

AGGREGATE CONSEQUENCES OF INNOVATION AND INFORMALITY

by

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DISSERTATION ABSTRACT

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Title: Aggregate Consequences of Innovation and Informality

The fundamental question in development economics is what causes some countries to become more prosperous than others. The literature, starting with Hall and Jones (1999), has identified differences in total factor productivity (TFP) as being the driver of cross-country income differences. I investigate policies that may give rise to these differences in TFP. I pay particular attention to the influence of informal economies in developing countries and how macroeconomic policies can distort firm-level incentives to innovate and operate formally.

To address these questions, I construct a series of macroeconomic models which have several common elements. First, I model firm-level decisions with regard to innovation. These firm-level decisions ultimately give rise to differences in productivity across countries. Second, I embrace the role of firm heterogeneity in productivity to examine the dynamics of firm choice. Finally, through the use of computational methods, I simulate these models to evaluate the macroeconomic effects of policy distortions on firm-level decision making.

Subject to the common elements above, each chapter answers a specific policy question. Chapter II asks whether size-based tax distortions can generate firm-size distributions often observed in developing countries. I find that a model with innovation and firm-level heterogeneity can explain the prevalence of large firms in response to tax distortions, but additional frictions are necessary to explain the ubiquity of small firms in most developing countries. It also illustrates tax distortions may have little impact on aggregate output while dramatically reducing innovation. Chapter III documents that tax rates can negatively affect growth by inducing firms to participate in the informal sector rather than the formal sector. Finally, Chapter IV shows how tax revenues are affected by changes in tax rates given the provision of a productive public good.

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CHAPTER I

INTRODUCTION

The fundamental question in development economics is what causes some countries to become more prosperous than others. The literature, starting with Hall and Jones (1999), has identified differences in total factor productivity (TFP) as being the driver of cross-country income differences. I investigate policies that may give rise to these differences in TFP. I pay particular attention to the influence of informal economies in developing countries, and how macroeconomic policies can distort firm-level incentives to innovate and operate formally. More recently, work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) illustrates the importance of policy distortions in misallocating firm inputs. These misallocations are shown to have important implications in terms of total factor productivity.

To address these questions, I construct a series of macroeconomic models which have several common elements. First, I model firm-level decisions with regard to innovation. These firm-level decisions ultimately give rise to differences in productivity across countries. Second, I embrace the role of firm heterogeneity in productivity to examine the dynamics of firm choice. Representative agent models simply cannot capture the inefficient redistribution of resources across firms that often results from government policy. Finally, through the use of computational methods, I simulate these models to evaluate the macroeconomic effects of policy distortions on firm-level decision making.

Chapter II is titled “The Decision to Innovate: Aggregate Implications of Size-Based Distortions.” It explores how size-based policy distortions affect firm level decisions in the context of a partial-equilibrium model. Aggregate productivity is improved through heterogeneous firms making investments in innovation. I calibrate and solve the model for different tax distortions, focusing on tax schemes that fluctuate based on firm size. I find that under some size-based distortions, aggregate output is similar to aggregate output in an undistorted economy. However, firms pursue less innovation and aggregate TFP is lower. This result is partially driven by the exogenous wage rate, an element that is endogenized in Chapter III. Output per worker is considerably lower, echoing previous work that underscores the importance of TFP in explaining cross-country differences in income.

Chapter III is titled “Informality, Innovation, and Aggregate Productivity.” It investigates how the ability to innovate affects firms’ decisions to operate informally and the aggregate consequences of their sectoral choice. I embed a sectoral choice model, where firms choose to operate in the formal or informal economy, into a richer general equilibrium model to analyze the effect of firm-level decisions in response to government constraints in the form of taxation. The model expands upon Chapter II by incorporating an endogenous wage rate. I calibrate the model and conduct simulations to quantify the impacts of firm decisions on the aggregate economy. I find that a change in tax rates from 50% to 60% leads to a 20.9% reduction in the size of the formal sector. This large change is accompanied by a 0.07 percentage point reduction in TFP growth. Given that countries like Mali, Mexico, and Sri Lanka impose total tax rates near 50%, these findings have significant and applicable policy implications in lesser developed countries. Even moderate changes in tax rates, for instance, 10% to 20%, decrease the size of the formal sector by more than 7.7% and decrease TFP growth by 0.09 percentage points.

Chapter IV is titled “Informality, Tax Policy, and the Provision of Public Goods.” This chapter looks at how the provision of public goods affects the decision of firms to operate in the formal sector. While previous chapters underlined the distortionary effects of tax policies, this chapter takes a more holistic approach to government policy. In the formal sector, firms face taxation; however, in addition to their ability to innovate, their production processes are augmented by the provision of a public good. Informal firms evade taxes, but do have limited access to these public goods. I construct and calibrate a general equilibrium model to investigate revenue-maximizing tax rates. Under a variety of parameter values, I find revenue-maximizing tax rates from 39% to 48%. The range of these tax rates suggests that many developing countries operate at tax rates beyond what maximizes total revenue. Coupled with the findings in Chapter III, these sub-optimal tax rates are also accompanied by increasingly large informal sectors. The results in Chapter IV are of further importance because they illustrate an alternative mechanism for the Laffer Curve: the decision to be informal.

While each chapter has its own policy conclusions, there are several common themes. First, the chapters illustrate the importance of government policy, mainly tax policy, in distorting firm-level decisions. This in turn has important implications for improvements in TFP. Secondly, firm-level heterogeneity is shown to be a necessary modeling element to capture and illustrate

the implications of distortionary policies. Finally innovation, as one avenue by which TFP is accumulated, is an important margin for determining entry into the formal sector. By capturing the role of innovation and distortionary policies on TFP, this research makes a significant contribution to understanding the broader question of cross-country differences in income per capita.

CHAPTER II

THE DECISION TO INNOVATE: AGGREGATE IMPLICATIONS OF SIZE-BASED DISTORTIONS

Introduction

Differences in technology and total factor productivity (TFP) are familiar refrains for explaining cross-country differences in income. Both exogenous and endogenous growth models accept the importance of technology and innovation, yet there still remain questions about why technology improves productivity so differently across countries.¹ Recent work investigating the extent of these differences has focused on the role of macroeconomic distortions, which cause inefficiencies in the allocation of resources.² Thus far, no work has explicitly linked the process of technological change that is vital in determining aggregate TFP with the macroeconomic distortions that have been studied extensively in the literature. This chapter seeks to connect technological change with size-based distortions to study aggregate outcomes.

Early work on explaining TFP changes has been directed in two principal directions. Howitt (2000) and Klenow and Rodríguez-Clare (2005) document how TFP differences can arise from differences (in growth rates and levels, respectively) in innovation processes across countries. Alternatively, Restuccia and Rogerson (2008) began an important second avenue of research on TFP differences.³ Specifically, they look at misallocations within an economy that may be unfortunate results of policy. These two areas of investigation, and the host of papers that have followed, need to be connected.

The primary research question of this chapter is how size-based policy distortions affect a firm's innovation decisions and the aggregate effects of those decisions. To answer this question, it is important to have a model that includes innovation along with heterogeneous firms that are affected differently by the imposed distortions. Innovation in the model should be thought of as more general than the process of invention. Productivity changes occur both through innovation

¹See Solow (1956) and Romer (1990) for seminal works.

²For instance Hsieh and Klenow (2009) and Alfaro et al. (2008).

³Guner, Ventura, and Yi (2006) pursue a similar point by looking at how firm-size distributions are affected by laws that place limitations on the size of establishments, mostly in the form of taxes or subsidies based on firm size.

as well as adoption. As an instructive example, firms can improve their productivity by adopting just-in-time inventory management just like they can from inventing new machinery to build their product. Since both of these changes may enhance productivity, or decrease it if they are unsuccessful, I consider both to be innovation in the model.

A central component of this model, and this line of research generally, is that there exist distortions that vary based on firm size.⁴ To be clear, distortions throughout this chapter refer to output distortions, not input distortions. Taxes are levied on firms' output rather than causing differences in input prices among firms. While the model developed subsequently will be used to test a variety of tax schemes, the most interesting scheme is motivated by Tybout (2000) in his survey of manufacturing firms in developing countries. He explains that small firms may be able to avoid costly regulations, corruption, and other costs of operation in lesser developed countries. On the other hand, large firms may benefit from these regulations and/or be able to influence them through the political process. It is the firms in the middle, that become just large enough to invite additional attention without reaping the benefits of being large, that are the least well-off. To mimic this story, I test a size-based distortion that takes a quadratic form. Both small and large firms face lower taxes than medium-sized firms.

I find that taxation schemes that are independent of firm size have predictable effects such as decreasing average firm size, innovation, and output growth. Size-dependent tax policies have a significant effect on the size-distribution of firms, but total output seems to be affected very little by a tax schedule that reflects Tybout (2000). Evidence suggests that firms substitute investment in innovation for labor resources to avoid increased taxes. I find that under the current calibration, a Tybout-style tax lowers aggregate TFP even though total output remains relatively the same.

