphing form to Standard form

By section 8.2.3, students distinguish the advantages to standard form (finding y-intercepts) and graphing form and standard form. From first converting from graphing form to standard form, students learn that graphing form is advantageous when finding the vertex of a graph. Additionally, students learn factored form is most efficient for finding the x-intercepts or roots of a graph. By learning the Zero Product Property, students practice creating corresponding forms between within the quadratic web.

In 8.2.1 students practice converting between all representations of the quadratic web through an activity called the “Water Balloon Contest”. This activity describes the trajectory of 4 water balloon tosses in the form of a situation, table, graph, and quadratic equation, and has students convert all descriptions to table and graphing forms. The graphing form helps students recognize that parabolas are symmetrical, and that vertices are located on the mid-point, or line of symmetry between 2 parabolas. The table of values also solidifies the concept that y-intercepts have zeroes as the x-values, and x-intercepts have zeroes for the y-values.

SSE.B.3 Completing the Square

After learning how to find the roots and vertex of a quadratic algebraically, 8.2.3 uses an area model to describe the complete-the-square method. Having recognized that the vertex is on the line of symmetry from section 8.2.1, students are able to

algebraically determine the vertex coordinate by plugging in the x-value symmetrically between the two roots.

SSE.A.2 Shortcuts to using generic rectangles

In section 8.1.5, students learn how to recognize the difference of squares ($a^2x^2 - b^2$) and the perfect square trinomial patterns ($a^2x^2 + 2abx + b^2$) to convert between factored and standard form without using a generic rectangle. While students are learning to recognize these patterns, the text also provides unfactorable quadratics to reinforce the fact that not all quadratics have shortcuts, and that one must carefully decide if a shortcut can or cannot be applied to this expression (CPM, Teacher's Note, Section 8.1.5).

HSF.IF.C.7 Converting between rule and graph

In 8.2.2, the textbook describes the zero product property as a way to find the roots of a factored equation. Students learn that the y-intercept is easy to find in standard form, while the roots are readily found in standard form. These three pieces of information are sufficient to sketch a quadratic equation without the use of a table.

Standards for Mathematical Practice

The CPM textbook can greatly improve in identifying when students are using the 8 mathematical practices. The teacher's edition does nothing except to state which practices are being used throughout the lecture, with no connection to the daily content.

Although the 8 Standards for Mathematical Practices are not explicitly addressed in the unit, I was able to gauge how central the practices are to the content through my 20 days of classroom observation. Because I went into each class unaware

of what was to be taught that day, I was able to perceive the class as both a student and outside observer. Thus, my dataset on how students demonstrated the SMP's is a good gauge of the connectedness between the SMP's and the CPM content, as the CPM textbook did not explicitly connect the SMP's to each day's lesson.

Overall, CPM's emphasis on group work often prompts students to construct viable arguments and critique the reasoning of others (SMP 3). In fact, the teacher's edition of the text includes a list of activities that encourages student communication. For example, in 8.1.4, the text has students participate in a "Look at It" activity, where each student looks at a problem, and writes down a portion of the solution before passing it on to the next person. The second person then reads over the first line, edits it if necessary, and then proceeds with the second line of the proof. This cycle is continued until all students have written a line of the solution. This activity is a great way for students to warm up to their table mates, and get practice justifying steps and correcting the reasoning of others. In another example, students participate in a "Reverse Teaching" exercise, where students work in groups to solve one problem, and then diffuse into other groups to teach their problem to others. By having students explain the solution pathway that they took to reach their answer, the 'teachers' get to solidify their understanding of the problem's underlying concepts, which also makes students feel more comfortable talking with each other.

The use of modeling (SMP 4) was also prevalent in many of the lessons. Students modeled polynomial multiplication using algebra tiles in 8.1.1, and used the tiles again to model the 'Complete the Square' method in 8.1.5. In 8.2.1 and 8.2.4, students are given real world problems such as a Water Balloon toss and must convert

quadratic equations into other representations such as a table or graph. While CPM also assigned homework questions that required students to model real life situations (8-35, 8-55), these same problems were answered during the homework review of the next class day because students viewed them as too difficult.

Modeling with algebra tiles was also an exercise that encouraged students to “Look for and make use of structure” (SMP 7). As one of the three CCSSM standards targeted in this lesson involved “seeing structure in expressions”, it is clear that students had ample opportunity to exemplify this practice. In particular, Section 8.2.4, where students learned how to take the x - and y -intercepts from a graph to make an equation is a strong example of using the structure of a graph to create a different representation. However, it is important to note that even when this SMP was easily spotted during a classroom lesson, CPM still did not advertise the use of SMP’s to students in any part of the unit.