The research question naturally links the endogenous growth work with the more recent emphasis on national policy distortions focused on firm size. This link is important on several fronts. The innovation process itself offers little in terms of specific policy proposals and testable implications at the national level. Additionally, current work on firm-size distortions does not focus on the mechanism through which technology is accumulated. Bridging these areas can

⁴While empirical evidence is scarce, Goyette (2009) documents audit probabilities (effective tax rates) that vary with firm size.

help us understand the effects of specific national distortions on the process of innovation and technology accumulation.

I use a simple framework in which firms are capable of innovating and raising their productivity in future periods. Firms are heterogeneous in their initial productivity as in Melitz (2003). Thus, the model generates firm-size distributions where firm size is correlated with, but not determined by, the productivity level of firms. Different distortions (implicit tax schemes) change the incentives for innovation and the resulting firm-size distributions.

There is a well-developed literature documenting the tax schedule and regulatory environments across the distribution of firm sizes. Unfortunately, these studies have usually focused on segments (i.e. large firms or small firms) of the distribution. The benefits of being a small firm are well documented. The seminal work of Rauch (1991) shows that there is a missing middle in the distribution of employment because small firms did not want to become large enough to attract attention.⁵ By staying small, and informal, firms avoid costly regulation and corruption. Qualitatively, de Soto (1989) tells the same story in Peru where entrepreneurs sought to stay small to avoid additional costs of being formally recognized and regulated.

Newer work has underscored the theory and observations of Rauch (1991) and de Soto (1989) with quantitative results. Gallipoli and Goyette (2011) show that the audit probability in Uganda is based on firm size (number of workers) rather than any level of capital or revenue. Specifically, they find that the probability of being audited increases substantially for firms with more than 30 employees. Importantly, none of the works above discuss the potential benefits of being a large firm. In order to generate a “missing middle,” there must be incentives to, at some point, grow.

This incentive is documented by Levenson and Maloney (1997) who find that large firms often benefit from regulatory regimes. If regulation poses some form of fixed cost, increasing firm size decreases average costs. Further, the literature on lesser-developed countries is rife with examples of how political power and leverage are endowed to large firms. For example, Faccio (2006) documents the widespread nature of political connections and the strong correlation between firm size and political connections.

⁵The missing middle refers to employment as a percentage of total workers in medium-sized firms being less than employment in small and large firms, creating a missing middle. See Tybout (2000) for a more extensive discussion.

Certainly, some distortions may not fit well into an implicit tax rate as done in this work. For instance, García-Santana and Pijoan-Mas (2014) evaluate the effects of the Small Scale Reservation Laws in India. These laws reserve certain goods for production by small firms. While these laws may not be best modeled as a simple distortionary tax as done here, they certainly represent a policy that is size-dependent and is consistent with the story presented by Tybout (2000) in that firms prefer to stay small in order to produce certain goods.

Unfortunately, no work documents the actual implicit taxes faced by firms over the entire distribution of firms. The difficulty in doing so is two-fold. First, the data on taxes and regulations that firms face is difficult to collect over the entire distribution of firms, especially for small informal firms.⁶ The data that has been collected is mostly survey data that is suggestive of the types of patterns discussed above, but it is extremely difficult to work with quantitatively due to the subjective nature of the responses.⁷ Second, while authors like García-Santana and Pigoan-Mas (2014) and Gallipoli and Goyette (2011) look at the effects of specific policies that vary across firms, they do not capture the aggregated distortions that are found to exist in Hsieh and Klenow (2009). Firm-size distributions in lesser-developed countries are a result of a conglomeration of policies that are difficult, if not impossible, to derive.

Despite its limited quantitative use, it is worthwhile to use the available survey data to motivate the use of size-based distortions. For instance, Table 1 illustrates how both the incidence of bribes and the percentage of sales paid in bribes initially increases with firm size, but eventually decreases for larger firms in the formal sector. Informal sector firms tell a slightly different story, as bribes increase with firm size in the informal sector. Informal firms pay bribes less often, but when they do, they pay a larger portion of sales. Note that the shape of bribery rates in Table 1 is largely consistent with the pattern described by Tybout (2000). Bribes as a percentage of sales tend to be highest for medium-sized firms.

Data from the World Bank's Enterprise Surveys also appears roughly consistent with macroeconomic distortions that vary by firm size. Table 2 shows the mean survey response for each type of firm. Generally it appears that firms are more concerned about taxes, regulation, and

⁶The World Bank conducts surveys for both formal and informal firms; however, splicing the surveys together into a coherent picture of the universe of firms remains a daunting project.

⁷For example, in Table 2 there is no objective mechanism to ensure that a firm that sees regulation as a "2" represents the same degree of constraint as another firm that rates regulation as a "2." The two firms could face drastically different regulatory environments, but their baseline for regulation is different.

TABLE 1. Bribes by Firm Size

Sector	Firm Size	Firms Reporting Bribes (%)	As a Share of Sales (%)
Formal Sector		55.5	3.9
	Micro (≤ 10 employees)	49.9	4.4
	Small (10-19)	56.7	4.8
	Medium (20-49)	57.6	4.0
	Large (50-249)	58.5	3.4
Informal Sector	Very Large (250+)	55.7	3.0
		27.4	8.6
	Small (≤ 10 employees)	25.5	8.5
	Large (10+)	49.1	9.3

Source: World Bank's 2005 World Development Report

corruption as they grow larger; however, there is likely an under-sampling of informal firms (small firms), and small firms are less likely to not have an opinion or not know the answer to each of the survey questions.⁸ In totality there seems to be sufficient evidence for studying the effects of size-based distortions. Part of the impetus for doing so in this chapter, is to develop a framework that can be calibrated to individual countries whose specific distortions may vary substantially as exhibited in Table 2. Given the difficulty of quantifying “aggregate” distortions, future calibration exercises testing individual distortions, as in García-Santana and Pijoan-Mas (2014), may be more productive.

⁸Non-responses are excluded from the data presented in Table 1.

TABLE 2. Survey Results for Firms' Perceived Constraints

Country	Firm Size	Taxes	Regulation	Corruption
Mexico				
	Small	2.39	1.71	2.49
	Medium	2.46	1.78	2.55
	Large	2.31	1.84	2.54
Ecuador				
	Small	1.97	1.73	2.29
	Medium	2.02	1.68	2.52
	Large	2.08	1.67	1.88
DRC				
	Small	2.27	1.93	2.74
	Medium	2.40	1.88	2.97
	Large	2.47	2.21	3.21
Angola				
	Small	2.20	1.98	1.40
	Medium	2.44	1.99	0.27
	Large	2.41	2.03	2.10

Figures above represent the mean firm response to how large an obstacle was posed by taxes, regulation, and corruption. Rankings are on a scale from 0-4 with 0 representing “no obstacle” and 4 representing “very severe obstacle.” Firms are characterized as small if they have fewer than 20 workers, medium if they have 21-100 workers, and large if they have more than 100 workers. Note that the data used to produce these figures does not generally include firms with fewer than 5 workers which may significantly change some of the averages for small firms. The data is from the World Bank’s Enterprise Surveys for Mexico, Ecuador, Democratic Republic of Congo, and Angola in 2010.

Two additional papers warrant greater elaboration. Restuccia and Rogerson (2008) established several precedents in the literature and began the emphasis on evaluating the effect of distortions on production with heterogeneous firms. They created an environment where heterogeneous productive units faced different prices due to policy-induced distortions.

Unlike later authors who study specific policies, Restuccia and Rogerson (2008) develop an undistorted economy and then test the effects of general distortions on productivity and firm-size distributions.⁹ Their calibration exercise utilizes U.S. data, establishing the current practice of considering the U.S. economy to be the undistorted case. Ultimately, the authors find that price heterogeneity caused by distortions can decrease output and TFP by 30% to 50%.

Following soon after, Hsieh and Klenow (2009) has become the *de facto* standard for comparison in the literature. They follow Restuccia and Rogerson (2008) in considering the U.S. economy to be an undistorted economy, but they specifically calculate TFP dispersion for India and China from micro-level data. They then ask the question, what would be the TFP gains if capital and labor in India and China were allocated similarly to the United States? In this sense, rather than distorting an economy, they show the benefits of policies that would remove distortions already in place. Their results show that TFP gains for China are in the range of 30% to 50% and 40% to 60% in India.¹⁰ While my theoretical model borrows significantly from Hsieh and Klenow (2009), the exercise pursued is certainly different. One of the principal shortcomings in the methodology of Hsieh and Klenow (2009) in the current context is their reliance on survey data. Specifically, the authors are forced to neglect the smallest firms in the firm-size distribution. India's Annual Survey of Industry omits firms with fewer than 50 people, and China's Annual Survey of Industrial Production only surveys firms with revenue above \$600,000 or that are state-owned. This chapter emphasizes the existence (or lack thereof, as the case may be) of relatively small firms that are excluded from Hsieh and Klenow (2009).

The remainder of the chapter proceeds as follows. The next section develops the theoretical model. The following two sections lay out the scope of the numerical application and present the results, respectively. I then discuss the key results. The final section provides conclusions and discusses areas for future work.

The Model

This section details the theoretical model. Heterogeneous firms make decisions about how much to invest in innovation and how much labor to hire. Firms have perfect knowledge with

⁹The authors do not claim that the U.S. economy is truly undistorted, but rather that their results should be viewed relative to the U.S.

¹⁰Notice how close these estimates are to those of Restuccia and Rogerson (2008).

regard to the tax rates they face and the results of their investment in innovation. The key object of interest is how firms react to higher tax rates that are specifically tied to the size of their workforce. Intuitively, this changes the incentives for firms to hire workers by changing the marginal profits of each worker, depending on the exact specification for taxes.

The Economy

The aggregate economy is similar to single-sector version of Hsieh and Klenow (2009). Aggregate output is a CES aggregation of M firms producing differentiated products:

$$Y_t = \left(\sum_{i=1}^M y_{it}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1. \quad (2.1)$$

Output of firm i at time t , is y_{it} , and σ is the elasticity of substitution between intermediate goods. Cost minimization in the production of the final good implies that firms in the intermediate goods sector face a demand curve for their products:

$$p_{it} = \left(\frac{Y_t}{y_{it}} \right)^{\frac{1}{\sigma}}. \quad (2.2)$$

Firms

Firms produce differentiated products and compete in monopolistically competitive markets, hence the parameter σ for the substitutability of firms' output in the creation of the final good. Higher levels of σ correspond to lower levels of market power for intermediate good producers. Each firm has access to the production technology

$$y_{it} = \varphi_i A_{it} l_{it}^\alpha, \quad (2.3)$$

where φ represents a time invariant productivity draw from a stable distribution as in Melitz (2003). This productivity represents firm specific characteristics like entrepreneurial talent that cannot be improved upon through innovation. Other work in the literature, notably Guner, Ventura, and Yi (2008) and Garcia-Santana and Pijoan-Mas (2014) utilize a Lucas (1978) span-of-control framework to model firm heterogeneity.

I use elements of Melitz (2003) for its simplicity and for comparative purposes with Hsieh and Klenow (2009).¹¹ Unlike previous works, like Jovanovic (1982) and Hopenhayn (1992), that studied firm heterogeneity, Melitz (2003) abstracts away from the complicated entry and exit process of firms. Specifically, the use of an exogenous death rate of firms greatly simplifies firm optimization while still allowing for heterogeneous production units. Further, since the current chapter seeks to address differences in growth, Melitz (2003) offers an advantage over a Lucas (1978) span-of-control model because there is no decision about how to allocate the population between entrepreneurs and laborers. This eliminates a margin that is not of principal interest here. The framework used here may be of further use in an investigation of the productivity levels required to enter international markets as in Melitz (2003).

Firm production depends on A_{it} , which represents the technology level at firm i . This level of technology can be improved over time by innovating (described in Section 3.2). Firms hire labor from an infinite pool of workers who supply their labor inelastically at a fixed wage rate, w . The assumption of a fixed wage limits the model to partial equilibrium results; however, Section 3.5 discusses the implications of endogenizing the wage rate. The exogenous wage rate should be seen as an intermediate step in modeling the aggregate environment, and future work will seek to endogenize it.¹²

Timing in the model proceeds as follows (each stage discussed in-depth below). At the beginning of each period, M firms enter the market based on a previous determination that their discounted profit stream from entry exceeded the entry costs.¹³ Since each firm, i , produces a unique good, changes in the number of firms would complicate the model by adding *product innovation*. Taking aggregate output and the tax rate as given, firms make their production decisions. Aggregate variables are determined by firms' production decisions, and taxes are collected. At the end of the period, firms face an exogenous probability of death.

¹¹They also show how their model operates similarly with a Lucas (1978) set-up.

¹²The wage rate remains fixed here due to computational complications. Specifically, since each firm faces a different tax rate under size-dependent distortions, aggregate labor demand is the sum of M firms' non-linear labor demands. Additionally, adding w as an endogenous variable also requires iteration on both Y_t and w_t which slows convergence.

¹³The number of firms, M , is not time dependent due to the assumptions made with regard to the fixed cost of entry. Fixing the number of firms limits innovation in the model to *process innovation*.

Technology and Innovation

Firms are able to improve their technology level, A_{it} by investing in innovation. Innovation follows a process similar to innovation in Atkeson and Burstein (2010); however, in this case there is no uncertainty with respect to the outcomes of innovation. Firms choose a level of $q_{it} \in \{0, 1\}$, which determines both the costs and returns to innovation. Firms invest $A_{it}c(q)$, where $c(q)$ is the cost function of innovation, which is increasing in the firm's choice of q_{it} .¹⁴ The returns from this investment are a linear combination of a step-size increase and step-size decrease in the level of technology. That is, technology evolves according to

$$A_{it+1} = q_{it}(A_{it} + \Delta A) + (1 - q_{it})(A_{it} - \Delta A). \quad (2.4)$$

This equation can be simplified such that

$$A_{it+1} = A_{it} + \Delta A(2q_{it} - 1). \quad (2.5)$$

This specification for the evolution to technology has several desirable features. First, it captures the fact that firm-level technology can depreciate over time without sufficient levels of investment in innovation. Secondly, the discrete step-size, ΔA , has clear estimated values in the literature. This allows a key driver of innovation to be closely aligned with the data. Finally, notice that the costs of innovation scale up with the level of technology. This captures the fact that innovation is more costly at higher levels of technology, and it ultimately leads to a steady state in the model. Innovation in the model is best considered *process innovation* where firms improve their production processes rather than *product innovation* where firms increase the variety of goods that they produce.¹⁵

The Firm's Problem

Firms seek to maximize their discounted stream of profits. Firms discount the future by $\beta(1 - \delta)$, reflecting both a standard discount rate β and the probability of firm exit, δ (explained

¹⁴Note that in steady-state this "choice" is fixed at $q = \frac{1}{2}$.

¹⁵Process innovation in the model operates in many ways like a Schumpeterian quality ladder model as described in Barro and Sala-i-Martin (2004) in Chapter 7.

further in Section 2.2). Each firm, i , solves their maximization problem by choosing their price, the amount of labor to hire and how much to invest in innovation according to

$$\max_{q_{it}, l_{it}} \sum_{t=0}^{\infty} (\beta(1-\delta))^t \{(1-\tau_{it})p_{it}y_{it} - w_t l_{it} - A_{it}c(q_{it})\}, \quad (2.6)$$

subject to the evolution of technology in (2.4), the demand curve for intermediate goods in (2.2), their production function in (2.3), and firm characteristics φ_i and $A_{i0} > 0$. Distortions in the economy occur through the parameter τ , which represents an implicit tax rate or distortion. These distortions are meant to model the multi-dimensional nature of constraints felt by firms. They represent an aggregation of tax rates, regulation, and corruption. Note that τ can vary across firms, since the tax rate will later be tied specifically to firm size such that $\tau_{it} = \tau_{it}(l_{it})$.

Firms' first order conditions are derived for the case where distortions are tied to firm size. In cases where the tax rate is independent of firm size, $\tau'(l_{it}) = 0$. Taking derivatives with respect to the firm's choice variables, l_{it} and q_{it} , results in two first order conditions:

$$w_t = (1 - \tau(l_{it}))\kappa Y_t^{\frac{1}{\sigma}} (\varphi_i A_{it})^{\zeta} l_{it}^{\xi} - \tau'(l_{it}) Y_t^{\frac{1}{\sigma}} (\varphi_i A_{it} l_{it}^{\alpha})^{\zeta}, \quad (2.7a)$$

$$A_{it}c'(q_{it}) = 2\Delta A\beta(1-\delta) \left((1 - \tau(l_{it+1})) Y_{t+1}^{\frac{1}{\sigma}} (\varphi_i l_{it+1}^{\alpha})^{\zeta} \zeta A_{it+1}^{-\frac{1}{\sigma}} - c(q_{t+1}) \right), \quad (2.7b)$$

where the scalars $\kappa = \left(\frac{\alpha(\sigma-1)}{\sigma}\right)$, $\zeta = \frac{\sigma-1}{\sigma}$, and $\xi = \left(\frac{\alpha(\sigma-1)-\sigma}{\sigma}\right)$, are used for expositional convenience. Equation (2.7a) maintains its standard interpretation where the cost of hiring additional labor, w , is equated to the marginal increase in profits. Note, however, the second term on the right-hand side. This term measures the change in the tax rate faced by a firm that chooses to hire at labor level l_{it} . Equation (2.7b) equates the marginal cost of innovating today with the discounted marginal profits tomorrow. Since the costs of innovating scale with the technology level, the second term on the right-hand side is necessary because firm investments in innovation today lead to higher costs for innovating in the future. These two first order conditions, along with Equation (2.5), are sufficient to solve the model in steady-state.