The two SMP’s that received the least recognition in Chapter 8 was SMP 5, “Use appropriate tools strategically”, SMP 6 “Attend to precision”. In all of my observations, section 8.2.1, where students used calculators and colored pencils to graph the trajectory of water balloons, was the only lesson where students specifically targeted SMP 5. Such lack of examples may stem from the fact that (at least in this class), students were unclear about whether they were allowed to seek out tools themselves. Thus, if Cornelia wanted students to use a calculator or other tool, she either told them directly, or mentioned the option so many times that students had no choice in using tools like tiles or calculators.

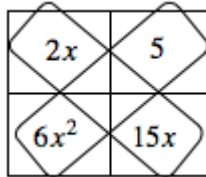
Similar to SMP 5, there were never any examples throughout my observation that explicitly focused on SMP 6, “Attend to precision”. While part of these findings may be because of my inability to imagine ways in which students would demonstrate precision with quadratic equations, such a conclusion also validates either a lack of student awareness about SMP 6 or a general confusion about how to demonstrate precision. While using proper definitions is a way to communicate precisely, there were just as many definitions that students used than did not use, including terms like monomial, binomial, and trinomial, as well as the formal names of quadratic patterns. Then again, these findings may also speak more to the fact that by the high school level, SMP 6 should not be considered a separate math practice, as precision is readily incorporated into other SMP’s. For example, in using a calculator to estimate the vertex of a graph, students are practicing both SMP 5&6.

Balance of Procedural Fluency and Deeper Conceptual Understanding

Chapter 8 provides ample opportunities to both practice mathematical procedures and demonstrate conceptual understanding. However, when placed in a timed lesson plan, the emphasis of procedural mastery may diminish conceptual understanding among students. For example, while Learning Logs act as a great resource to record and justify patterns, students may write procedures in their Logs before fully understanding why this shortcut works. The fact that students are able to use their Learning Logs during quizzes, tests, and homework makes this practice even more problematic, as students should always justify why a process works before using it.

One example of generalizing a pattern preemptively resides in section 8.1, where students create a hypothesis (that the products of diagonals in a generic rectangle are equivalent) from the image below.

Casey's Pattern (CPM)



CPM, Section 8.1, Problem 8-4

After only after having verified that the pattern holds for 3 other quadratic equations, students are prompted to write down Casey's pattern in their Learning Log: "Does Casey's pattern always work? Verify that her pattern works for all of the 2-by-2 generic rectangles in problem 8-3. Then describe Casey's pattern for the diagonals of a 2-by-2 generic rectangle in your Learning Log."

In assuming that verifying a pattern with 3 examples is sufficient evidence to generalize a rule, the textbook not only prevents students from thinking to their fullest, but indicates that as long as you provide three examples, any pattern is generalizable. The fact that students were asked to verify if Casey's pattern works in another quadratic expression in section 8.1.2 is proof that the kids generalize patterns too soon.

Focus & Coherence

The chapter provides a logical progression of lessons that develop knowledge built from previous understandings. For example, in Section 8.1, students use their previous experience multiplying polynomials with algebra tiles in Chapter 3 to reverse

the process, and convert between standard and factored form. Students similarly rewrite sums as a product by finding the Greatest Common Factor with polynomials. (8-10).

One suggestion to strengthen Chapter 8 is the incorporation of common factors into the general generic rectangle procedure. In 8.1.4 students learn about the Common Factor and how to “factor completely” by discovering the a value in standard form $a(x+b)(x+c)$. However, in a summary titled “Factoring Quadratic Procedures” provided directly afterwards, it fails to mention anything about the GCF.

Math Notes, Factoring Quadratic Expressions (CPM)

METHODS AND MEANINGS

MATH NOTES

Factoring Quadratic Expressions

Review the process of factoring quadratics developed in problem 8-13 and outlined below. This example demonstrates how to factor $3x^2 + 10x + 8$.

1. Place the x^2 and constant terms of the quadratic expression in opposite corners of a generic rectangle. Determine the sum and product of the two remaining corners: The sum is simply the x -term of the quadratic expression, while the product is equal to the product of the x^2 and constant terms.
2. Place this sum and product into a Diamond Problem and solve it.
3. Place the solutions from the Diamond Problem into the generic rectangle and find the dimensions of the generic rectangle.
4. Write your answer as a product: $(3x + 4)(x + 2)$.

2	6x	8
x	3x ²	4x
	3x	4

Rigor: Application, Conceptual Understanding, Procedural Skills & Fluency

Application: Sections 8.2.2 and 8.2.4 are particularly strong in their use of real-world problems. In these lessons, students convert between all aspects of the quadratic web in the context of a Water Balloon toss, and create proper parabolic expressions to angry customers as Customer Service employees at a Quadratic Factory. However, the chapter could improve by including contextual problems in the other 8 sections.