Entry and Exit

Since I am primarily concerned with the steady-state behavior of firms under different tax distortions, I simplify the entry and exit process of firms in a manner similar to Melitz

(2003). Firms face a constant probability of exit, δ , each period. Firms that die are replaced with identical firms.¹⁶ Functionally, the exit process of firms serves to further discount future profits of firms, without adding stochasticity to the steady state. Firm entry typically requires firms to forecast their profits and enter the market if their expected discounted profits exceed the fixed cost of participating in the market. I assume that the M firms in the quantitative application have already undergone this entry process. Imposing an endogenous entry process adds little to the steady state analysis. Predictably, average productivity in the market will be higher, as the least productive firms have expected discounted profits that are less than the cost of entry.

Steady State Equilibrium

Definition A steady-state equilibrium in the model is composed of aggregate quantities, $\{Y^*, T^*\}$, aggregate prices, $\{w, P\}$, firm-level decisions, $\{y_i^*, p_i^*, A_i^*, l_i^*, q_i^*\}$, firm-specific tax rates $\{\tau_i^*\}$, and innovation costs $\{H, b\}$ such that all firms solve their profit maximization problem and the labor market clears.

Imposing the steady state definition on Equation (2.5) implies

$$A^* = A^* + 2\Delta A(2q^* - 1). \quad (2.8)$$

Solving for q_i^* implies that in steady state all firms choose $q_i^* = q^* = \frac{1}{2}$. Using this condition, and the steady state definition, firms' first order conditions become

$$w^* = (1 - \tau(l_i^*))\kappa Y^{*\frac{1}{\sigma}} (\varphi_i A_i^*)^\zeta l_i^{*\xi} - \tau'(l_i^*) Y^{*\frac{1}{\sigma}} (\varphi_i A_i^* l_i^{*\alpha})^\zeta, \quad (2.9a)$$

$$A_i^* c'(q^*) = 2\Delta A\beta(1 - \delta) \left((1 - \tau(l_i^*)) Y^{*\frac{1}{\sigma}} (\varphi_i l_i^{*\alpha})^\zeta \zeta A_i^{*\frac{1}{\sigma}} - c(q^*) \right). \quad (2.9b)$$

where $q^* = \frac{1}{2}$. Given the initial time-invariant productivity draw for the firm, (2.9a) and (2.9b) are sufficient to solve for the steady state equilibrium of the economy.

¹⁶This can be thought of as a perfect markets assumption for firm technology.

Quantitative Application

This section details the parameter values used to numerically solve the model. Each application uses the same parameter values with the exception of the type of implicit tax or distortion that firms face. Note that the purpose of this chapter is not to calibrate results to any particular economy, but to look generally at the effect of different distortions on growth and the innovation process. Further work could evaluate specific distortions and pursue a more detailed calibration exercise. Table 3 serves as a guide to the parameters in the model, and Table 4 provides their specific values used in the calibration.

TABLE 3. Parameters of the Model

Parameter	Definition	Parameter	Definition
σ	Elasticity of substitution	α	Labor's share of income
δ	Firm discount rate	b	Innovation costs
τ	Tax rate on output	ΔA	Innovation step-size
M	Number of firms	w	Wage Rate

TABLE 4. Parameters Used in Quantitative Application

Parameter	Value	Parameter	Value
σ	3	α	0.33
δ	0.10	b	10
τ	Varies	ΔA	0.1
M	1000	w	1
φ	$\sim \mathcal{N}(10, 5.9234)$		

After generating the firm-size distribution for the undistorted economy, I create two tax distortions to evaluate the hypothesis of Tybout (2000). First, I create a linear tax environment where the tax rate is increasing in the amount of labor that firms hire. I set the tax rate so that all firms face a minimum tax rate of 10% (the lowest tax rates observed in the data) and the largest firms in the undistorted economy would face a tax rate of 40% (the approximate mean in the data).¹⁷ This tax scheme runs counter to the intuition of Tybout (2000) regarding the phenomenon of the missing middle. This process yields a linear tax schedule of the form $\tau(l_{it}) = .1 + (7.4909 \times 10^{-6})l_{it}$.

¹⁷Note that this does not imply the largest firms in the linear distortion environment face a 40% tax rate. Firms in the linear tax environment operate on a smaller scale. If a firm in the linear tax environment decided to utilize the same amount of labor as the largest firm in the undistorted economy, it would face a 40% tax rate.

Secondly, I create a quadratic tax schedule to replicate the story of Tybout (2000). The tax rate is increasing for small firms and decreasing for the largest firms. Medium-sized firms face the highest tax rate of 40% (the mean in the data). The smallest firms continue to face a 10% tax rate to capture the lowest rates observed in the data. I fit the curvature of the tax schedule to make sure that no firms receive a subsidy (negative tax rate). The quadratic environment (Tybout-style tax) imposes a tax schedule of the form $\tau(l_{it}) = .1 + (3.1707 \times 10^{-5})l_{it} - (8.3284 \times 10^{-10})l_{it}^2$. For reference, Figure 1 plots total tax rates for a number of different developing countries. It illustrates that the tax rates imposed in the quantitative exercise are well within the range of tax rates observed in lesser developed countries. The tax rates, however, include taxes both on profits and on fixed assets. The model imperfectly captures this type of tax by only taxing firms' output. If taxes were to be levied on the technology level of the firm (its only fixed asset), this would likely further decrease TFP as it would increase the expected costs of innovation. Figure 2 plots the exact form of taxes in the linear and quadratic tax environments.

I calibrate the dispersion of time-invariant productivity to approximately match the distribution of educational attainment in China in 2010 for 20-24 year-olds as reported by Barro and Lee (2013). I generate a normal distribution with mean 10 and standard deviation based on the Barro and Lee (2013) data. While this may be a rough proxy for time-invariant productivity or entrepreneurial talent that is not tied to firm technology, it seems more natural than calibrating the distribution of φ to the dispersion of U.S. TFP. That distribution undoubtedly reflects its own long-run adjustment process and may not accurately reflect the distribution of talent in other countries. The number of firms, M , balances the desire for a large economy with the computational intensity of numerically solving the model. The probability of exogenous exit is set at a standard rate of 10%.

The innovation cost function, $c(q_{it})$, is bq_{it} . Importantly, costs are linear in q_{it} , and functional form of $c(q_{it})$ is inspired by Atkeson and Burstein (2010). I choose a level of b such that the economy converges to a steady state equilibrium. To be sure, the level of b chosen here is not the only level of innovation costs that generates a steady state; however, costs that are too low lead to all firms choosing to innovate with $q = 1$, and costs that are too high lead to all firms choosing to pursue no innovation ($q = 0$), neither of which implies a balanced growth path. Brandt et al. (2012) estimate firm-level TFP growth in China to be 2.7%. Considering that this

is an upper bound for lesser-developed countries, I target ΔA more conservatively, so that ΔA is approximately 1% of the average level of A^* . Since the equilibrium level of A^* depends on ΔA , I iterate over values of ΔA until I meet this criteria. Ultimately, this requires that $\Delta A = 0.1$.

FIGURE 1. Global Total Tax Rates (2012)

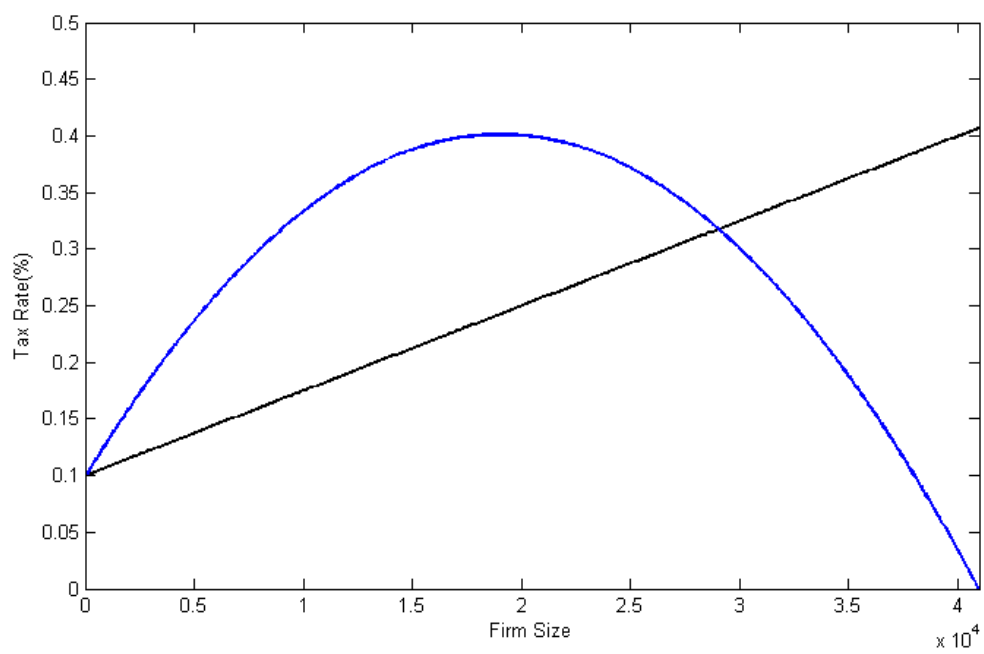


Source: World Development Indicators 2012. The scatter plot includes 81 randomly chosen countries that have available data on total corporate tax rates and GDP growth for 2012. Countries with total tax rates above 100% are dropped.