Chapter 8 challenges students’ conceptual understanding of quadratic equations by translating expressions into tables, graphs, situations, and vice versa. Students also develop processes to convert quadratics into 3 forms (standard, factored, and graphing

form), and learn the advantages of each. In every section, students are faced with counter examples that do not fit the pattern of the lesson, and must state why or why not a particular procedure can be used. In addition to daily verbal justification, students are given 6 Learning Log assignments in the course of the unit to write down generalizations they make in class. Suggestions to strengthen the unit would include pushing the Learning Log assignment in 8.1.2 to a later day to prevent generalizing “Casey’s pattern” without proper justification, as well as redefining what “sketching a parabola” requires in 8-64.

Regarding Procedural Skill and Fluency, students are given ample opportunities to practice their various factorization strategies (8.1.1-2), convert between quadratic forms (8.1.4, 8.2.4-5), as well as switch between representations (8.2.1, 8.2.2). Although problems are presented with more context than merely stating a procedure, the chapter can be strengthened by providing more problems that do not dictate which procedure to use to find a solution. Even by the end of the unit (Problem 8-112), students are asked to “Factor and use the Zero Product Property to find the roots of the following quadratic equations”². While conceptual understanding of the Zero Product Property is key to connecting the relationship between factored form of a quadratic with finding the roots on its graph, this procedural emphasis does not challenge students to “plan solution pathways” as encouraged in the first mathematical practice, but merely jump into a solution attempt.

Therefore, upon comparing the amount of opportunities of conceptual understanding and procedural fluency, the emphasis of procedural mastery may diminish conceptual understanding among students.

Improvements to the CPM textbook

While the CPM textbook is more closely aligned than M&L is to 2009 standards, there are a few ways the text can be improved to be better aligned with the CCSSM:

While it is good to write things in Learning Logs, CPM should take care to make sure students can fully justify a pattern before writing it in a notebook that students can use for other assessments. As I mentioned before, the fact that students were asked to verify if Casey's pattern works in another quadratic expression in section 8.1.2 is proof that the kids generalize patterns too soon. Additionally, student's proof for the Zero Product Property can also be strengthened before writing the property in their Learning Logs.

The text can also improve by explicitly connecting the use of SMP's in daily lessons. While most of the content in Chapter 8 provided a foundation for which students could demonstrate SMP 3, 4, & 7, care should be taken in specifying how SMP 5 & 6 appear in the Chapter.

Another suggestion to strengthen Chapter 8 is the incorporation of common factors into the general generic rectangle procedure. In 8.1.4 students learn about the Common Factor and how to "factor completely" by discovering the a value in standard form $a(x+b)(x+c)$. However, in a summary titled "Factoring Quadratic Procedures" provided directly afterwards, it fails to mention anything about the GCF.

Lastly, Section 8.2.4, which has students investigate how many points are needed to sketch a unique parabola, should be re-worded to define what distinguishes a

sketch from a graph. This section can also be improved to include a more rigorous verification of the uniqueness of a parabola.

Comparing the M&L and CPM textbooks

Overall, the shifts between Chapter 10 of M&L and Chapter 8 of CPM are moderately aligned to the change in content standards. Because while there is an increase in student justification and student reasoning, many improvements are needed in order for the CPM textbook to be fully aligned to the CCSSM.

To best model the shifts in the M&L and CPM lessons regarding the level of student justification, of conceptual understanding, and student reasoning, I am going to compare the M&L lesson 10.4 and CPM lesson 8.1.5, which focus on recognizing factoring shortcuts. I chose to compare these two lessons because they are the most closely aligned lessons in terms of content and structure, and therefore will most clearly demonstrate the differences between lessons.

Overview

For both sections, the lesson's objective is to have students recognize three patterns of quadratic equations that can be quickly factored with shortcuts. The patterns include quadratics of the form perfect-square trinomials $[(ax + b)^2 = a^2x^2 + 2abx + b^2$, $(ax - b)^2 = a^2x^2 - 2abx + b^2$], and the differences of squares $[a^2x^2 - b^2 = (ax - b)(ax + b)]$.

Questions that I kept in mind when creating my comparison were: How does each textbook develop conceptual understanding? Where does the book take steps to verify student understanding? At what level are students challenged when solving problems?

Levels of student justification

In the CPM text, students are placed in teams and given quadratics in standard form and then must “look for similarities and differences among the expressions and their corresponding factored form.” (CPM 8.1.5 8-45). In addition, students must “Be prepared to share your factored form with the class, then work as a class to sort them into groups based on the patterns you find.” (et al)

Factoring Shortcuts (CPM)

8-45. SPECIAL QUADRATICS

Your team will be assigned several of the quadratic expressions below to factor (if possible). Look for similarities and differences among the expressions and their corresponding factored forms. Be prepared to share your factored results with the class. Then work as a class to sort them into groups based on the patterns you find in their factored forms.

a. $x^2 - 49$

b. $x^2 + 2x - 24$

c. $x^2 - 10x + 25$

d. $9x^2 + 12x + 4$

e. $5x^2 - 4x - 1$

f. $4x^2 - 25$

g. $x^2 - 6x + 9$

h. $x^2 - 36$

i. $7x^2 - 20x - 3$

j. $4x^2 + 20x + 25$

k. $x^2 + 4$

l. $9x^2 - 1$



Chapter 8, Section 8.1.4, Problem 8-45 (CPM)

As the text specifically dictates that students work in groups, there is an explicit emphasis on student communication as members must check-in with each other to verify their factorization. What’s more, as the number of the categories aren’t specified, students are challenged to justify why each expression fits into its given group. Lastly, the instructions warn that some equations may not even be factorable, which re-enforces the idea that the inability to use a procedure on a problem does not prove it is unsolvable.

While in the M&L text, 12 quadratic equations are presented in *factored form* and are already grouped based on the patterns students are investigating.

Factoring Shortcuts (M&L)

ACTIVITY
Developing Concepts

Investigating Special Product Patterns

1 Use the FOIL pattern to find the products in each group.

1. $(x - 2)(x + 2)$	2. $(x + 3)^2$	3. $(x - 2)^2$
$(2n + 3)(2n - 3)$	$(3m + 1)^2$	$(6x - 4)^2$
$(4t - 1)(4t + 1)$	$(5s + 2)^2$	$(5p - 7)^2$
$(x + y)(x - y)$	$(x + y)^2$	$(x - y)^2$

2 Describe any patterns that you see in each group.

Chapter 10, Section 8.1.3, (M&L)

As the text does not specify that student work together, we cannot assume that students will communicate with other to verify their answer after using the FOIL pattern. The fact that the expressions are already categorized into 3 intentional groups removes the challenge for students to justify how specific they must be in determining if one quadratic is distinct from another. Lastly, as the equations are already correctly categorized, students' expectation to recognize these patterns becomes demoted from 'justifying' as with the CPM text, to merely 'describe'. Thus, at this point in the lesson, the level of student justification are much stronger in CPM than the M&L lesson.

CPM

After students have sorted the first 12 quadratics into groups, they are given 6 more equations to categorize before receiving the formal definition of these patterns. However, two out of the six equations do not fall into any special pattern. This is to

“reinforce the fact that not all quadratics are either perfect-square trinomials or a difference of squares and that students will need to carefully decide if a quadratic can (or can *not*) be factored using one of these shortcuts.”(CPM Teacher’s Notes, Section 8.1.5) In including unfactorable quadratic equations, students are reminded what qualities an expression must have to warrant the use of a shortcut, thus justifying when these shortcuts can be used.

M&L

In comparison, the M&L chapter does not provide equations for students to categorize after the initial twelve, but jumps right into formal pattern definitions. In skipping this step, M&L misses an opportunity to challenge students to recognize what qualities allow an equation to be factored with a shortcut. After the formal definitions, Section 10.3 provides a formal proof for one of the special patterns via an area model. So, while the proof for one of the patterns is included in the book, students merely read the proof, and do not actively participate to show their understanding. The lack of student participation required at this point is a huge variable in ensuring that students understand what they are learning. Additionally, the book only provides the proof for one of the two patterns, and so even if students were to understand the proof, there is no saying if they could conceptualize the other pattern. Unlike the CPM lesson, which has students prove their understanding of the patterns by drawing area models for both, it is feasible that some students still would not understand why the patterns work using the M&L textbook if they are only required to read examples.

CPM

It is not until after students group the additional 6 quadratics that CPM introduces the formal definitions of the two patterns in problem 8-47. However, students are not allowed to use the factoring shortcut until they verify the proof using a generic rectangle: “Use a generic rectangle to prove that $a^2x^2 - b^2 = (ax - b)(ax + b)$. Be ready to share your work with the class.” (CPM, Section 8.1.5 8-47 a). In having students write a formal proof of this pattern, they are able to justify why this pattern works for any specific input.

M&L

After providing the single proof of the perfect square trinomial pattern, M&L includes 5 example problems that use the patterns to factor quadratics but provide full solutions. Again, the fact that students only have to read the examples and not solve them themselves prevents students from proving their understanding of the underlying concepts. What’s more, as I mentioned in my previous M&L close analysis, these examples so closely mirror the upcoming homework problems, that students can look back to these examples and merely substitute different values to get the answer. Thus, since many homework problems are reduced to plug and chug methods, the procedure inherently becomes the focus of the lesson.