Results

The quantitative application centers on testing five types of distortions. As a baseline, there is an undistorted economy such that $\tau = 0$. There are two exercises with tax rates of 20% and 40% across the board (referred to as size-independent taxes). Additionally, there are two exercises with tax rates that depend linearly and non-linearly with firm size (size-dependent taxes). This section begins with size-independent tax environments to establish a baseline and develop intuition and then proceeds to the primary results of interest, tax-dependent distortions.

FIGURE 2. Size-Dependent Distortions



Linear and quadratic tax distortions based on firm size. The quadratic environment follows a story similar to Tybout (2000), while the linear environment causes disincentives to hire labor all firm size levels.

Size-Independent Distortions

Table 5 shows the steady-state levels of aggregate output and aggregate TFP. As expected, the undistorted economy has higher levels of aggregate output and aggregate TFP. The results suggest that imposing a 20% tax rate on a previously undistorted economy would result in a 5.9% loss in log output and a 5.6% loss in log TFP. Similarly, imposing an additional 20% tax on an economy that is in steady state with a 20% tax rate, implies a loss of 8.5% in log output and a 7.5% loss in aggregate TFP.

TABLE 5. Aggregate Statistics - Size-Independent Distortions

Tax Rate	Log Output	Log TFP
Undistorted	18.12	11.44
$\tau = .2$	17.05	10.80
$\tau = .4$	15.60	9.93

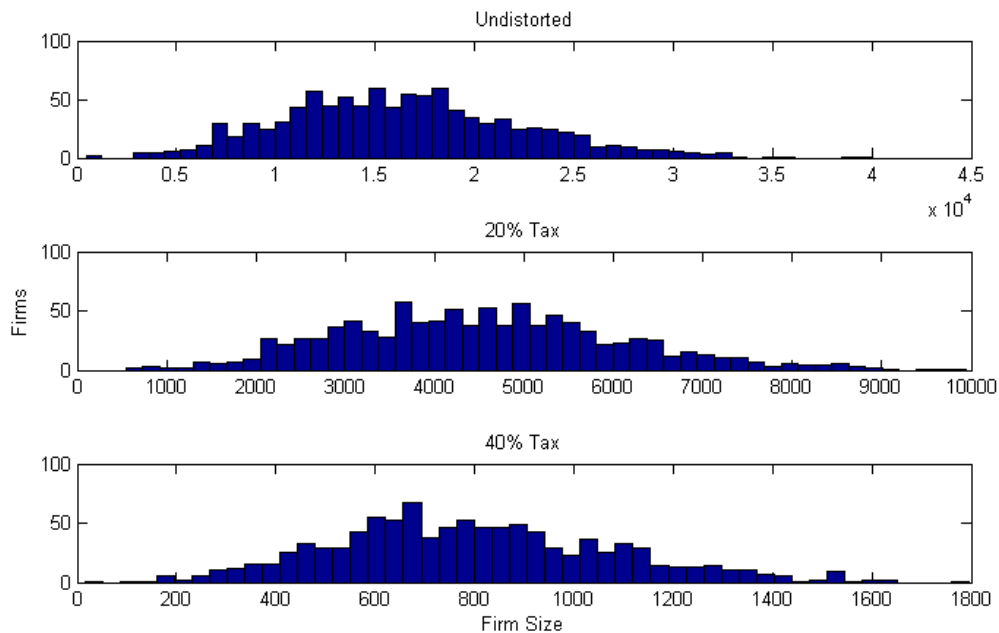
Aggregate TFP is calculated as $\sum_{i=1}^M \varphi_i A_i^*$.

Economies with larger distortions have smaller, less technologically advanced firms. Figure 3 illustrates the distribution of firm size across the different size-dependent tax environments, and Table 6 provides statistics on firm characteristics of each environment. The effect of firm size can be seen directly from the firm's first order conditions. In the size-independent environments, there is no marginal change in the tax rate from hiring more labor; therefore, the second term in Equation (2.7a) is zero. For a given level of technology and a fixed wage rate, an increase in τ requires a decrease in the amount of labor that the firm hires. Although size-independent taxes have significant effects on the accumulation of technology in steady state, they generally do not distort the firm size distribution in Figure 3. The firm size distributions tend to simply shift to a smaller scale as the tax rate increases.

TABLE 6. Size-Independent Distortions

Statistic	Avg. Technology	Avg. Firm TFP	Avg. Firm Size
Undistorted	9.02	93.20	16,439
$\tau = .2$	4.72	49.24	4,526
$\tau = .4$	1.96	20.62	796

FIGURE 3. Firm Size Distribution across Size-Independent Distortions



Each histogram is created using 50 bins. Note that the x-axis is scaled differently across histograms.

Size-Dependent Distortions

More interesting are the size-dependent distortions. These distortions seek to mimic (quadratic tax) or contrast (linear tax) with the conditions described by Tybout (2000). Specifically, the quadratic distortion is intended to capture the desire for firms to be either small enough to avoid taxes or large enough to influence policy (tax rates). The other size-dependent distortion is linear in firm size. That is, firms face a linearly increasing tax rate as they hire more workers. Figure 2 illustrates the two size-dependent distortions. While these distortions are admittedly somewhat *ad hoc*, they capture important features to test whether the phenomenon of the missing middle can be captured by size-based tax distortions.

The striking feature of the size-dependent applications is illustrated in Table 7. It illustrates that the size-dependent distortions have much smaller output losses than the size-independent distortions. The imposition of the linear tax rate results in a 6.0% loss in log output, which is very similar to the size-independent distortion of $\tau = .2$. Even more surprising, the quadratic environment only entails a .9% loss in log output. Importantly, however, notice that

TABLE 7. Aggregate Statistics - Size-Dependent Distortions

Tax Rate	Log Output	Log TFP
Undistorted	18.12	11.44
Linear	17.04	5.06
Quadratic	17.95	6.90

Aggregate TFP is calculated as $\sum_{i=1}^M \varphi_i A_i^*$.

there are losses in terms of log TFP. The linear distortion implies a 55.8% loss in aggregate TFP, and the quadratic distortion implies a 39.7% loss in TFP.

Table 8 highlights the key differences among firms in the size-dependent distortion economies. The linear economy behaves much the same as the size-independent environment with a tax rate of 20%. Output, technology, and firm size are all significantly lower than in the undistorted economy. However, there is an interesting trade-off between technology and labor in the quadratic environment. Firms in the quadratic tax environment are, on average, larger in steady-state than their counterparts in the undistorted economy. This results because the demand for labor is augmented by tax incentives (i.e. less effective tax) for increasing firm size. This comes at the expense of technology accumulation. The quadratic tax environment has lower accumulated technology and firm level TFP than the undistorted economy. Additionally, it outperforms both size-independent tax schemes along all the dimensions of interest.

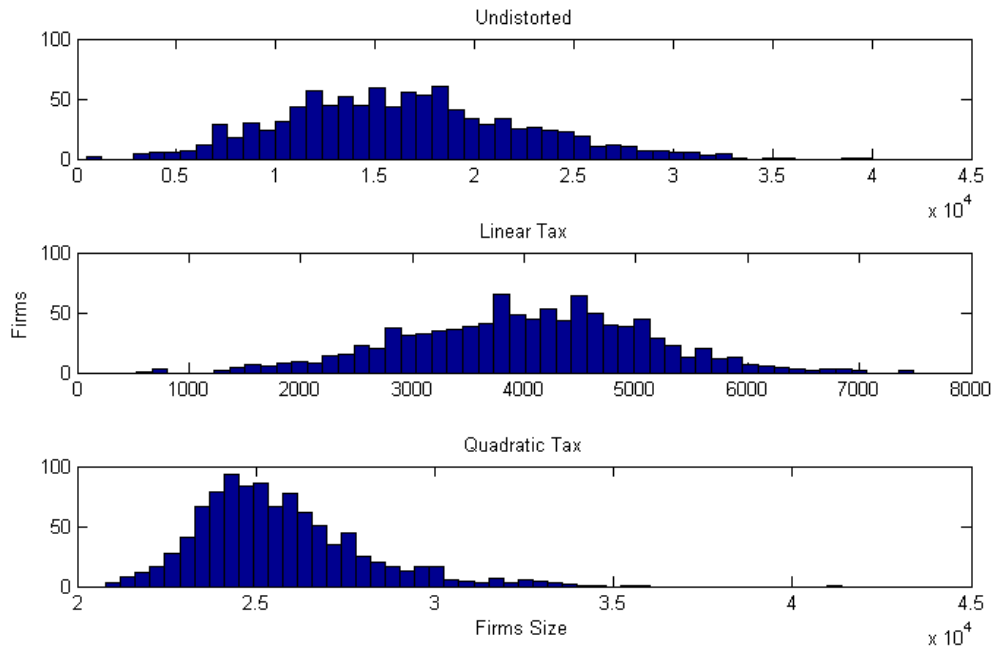
TABLE 8. Size-Dependent Distortions

Statistic	Avg. Technology	Avg. Firm TFP	Avg. Firm Size
Undistorted	9.02	93.20	16,439
Linear Tax	4.90	50.63	4,026
Quadratic Tax	6.71	69.01	25,669