It is not until after reading through 5 example problems that students are tested to verify their conceptual understanding. Labeled the Guided Practice section, every lesson includes three “Concept Check” questions, which are presented in the last section before homework problems. However, the majority of these questions could be strengthened to require more justification or generalizations. Out of the three questions,

one asks to write 2 expressions for a generic rectangle with dimensions $(2x+2)$, and another asks to explain whether “*The product of $(a+b)$ and $(a+b)$ is $a^2 + b^2$* ”. (M&L, Section 104, p.593)

While these questions have the potential to challenge student’s understanding of quadratic patterns, the first could be improved by replacing $2x+2$ with a general the general equation $(a+b)$ as the CPM text does, and the second question could test students’ knowledge of the differences of squares pattern, instead of having both questions focus on the perfect square trinomial form. Thus, as the Guided Practice section lacks student justification, and it is the only place that has students actively demonstrates their understanding, the M&L lesson does not provide a sufficient level of conceptual understanding in its lessons. As we will see, the addition of homework problems, which is majorly procedure focused, will only add to this imbalance of procedural fluency versus conceptual understanding.

CPM

After students prove the difference of square and perfect square trinomial patterns with generic rectangles, students are told to write down their conclusions in their Learning Logs. This physical description is another way to solidify student understanding, particularly when they are encouraged to “include an example of each type.” (CPM,8.1.4, 8-48).

Homework:

CPM

CPM includes an average of 6 homework problems a lesson. In general, homework assignments are used as a way to review concepts from previous chapters, with procedural practice only acting as the secondary intention. For example, while 8-49 requires students to factor 4 polynomials (which can be solved with the shortcuts of today's lesson), the rest of the problems involve skillsets from previous chapter, including using exponent properties, creating geometric and arithmetic sequences, and using the substitution and elimination methods to solve for x .

In comparison, M&L includes upwards of 60 homework problems a lesson. Broken down into procedural practice, real world problems, and mixed review, these assignments were extremely procedural-heavy, as seen in the figure below.

PRACTICE AND APPLICATIONS

STUDENT HELP

→ **Extra Practice** to help you master skills is on p. 806.

SUM AND DIFFERENCE PATTERN Write the product of the sum and difference.

15. $(x + 3)(x - 3)$ 16. $(y - 1)(y + 1)$ 17. $(2m + 2)(2m - 2)$
 18. $(3b - 1)(3b + 1)$ 19. $(3 + 2x)(3 - 2x)$ 20. $(6 - 5n)(6 + 5n)$

SQUARE OF A BINOMIAL Write the square of the binomial as a trinomial.

21. $(x + 5)^2$ 22. $(a + 8)^2$ 23. $(3t + 1)^2$
 24. $(2s - 4)^2$ 25. $(4b - 3)^2$ 26. $(x - 7)^2$

SPECIAL PRODUCT PATTERNS Find the product.

27. $(x + 4)(x - 4)$ 28. $(x - 3)(x + 3)$ 29. $(3x + 1)(3x - 1)$
 30. $(6x + 5)(6x - 5)$ 31. $(a + 2b)(a - 2b)$ 32. $(4n - 8m)(4n + 8m)$
 33. $(3y + 8)^2$ 34. $(9 - 4t)(9 + 4t)$ 35. $(2x + \frac{1}{2})(2x - \frac{1}{2})$
 36. $(-5 - 4x)^2$ 37. $(3s + 4t)(3s - 4t)$ 38. $(-a - 2b)^2$

CHECKING SOLUTIONS Tell whether the statement is *true* or *false*. If the statement is false, rewrite the right-hand side to make the statement true.

39. $(9x + 8)(9x - 8) \stackrel{?}{=} 81x^2 - 64$ 40. $(6y - 7w)^2 \stackrel{?}{=} 36y^2 - 49w^2$
 41. $(\frac{1}{3}a + 3b)^2 \stackrel{?}{=} \frac{1}{9}a^2 + 2ab + 9b^2$ 42. $(\frac{2}{7}n - 3m)(\frac{2}{7}n - 3m) \stackrel{?}{=} \frac{4}{49}n^2 - 9m^2$

MENTAL MATH Use mental math to find the product.

43. $26 \cdot 34$ 44. $45 \cdot 55$ 45. 16^2 46. 41^2

STUDENT HELP

→ **HOMEWORK HELP**

Example 1: Exs. 15–20, 27–42
 Example 2: Exs. 21–42
 Example 3: Exs. 43–46
 Example 4: Exs. 47, 48
 Example 5: Exs. 49–52

10.3 Special Products of Polynomials **593**

As we can see in problems 15–38, correct solutions can be achieved entirely by memorization. Students were not even challenged to determine which shortcut to use, as the problems were organized by pattern. What’s more, based on interviews with Cornelia, she usually assigned 30 homework problems a day, over 90% of which were taken from the Practice and Application Section.

Conclusion

Thus, by the end of the day, CPM has taken many more steps for students to justify their conceptual understanding in comparison to M&L. First, by beginning with an open ended problem that called to group quadratics in an unspecified number of categories, CPM challenged students to justify why each expression fits into its given group but not any others. Then, after recognizing the three patterns, CPM provided

additional expressions, (2 of which were unfactorable) to have students justify what qualities allow for a factoring shortcut to be used. Additionally, students did not merely read a proof of the shortcuts, but drew area models for both general formula patterns $[a^2x^2 - b^2 = (ax - b)(ax + b)]$, $[(ax + b)^2 = a^2x^2 + 2abx + b^2]$ to prove how the formula will work for any specific input of a and b . Lastly, CPM's explicit directions for students to work in groups provides a means for students to share reasoning with peers throughout the class.

Overall, these shifts between Chapter 10 of the M&L textbook and Chapter 8 of CPM are moderately aligned to the change in content standards, which require a deeper level of conceptual understanding and extended procedural applications, as well as higher levels of demonstrated student thinking. However, because CPM is not fully aligned to the CCSSM, the shifts in texts do not highly correlate to the change in standards. Now that I have compared both textbooks, I will observe the shifts in teacher practice and gauge which practice, if any, can be explained by the shifts in standards, or indirectly by the shift in textbooks.

Past and Present Teaching

Methodology for Past Teaching

Without access to classroom observation videos and student work from 3 years ago, I relied on Cornelia's own reflections on her teaching style from 3 years ago to gauge how her teaching practices have changed. I also interviewed Arthur, the principal of the school where Cornelia worked, to learn about (in his opinion) how typical Algebra teachers from his school taught during 2012. Lastly, while I couldn't access student work, I was able to access all of the old homework assignments and quizzes that Cornelia used during her classroom.

Analysis of Past Teaching

Past teaching

The three things I looked for in evaluation effective "Facilitation of Mathematical Discourse" includes the levels at which Cornelia created an encouraging environment, based discussion off of student work, and built a developed sequence of student arguments and strategies.

Facilitating Mathematical Discourse

Back in 2012, Cornelia was actively creating an encouraging environment for her students. In particular, she would encourage students to get to know each other by writing down interesting facts about themselves (such as their favorite dessert, or how many languages they spoke) for students to find things to bond over. Cornelia also had a "Tag out" system which encouraged students to share ideas in front of the class.

Whenever one student would volunteer to go up to the board, they had to high-five their

colleagues (akin to a tag team in wrestling) when they needed help. Students' excitement to imitate a wrestling duo's handshake distracted students from feeling ashamed should they not be able to solve the problem by themselves.

In terms of basing discussion off of student work, Cornelia always responded to student questions with another question, and would not state information unless other students had already shared the answer with the class. As you will see in the following pages, she could have increased student discussion by spending less class time with direct instruction.

While Cornelia always tried to base discussion off of student work, there was little to no development in the way she sequenced student ideas. However, as the M&L content frequently lacked high levels of student justification, students were often presented with problems that could be solved with a single procedure.

Building Procedural Fluency from Conceptual Understanding

The three main things I looked for in Building Procedural Fluency from Conceptual Understanding is the level of student centered investigation, and at what point and how often do teachers “ask students to discuss and explain why the procedures they are using solve particular problems?” (8 Principles to Action, 45).

Level of Student Centered Investigation:

One change in in Cornelia's classroom in the last 3 years is group seating arrangements. While seating arrangement may seem like a trivial aspect, Principal Arthur stated in his interview that there was much less peer interaction amongst student when desks “were arranged in silent rows of desks—called “cemetery style”. As students say in rows, they worked silently, took notes, or in some cases worked with

partners. The emphasis on note taking and little student interaction implies that at least for a part of class students would listen to the teacher lecture rather than investigate problems themselves.

In terms of student centered investigation, Cornelia structured her class in a way that presented more opportunities for students to investigate patterns than the textbook provided. By rearranging each section of the textbook so that students would make conjectures about a pattern before she lectured over the procedure, students practiced reasoning and problem solving at a higher frequency than had she just followed the textbook. However, more student led investigation could have been incorporated had students had more open ended, contextual problems.

One byproduct of student investigation is that students are given more opportunities to verify their conceptual understanding. However, because M&L textbook heavily emphasize procedures and lacked verification for opportunities of student justification, Cornelia's heavy reliance on the text negatively affected how often she was able to ask students to connect the procedures they were learning in class.

Current Teaching Methodology

Below is my methodology detailing how I observed Cornelia's current teaching practices:

Student observation

Every day I came to class and sat in the same location. I did not interact with students, and observed as silently as possible so that the students would not act out due to an observer being present. Cornelia, purposefully never acknowledged my presence to limit expectancy efforts.

Before class began I would select two students who were shoulder partners, and observed the SMP's that each demonstrated throughout class. Choosing two different students every day allowed me to observe student interactions in depth, but by selecting a different pair every day, I ensured myself that I would get a complete representation of the classroom.

Observing students in pairs was ideal because student interaction makes up a heavy component of the 8 SMP's. Therefore, observing a single student would be illogical, yet observing more than 2 would make it more difficult to capture all interactions. The fact that I chose shoulder partners helped in the sense that there was already an expectation that they would work together.