Despite the higher initial tax rates faced by firms in the quadratic tax environment, the economy performs similarly to the undistorted economy in terms of total output. The optimal response of firms in the quadratic tax environment is to become larger firms that incur lower tax rates. The distribution of firm size for the size-dependent distortions is presented in Figure 4. It is striking that the economy has no small firms, which is at odds with observations made in lesser developed countries. A discussion of the dynamics that generate these results is reserved for the next section. A clear implication of Figure 4 is that additional frictions are required to incentivize

some firms to stay small. Small firms are a crucial part of the “missing middle” phenomenon. Firms in the quadratic environment operate with much higher levels of labor to take advantage of lower tax rates. With larger firms, and less variance in firm size, the quadratic economy shows little loss in terms of log output despite utilizing far less technology in steady state.

FIGURE 4. Firm Size Distribution across Size-Dependent Distortions



Each histogram is created using 50 bins. Note that the x-axis is scaled differently across histograms.

Table 9 illustrates an additional facet of the results: the dispersion of productivity. The model generates firm TFP levels, $\varphi_i A_{it}$, that are less dispersed than has been observed in the literature. Specifically, Hsieh and Klenow (2009) report the dispersion of productivity for India and China. Their estimates are likely a lower bound for dispersion since their data does not include small firms. Firms in the quadratic economy show less dispersion in both firm size and TFP. This result is driven by their desire to take advantage of lower taxes on the right-hand side of the quadratic distortion.

Despite the lack of small firms, and seemingly competitive performance of the quadratic tax environment to the undistorted economy, it is important to realize the implications of the differences in aggregate TFP. As the seminal work by Hall and Jones (1999) documents, it is

TABLE 9. TFP Dispersion Measures

Statistic	Undistorted	Quadratic	India	China
90th to 10th	3.31	2.30	5.0	4.9
75th to 25th	1.81	1.53	2.2	2.3

90th to 10th is the ratio of firm productivity in the 90th percentile to that of the 10th percentile, among entering firms. Likewise, 75th to 25th is the ratio of productivity between the 75th percentile and 25th percentile. The final two columns present the results for India and China in Hsieh and Klenow (2009) for total factor revenue productivity.

differences in TFP that drive differences in output per worker and income per worker. Table 10 illustrates that the quadratic economy has comparable output in terms of the final good because it utilizes a larger labor force, which decreases output per worker below that of the undistorted economy.

TABLE 10. Output per Worker

Distortion	Output per Worker
Undistorted	4.50
Quadratic Tax	2.45

Discussion

The results section of the model underscored the potential effect of taxes, both size-independent and size-dependent, on the accumulation of technology. Despite the fact that total output is similar between the undistorted economy and the quadratic tax environment, the accumulation of technology suffers and firms become dependent on labor. This section explores the principal drivers of these results and explores possible extensions.

The assumption of an infinite labor supply and fixed wage rate are fundamentally important in the comparison between the undistorted economy and quadratic tax economy. Suppose that instead, there was a finite labor supply with a standard labor market-clearing wage rate. As firms grow in the quadratic tax environment and take advantage of lower tax rates, they hire increasing amounts of labor. In a well-functioning labor market, this would drive up the real wage, partially offsetting the desire of firms to become large, and lowering total output.

The current set-up of the model is interesting in terms of cross-country comparisons. It suggests that given a low wage rate (i.e. relatively fixed because the supply of labor is large),

firms that face size-dependent tax schedules may choose to increase output through labor rather than invest in innovation. This outcome may be reflective of large firms in developing countries. Larger firms, especially national champions, enjoy lower tax rates and special protections that accompany their over-sized influence in the economy.

The presence of large firms in the quadratic tax environment is in line with the story of Tybout (2000); however, there are no small firms at the lower end of the distribution. This result is driven partially by only analyzing the steady state. Outside of the steady state, firms could be observed in different stages of growth, presumably implying that there would be some small firms. A more complex entry and exit process, or other additional friction, is necessary to populate the lower tail of the firm-size distribution as observed in the data.

Finally, the distribution of time-invariant productivity is important in determining the final distribution of firm sizes and TFP. The choice of Chinese educational attainment is made because it should roughly capture or be correlated with entrepreneurial characteristics that cannot be improved by innovation. The data for 20-24 year-olds in China, however, may not be indicative of other countries. A large percentage of 20-24 year olds in China complete secondary school (71.18%), a feature that is not typical of developing countries. In order to generate more realistic firm-size distributions and firm-level TFP distributions the time-invariant distribution must have more weight toward the lower end of the distribution. Other lesser-developed countries' educational attainment profiles may better fit this requirement.

Conclusion

This chapter represents a first step in explicitly modeling innovation and how it is affected by size-based distortions. In doing so, it links the innovation and technology evolution process central to many growth models with the expanding literature on size-based distortions in developing countries. Combining heterogeneous production a la Melitz (2003) with innovation is a necessary step to evaluate the changes in the process of accumulating technology and how it interfaces with aggregate outcomes.

While further research is certainly required, these results suggest tax schemes that take a quadratic form negatively affect the accumulation of TFP, even while maintaining a similar level of aggregate output compared to the undistorted economy. It captures the incentives for

firms to become large and take advantage of lower tax rates. Further, it clearly illustrates that the “missing middle” cannot be explained by tax rates alone. In order to generate the “missing middle,” there must be additional incentives or constraints to keep firms small. Additional research should seek to address the absence of small firms. The results show that certain tax policies may drive total output, even while failing to increase TFP, which has been shown to be an important explanation of cross-country income differences.

CHAPTER III

INFORMALITY, INNOVATION, AND AGGREGATE PRODUCTIVITY

Introduction

The fundamental question in development economics is what makes some countries so much more prosperous than others. Hall and Jones' (1999) seminal work shows that it is not physical or human capital accumulation that primarily drives differences in income but rather differences in total factor productivity (TFP) resulting from country-specific institutions. In more recent work, Hsieh and Klenow (2009) show that much of this TFP difference arises from increased productivity dispersion in developing countries, presumably the result of policy distortions.

The goal of this paper is to understand how firm-level decisions regarding productivity affect economy-wide outcomes, and how those decisions depend on a country's tax policies. Firm productivity depends, among other factors, on innovation at the level of the firm. Country institutions, on the other hand, determine policies on taxation and law enforcement. This paper analyzes how firm-level innovation decisions are affected by government policies, and how those decisions affect aggregate productivity (TFP).

I construct a general equilibrium model where firms choose whether to participate in the formal or informal manufacturing sector. In equilibrium, this decision depends on policy constraints in the form of taxation and law enforcement. I calibrate the model and conduct numerical experiments to estimate the effect of tax distortions on the size of the informal sector and on aggregate productivity growth. I find that a change in tax rates from 50% to 60% leads to a 20.9% reduction in the size of the formal sector. This change is accompanied by a 0.07 percentage point reduction in TFP growth per year. Given that countries like Mali, Mexico, and Sri Lanka impose total tax rates near 50%, these findings have significant and applicable policy implications in lesser developed countries. Even moderate changes in tax rates, for instance, 10% to 20%, decrease the size of the formal sector by more than 7.7% and decrease TFP growth by 0.09 percentage points per year.

The model operates with two central tensions. At the firm-level, firms that choose to operate in the formal sector have the ability to innovate and improve future productivity, but must comply with government imposed taxes. Alternatively, they can choose to avoid these

taxes in the informal sector, although they also forgo the choice to innovate. On the macro-level, fewer formal sector firms decreases innovation and TFP growth. Informal firms are counted in aggregate TFP, but since they do not innovate, they do not contribute to its future growth. This assumption should be thought of as a reduced-form mechanism to capture innovation in the informal sector. In reality, informal firms improve their productivity through a combination of innovation and imitation. Further, given that informal firms operate on a smaller scale and have less ability to diversify against risk, they pursue less innovation. This point is echoed in Rosenzweig and Binswanger's (1993) finding that poorer farmers are less likely to undertake risky investments. In addition, data from the World Bank Enterprise Survey indicate that smaller firms, which tend to be informal, are much less likely to license foreign technology.

This paper is related to a large literature in development economics and international trade. It can be viewed as a link between models of monopolistic competition with innovation that are commonplace in the international trade literature, with the literature on informal economies. The theoretical basis for firm-size heterogeneity and innovation is Atkeson and Burstien (2010). This foundation is augmented by the decision of firms to either enter the formal market where they face taxes or the informal sector where they face punitive fines and forced closure. This environment generates a rich set of predictions regarding what types of firms, in terms of productivity, enter each sector, and how the decision to be formal is affected by government policies.