How I selected students

Students were placed in table groups of 4 people each. Every two days I chose a random table group and observed the two pairs of students within that group for the following two days. The table groups were chosen by their proximity to my location. Throughout the lesson I focused on the conversations of my 2 students, and wrote a check whenever I witnessed either of them exhibiting a student practice. Later, I began recording why I thought they were exhibiting one of the student practices.

Limitations of classroom observations

The fact that I was not able to sit next to students and move about the classroom definitely hindered my ability to gauge the frequency of student practices being used. That being said, I view my observation data as a minimum bound of total SMP's; that is to say, while there may have been more instances of students observing the SMP's, I can say that students practiced at least the number that I observed each day.

New term, new classroom

One thing that I did not expect was that on day 10 of 20, a new term began where half of the previous students were replaced with new ones. Wanting to maintain a complete representation of the classroom by the end of the lesson, I gave priority to the new students that I had never observed before starting my cycle over.

Current Teacher Practice Analysis

Creating an encouraging environment

Compared to 2012, Cornelia continually encourages a comfortable space for students in her classroom. At the beginning of each term, she will pass out 3x5 cards and ask for students to write down interesting facts such as their least favorite dessert and number of languages spoken. These cards share a dual purpose—students get to learn about their table partners, and Cornelia can read them later on to help her learn her students' personalities.

Cornelia encourages students to talk and share ideas with creative warm up problems. Often, she provides an equation and makes students come up with a creative context to fit it. For example, on Day 6 she gave students the equation $5(2)^x$ and encouraged students to “Have fun with the situation! Mine had something to do with pimples...” After Cornelia shared her example, one student was not afraid to point out that Cornelia's example involved whole numbers, but wouldn't work for non-integer values. Indeed, Cornelia made a point that teachers will make mistakes, (sometimes purposefully), to test student understanding. Clearly if students are comfortable calling out the teachers' mistakes, they feel comfortable talking with their peers.

Basing discussion off of student work

During whole class discussions, Cornelia always encourages students to share how they got their answers. It was clear that Cornelia created a comfortable environment for students as many [albeit, often the brightest] volunteered to share their process and reasoning. If various students had solved a problem in different ways, Cornelia would make sure to share the arguments with the class with the precursor “Do you all agree with ___?” Conversely, if students had questions during the class discussion, she would make sure to share the answer with the entire group, and not just the student in question. Thus, while Cornelia based student discussion off of various approaches and arguments of students, she did not build a progression of student responses, but sequenced them according to whichever students were happy to volunteer their answers.

Procedural Fluency from Conceptual Understanding

Level of student investigation:

In terms of how much time students spend investigating questions versus listening to Cornelia lecture, she dedicates a much larger proportion to student led instruction. Part of this is because since 2012 the duration of class increased from 55 to 70 minutes. However, in looking at the parts of the lesson where Cornelia introduces new material, we see an increase in student autonomy. Cornelia’s decision to let students work in groups as opposed to lecturing in front of class is largely due to the CPM curricula.

For starters, Cornelia followed the textbook very closely as this was the first time she was teaching Algebra 1 with the CPM textbook. The Teacher’s Notes section, which includes a Suggested Lesson Activity for each day, not only emphasizes group

learning, but provides a variety of activities to encourage student communication. For example, in Section 8.1.2, when students are using algebra tiles to model quadratic equations, CPM suggests that students work in a Think-Pair-Share format. That is, students would build a rectangle using the algebra tiles, check-in with a partner to go over their combined answers, then come together as a table group to share their strategies.

This emphasis of group work was also apparent in the problems themselves. In almost every lesson, at least one problem stated to “Share your ideas with your teammates and be prepared to demonstrate your process for the class.” (8-25, 35, 45, 47, 90, 105). In conclusion, the CPM textbook clearly aligns to an increase of student investigation.

Aligning Shifts in Teacher Practice to Shifts in Math Content Standards

In evaluating the alignment in Cornelia's facilitation of math discussion and her development of procedural fluency from conceptual understanding, I found that the shifts in her teaching practices highly correlate to the change in textbooks. While the switch in textbooks led to an increase in student justification regarding conceptual understanding and communication, Cornelia's teaching practices have shifted so that she:

1. Encourages a higher frequency of student interaction throughout class; and
2. Ask students why a procedure is helping in solving particular problems earlier and more frequently, which consequently enables her to provide more opportunities for students to justify their work.

While an increase in student interaction acts as a medium for students to justify their work, student interaction serves as a distinct shift from student justification because it also provides outlets for students to ask for help, as opposed to communicate only when they know the answer. While the CPM curriculum was not directly responsible for encouraging students to ask for help from peers as well as herself, Cornelia's emphasis on communicating in groups acted as the foundation for students to feel comfortable asking for help when they did not understand something. Thus, because CPM emphasizes group work in every lesson, the CPM curricula indirectly aligns to this shift in Cornelia's teaching practice.