The informal economy in the present context refers to informal product markets. In this case, entrepreneurs make a decision whether to abide by laws and regulations governing firms in the formal sector, or operate in the informal sector to bypass these laws. Informal labor markets, on the other hand, refer to workers themselves who operate informally and often receive lower wages, worse working conditions, etc. Both Nataraj (2011) and Goldberg and Pavcnik (2003) discuss this distinction and opt to address informal product markets rather than informal employment. To be sure, the two formulations of the informal sector are not independent as workers at informal firms constitute informal employment. This paper does not address the changes on informal employment in formal sector firms.

Moreover, there is considerable divergence in what constitutes an informal firm, both in the literature and country-specific contexts. In the United States, informal firms are most often associated with the production of illegal goods like narcotics. In other countries, like

TABLE 11. Average Size of Informal Sector as Percentage of GDP in 2005

Region	Mean	Max
East Asia and Pacific	17.5%	51.0%
Latin America and the Caribbean	34.7%	66.1%
Middle East and North Africa	27.3%	37.2%
South Asia	25.1%	43.7%
Sub-Saharan Africa	38.4%	61.8%

Measurements are weighted by the size of GDP. Source: Schneider, Buehn, and Montenegro (2010).

India, informal firms are often firms that are not required to register given their size. Certainly different data sources utilize different definitions. Henly and Arabsheibani (2009) show that in the Brazilian context, these different definitions can be significant. Throughout, I define informality as suggested by Kanbur (2009): informality should be defined with regard to a specific policy or regulation. In the current context then, firms that choose to opt out of the formal tax environment are considered informal. In terms of policy applications, informal firms in the model likely represent firms that are informal on other margins as well, for instance in terms of the labor they hire or the goods they produce.

The informal economy accounts for a large share of economic activity in developing countries. This is evident from Table 11: in regions with a high concentration of developing countries, a large percentage of the aggregate economy is considered informal. Moreover, this pattern is persistent. Thomas (1992) shows that informal economic activity was prevalent in developing countries in Latin America, Africa, and Asia from 1950 to 1986. He also reports that most Latin American countries had 25% to 40% of their workforce participating in the informal sector. The percentages are even higher for many African and Asian countries. In India today, about 80% of the manufacturing sector is composed of informal firms (Nataraj (2011)) and accounts for 20% of value-added.¹

Several related papers explain the development and prevalence of informal firms in developing countries. Early theoretical justifications typically emphasize the existence of a wage-rate differential between the formal and informal sector created by the existence of an enforceable

¹The informal sector in India operates differently from the traditional notion of informal firms. Informality in India does not connote illegal activity, rather it simply means that the firm is not required by law to register given its size. Typically the cut-off is 10 employees for firms that use electricity and 20 employees for firms that do not use electricity.

minimum wage in the formal sector. The seminal work in this area is Rauch (1991). Rauch (1991) analyzes a model with heterogeneous agents (differing productivities) who make a sector choice as in Lucas (1978). A productivity threshold determines which sector entrepreneurs enter: more productive entrepreneurs enter the formal sector and less productive ones enter the informal sector. This results in a strict size dualism between the sectors as the smallest formal firm is necessarily larger than the largest informal firm.

Contemporary empirical work has identified the importance of institutions in determining whether firms choose to be informal. Dabla-Norris, Gradstein, and Inchauste (2008) show that the quality of the legal framework is fundamentally important in determining the existence of informal firms. Intuitively, a better legal system is able to better enforce laws regarding taxes and regulation. Not surprisingly, they also find that firms that face greater taxes and regulation in the formal sector are more likely to enter the informal sector. Given the empirical importance of these institutions, the model in this paper captures both the role of tax enforcement and taxation in firms' profit maximizing decisions. While the authors' findings are informative, they do not include any measure of how firms' decisions are affected by productivity and innovation. Additionally, and similar to many studies on the informal sector, the authors are forced to use data that may not truly be indicative of the decision-making of firms.²

Despite the realization that informal firms constitute a significant part of most developing economies and have been studied extensively, there has been little research on the sectoral choice of firms in a dynamic environment. Outside of the literature on informality, there is a large and evolving literature documenting important aspects of firm dynamics. These works generally use Lucas (1978) or Melitz (2003) as a starting point for modeling heterogeneous firms and their decisions. The dynamics in these papers have become increasingly complex and have been used to study everything from economic growth as in Luttmer (2007) to inefficient allocation of resources as in Hsieh and Klenow (2009).

The addition of innovation into firm-level dynamics is of principal importance to the question at hand. Atkeson and Burstein (2010) embed both process and product innovation into a model of monopolistic competition. Process innovation in their model allows firms to improve

²They are only able to capture the decision making of informal firms indirectly from formal firms based on a survey question which asks how much the typical firm keeps "off the books."

their productivity through time.³ While the authors use their model to study the effects of changes in marginal trade costs, their approach is more general and is useful for understanding how firms improve their production processes through time.

The implications of this paper, particularly on fiscal policy, have some analogs in the literature.⁴ Specifically, Auriol and Warlters (2004) analyze a model where governments may increase barriers to entry into the formal market in order to create larger taxpayers. These large firms, in turn, generate larger rents due to the lack of competition. The idea that higher tax rates may force more firms into the informal sector, lowering tax revenue, echoes Olson (1982). His work describes a revolving sequence where firms react to higher taxes through evasion which requires a higher tax rate to continue to provide a constant level of government services.

The remainder of this chapter proceeds as follows. The next section articulates the theoretical model, and it is followed by a documentation of the parameters and their values utilized in the simulations. The following section investigates the results of the simulations. I then discuss the key findings and limitations of the model. The final section concludes.

The Model

This section describes the theoretical model for the paper. The central item of interest is the decision of firms to participate in the formal or the informal sector, given the ability to innovate in the formal sector. At the firm-level, firms producing differentiated products make decisions about which sector to enter. Firms choosing to operate in the formal sector must pay taxes, but they can also improve their future productivity through innovation. Firms in the informal sector cannot innovate, although they completely avoid taxes. They do, however, face a probability of being caught, fined, and closed for operating in the informal sector. Firm-level decisions are aggregated using a standard CES production function, and firms' decisions with regard to innovation affect aggregate output and TFP growth.

³This process is similar to quality ladder models. For instance, see Grossman and Helpman (1991).

⁴There is a large literature in public finance and public economics on tax evasion and informality. Slemrod and Weber (2012) provide a discussion of the empirical issues associated with measuring the tax effects of the informal economy.

The Aggregate Economy

The aggregate economy is a standard model of monopolistic competition in a closed-economy with discrete time.⁵ Aggregate output is produced as a CES aggregate of M intermediate inputs:

$$Y_t = \left(\sum_{i=1}^M y_{it}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (3.1)$$

where y_{it} is the output of intermediate good producer, i , and $\rho > 1$ is the elasticity of substitution between goods. Final good producers choose Y_t and inputs, y_{it} , to maximize profits in a competitive market given input prices, p_{it} , and the price of the final good, P_t . The price of the final good is normalized to 1. Standard profit maximization dictates that in equilibrium, the demand curve for intermediate goods is

$$p_{it} = \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}}, \quad i = 1, \dots, M. \quad (3.2)$$

Intermediate Good Producers

Each intermediate good producer supplies a unique input into the production of the final good. These transactions occur in monopolistically competitive markets. Firms seek to maximize their discounted profits by making decisions about which sector, formal or informal, to enter, how much labor to hire, how much to invest in innovation, and what price to charge. These decisions are made subject to (3.1) and (3.2) taking the level of aggregate output as given as well as their initial productivity draw z_{i0} . Throughout, production decisions and innovation decisions are designated as $t \geq 1$ decisions that are based on the firm's sector choice in the first period.

Firms have access to constant returns to scale technology such that

$$y_{it} = e^{z_{it}/(\rho-1)} l_{it}, \quad (3.3)$$

where z_{it} is a firm specific productivity parameter and l_{it} is the firm's labor force.⁶ Productivity in (3.3) is scaled by $\frac{1}{(\rho-1)}$ for expositional ease, since it allows for firms' static (within-period)

⁵The structure of the model is similar to a closed-economy version of Atkeson and Burstein (2010) where all prices are in terms of the final good.

⁶Unless they are explicitly required, I drop the subscripts for firm and time on the productivity parameter z_{it} , such that $z_{it} = z$.

profits and labor hiring decision to be proportional to e^z as in Atkeson and Burstien (2010). Note that throughout this paper, firm productivity or firm TFP will refer to e^z . This distinction will be important in maintaining consistency between firm level measures of TFP and aggregate measures of TFP.