For example, throughout my classroom observation, Cornelia had students work in shoulder partners so frequently that when a student's partner was absent during Section

8.2.4, they changed seats so that they work on the classwork with a buddy. The fact that this student independently chose to move seats indicates that they preferred working with others as opposed to working alone. On Day 13, one of the two students I was observing went over his solutions at his table group, but then went over to a *different* table group to double check his solutions. While it is possible that students did not interact with each other this way 3 years ago because the seating arrangement provided no indication that students could work together, interviews with Principal Arthur verified that teachers throughout Cornelia's school switched to a group seating chart precisely because of the shift in textbook.

Having already established that the CPM curricula has students work in groups, there is an additional aspect to the CPM textbook that I did not mention in my previous textbook comparison that emphasizes CPM's use of group work. In the Teacher's Notes section of every lesson, CPM outlines four roles, consisting of a Facilitator, Recorder/Reporter, Resource Manager, and Task Manager, and describes ways in which a student could outline this role in today's lesson. In CPM's Team Support section, CPM describes the importance of roles in supporting productive teamwork, and "allow[ing] students to share responsibility for the effective functioning of the team and class." Although Cornelia rarely enforced students to fit one of these four roles, the fact that the CPM text assumed students would be utilizing these roles proves that CPM curricula encouraged group work. Cornelia's ability to ask "Why?" questions to her students earlier and more frequently goes hand in hand with her increased opportunities to provide student verification, and both shifts are largely due to the CPM curricula. Following my lesson comparison in 8.1.5, I proved that the problems in CPM required a

much higher level of student justification than the M&L class, and asked questions that solidified conceptual understanding throughout the lesson. Thus, as Cornelia followed the CPM curricula, it makes sense that Cornelia had more opportunities to ask students to justify their understanding. What's more, the Teacher's Note section of every lesson consistently provides activities that encourage student reasoning. For example, in addition to the Think-Pair-Share and Reverse-Teaching exercises I mentioned in previous sections, Cornelia also administered a "Participation Quiz" as outlined in the Teacher's Notes of Section 8.2.3. Instead of having students work on individual quizzes, Cornelia assigned one problem to the entire table group, and graded the quiz not based on getting the correct answer, but on how students communicated with each other. In valuing the collective journey students took to reach the answer instead of the answer itself, Cornelia provided more opportunities to focus on student justification, in large part to the CPM curricula.

Conclusion

In analyzing the shifts in teacher practices regarding facilitation of math discourse and building procedural fluency from conceptual understanding, there is a strong correlation between the change in teaching practices and shifts from the M&L and CPM textbooks. Cornelia's increased emphasis on encouraging student interaction, as well as providing opportunities for more student justification is largely due to the CPM textbook. Overall, the shifts between Chapter ten of McDougal & Littell textbook and Chapter eight of College Preparatory Mathematics textbook are moderately aligned to the change in content standards. While there is an increase in student justification regarding conceptual understanding and student reasoning, many improvements to Chapter 8 of the CPM text can be made to fully align the text to the CCSSM standards regarding quadratic equations. These changes include limiting the use of Learning Logs until students can fully prove a hypothesis, including more sections that begin with contextual problems like Sections 8.2.1 and 8.2.4, and better connecting the 8 Standards of Mathematical Practice to each lesson. Because the curriculum shifts are moderately aligned to the changes in content standards, we can conclude that the changes in student standards have made a moderate impact on teacher practices.

Given that the shifts between both standards emphasize a deeper conceptual understanding; this research indicates that the CPM unit has a better alignment to the CCSSM over the M&L unit because of the increased frequency and depth in which the text provides opportunities for students to justify their work. Regardless of the curricula used in Algebra 1 classrooms, teachers should strive to sequence their lessons to promote justification and conceptual justification throughout a lesson. What's more, in

order for all students to develop reasoning skills, it is important to create an encouraging environment where students feel comfortable asking for help. Although the CCSSM provides no outline for how to execute its student expectations, the steps the 6A school district has taken to aid its teachers has already make an impact in the classroom, and whether subconsciously or not, teachers will only continue to hone their practice to accommodate for these new changes.

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Glossary

CCSS/CC-- Common Core State Standards/Common Core
CCSSM—Common Core State Standards for Mathematics
CCSSO--Council of Chief State School Officers
CPM—College Preparatory Mathematics (high school textbook)
EQuIP rubric -- Educators Evaluating Quality Instructional Products rubric
M&L—McDougal and Littell (high school textbook)
NCTM-- National Council of Teachers of Mathematics
NGA--National Governors Association Center for Best Practices
ODE—Oregon Department of Education
OR-IMET— Oregon Instructional Materials Evaluation Toolkit
TIMSS--Third International Math and Science Study