Technology and Innovation

Firms producing intermediate goods can decide to innovate and improve their future productivity through process innovation.⁷ Firms choose an amount to invest in innovation. The likelihood of an innovation being successful increases with the level of investment. This probability of success, denoted q , provides a conveniently bounded choice for the firm's dynamic programming problem. Atkeson and Burstien (2010) use a similar set-up to model firms' innovation decisions.⁸ The costs for innovation are denoted $c(q)$, where $c(q)$ is an increasing and convex function of q .

A firm that invests $e^z c(q)$ has a probability q of having productivity $e^{z+\Delta z}$ and probability $1-q$ of having productivity $e^{z-\Delta z}$ in the following period. The costs of innovation are scaled by e^z to reflect the fact that innovation at higher levels of productivity is more expensive. Further, it is worth noting that productivity can only change by the fixed amount Δz . In this sense, the model operates much the same as a quality ladder with rungs at discrete intervals. This assumption is computationally helpful since the value of that state variable for each firm is always on the chosen grid, and there is no need to interpolate between grid points.

Firms must engage in research and development in order to maintain their productivity. Choosing not to engage in research, by choosing $q = 0$, necessarily leads to a firm's productivity decreasing in the next period. This process reflects a depreciation of productivity over time. For instance, this could reflect loss of market power due to a firm not adequately improving its supply chain. On an individual firm level, productivity necessarily changes each period to reflect both the depreciation of productivity and the stochastic nature of innovation.

⁷The firm's investment in innovation is best thought of as innovation in *process innovation*, rather than *product innovation*, or the introduction of new varieties of goods. This distinction is important in justifying entry and exit in the model (discussed below).

⁸Judd (1998) also provides a short discussion for this type of dynamic programming problem on page 406 of *Numerical Methods in Economics*.

Government

Government in the model has two roles: collect tax revenue, T_t , and fine and close informal firms. Firms are taxed τ percent of their profits each period in the formal sector. Firms that decide to operate in the informal sector face a probability, μ , of getting caught each period. If a firm is caught in the informal sector, it is fined its entire profits for the period and is forced to exit. The tax rate and probability of being caught in the informal sector are exogenous and known by all firms. Tax revenue is transferred back to households as a lump-sum payment.⁹ This formulation of the government, and the behavior of informal firms avoiding taxes, is consistent with the findings in Dabla-Norris et al. (2008). Specifically, the authors show that higher taxes and corruption increase the propensity of firms to operate informally, even when both variables are included in the same specification.

Entry and Exit

Each firm, i , is endowed with a firm-specific level of productivity z_{i0} . Given this level of productivity, the firm decides whether to operate formally, carrying z_{i0} into the first period and producing with productivity $z_{i1} = z_{i0}$, or operating informally and receiving a spillover of technology from the formal sector as described below. Informal firms receive and operate utilizing the technology spillover in the first period. Once firms decide which sector to operate in, they operate in that sector forever. This implies that formal sector firms will continue to see their productivity levels change in response to their decisions, while informal sector firms will receive an exogenous (to them) spillover from the formal sector each period.

Since my primary interest is in the firm's sectoral choice and the effect of innovation, I simplify the entry and exit of new firms. Firms face an exogenous probability of death, δ , each period which further discounts future expected profits. I begin with a large number of firms that are drawn from an initial distribution and make decisions about which sector to enter. Previous work has shown that the lowest productivity firms exit the market due to a fixed cost of entry (for instance Melitz (2003)) which pins down the size distribution of firms. Given that this fact is well

⁹Adding uncertainty to these parameters and specifically modeling the government's objectives as a revenue-maximizing agent are left as an avenue for future work. Other work, specifically Auriol and Warlters (2004) models the government as a revenue-maximizing agent. If firms have uncertainty regarding the probability of being caught in the informal sector, the government could maximize tax revenue by enforcing laws in the informal sector to an extent that exceeds firms' expectations.

documented, I assume that the initial draw of firms is done after the decision to operate. Firms that exit are replaced by identical firms, such that the mass of firms does not actually change from period to period.¹⁰ These assumptions on entry and exit isolate the channels for aggregate changes. Allowing *product innovation* in addition to *process innovation*, through the entry of an increasing number of firms, creates a different channel for TFP and tax revenue changes besides the choice of sector and obfuscates the role that sectoral choice and innovation play.

Consumers

Households have preferences of the form $\sum_{t=0}^{\infty} \beta^t u(C_t)$, where C_t is consumption of the final good and $\beta \in (0, 1)$ is the subjective discount rate. Households earn income from supplying their labor inelastically to intermediate good firms. The aggregate labor supply is denoted L and is fixed through time.¹¹ Households share ownership of all intermediate good producers. In addition to labor income, households receive income through lump-sum transfers, T_t , from the government and dividend streams, D_t , from the profits of intermediate goods producers. I assume that the final good is perishable and consumers have no ability to transfer wealth across time periods. They maximize their utility subject to the budget constraint $P_t C_t \leq w_t L + T_t + D_t$ given an initial level of assets, A_0 . Under the assumptions of the model

$$C_t = w_t L + T_t + D_t, \tag{3.4}$$

for all $t \geq 1$. The assumption of a perishable final good allows me to abstract away from the complications of saving.¹²

Firm's problem

Each firm faces the decision problem of which sector to enter, how much labor to hire, and how much to invest in innovation. In doing so, they take aggregate output, Y_t as given and take

¹⁰This can be thought of as a perfect markets assumption. When firms' die, their assets and processes are acquired by a new owner, rather than being deserted.

¹¹Labor growth could easily be implemented; however, fixing the labor supply isolates the source of growth in the economy to changes in TFP.

¹²Given the wide variety of savings methods and interest rates for informal and formal borrowers, it is best to leave the effects of different savings channels for future work.

into account the demand curve for their products in (3.2). Firms in both sectors are risk neutral. Firms that enter the formal sector (F) earn operating profits of

$$\Pi_t^F(z) = \max_{p_{it}, l_{it}} (1 - \tau) (p_{it} y_{it} - w_t l_{it}). \quad (3.5)$$

Firms that decide to enter the informal sector (I) earn operating profits of

$$\Pi_t^I(\bar{z}_t) = \max_{p_{it}, l_{it}} (1 - \mu) (p_{it} y_{it} - w_t l_{it}). \quad (3.6)$$

Since taxes are levied on total profits, the amount that formal firms invest also plays a crucial role in determining their total tax burden, as can be seen below in formal firms' value function.

While (3.5) and (3.6) appear similar, there is a fundamental difference with respect to the productivity level of informal firms. Informal firms are not able to innovate, and they instead receive a spillover of productivity, \bar{z} , from the formal sector. This parameter is meant to capture both imitation and innovation by firms in the informal sector. Even though informal sector firms may not choose to innovate in the model, they still improve their production processes by observing and adopting technologies seen in the formal sector. The productivity spillover acts as a reduced form method of capturing this process without further complicating the model by adding a dynamic decision for informal firms.

Evidence suggests that informal firms innovate substantially less than their formal sector counterparts. For instance, informal firms may adopt the use of e-mail upon seeing how formal sector firms integrate it into their production processes. The notion that smaller informal firms innovate differently from larger formal firms is supported by Rosenzweig and Binswanger (1993). While not explicitly modeled here, the ability of large firms to diversify against the riskiness of innovation likely plays a substantial role in their greater rates of innovation. The productivity spillover is determined by the entry of firms into either sector. It follows that firms must forecast the value of $\{\bar{z}_t\}_{t=1}^{\infty}$ before making their sector choice. The size of this spillover is time dependent, as the productivity level of firms in the formal sector fluctuates over time. A complete discussion of the size of this spillover is reserved for the quantitative application.

Additionally, firms in the informal sector face the prospect of being fined and closed for operating informally and avoiding taxes.¹³ This distinction is magnified through the process of innovation. Given the static level of profits in (3.5) and (3.6), and subject to (3.2) and (3.3), firms set their price as a constant mark-up over marginal cost:

$$p_{it} = \frac{\rho}{\rho - 1} \left(\frac{w_t}{e^{z_{it}/(\rho-1)}} \right). \quad (3.7)$$

Given the evolution of firm productivity as described above, the discounted present value of expected profits for all firms with an initial productivity draw z_0 satisfies a Bellman equation:¹⁴

$$V(z_0) = \max[V^I(\bar{z}_1), V^F(z_0)] \quad (3.8)$$

$$V^I(\bar{z}_1) = \max \sum_{t=1}^{\infty} (\beta(1 - \mu)(1 - \delta))^t \Pi_t^I(\bar{z}_t) \quad (3.9)$$

$$V^F(z_0) = \max_{q \in [0,1]} \left\{ \Pi_1^F(z_0) - (1 - \tau)e^{z_0}c(q) + \beta(1 - \delta) (qV^F(z_0 + \Delta z) + (1 - q)V^F(z_0 - \Delta z)) \right\}. \quad (3.10)$$

Equation (3.9) is the value function associated with entering the informal sector, where the firm would operate with a productivity of \bar{z}_1 , after opting to forgo producing using its endowed productivity level of z_0 . Equation (3.10) on the other hand, indicates that a firm that decides to enter the formal sector for the first period will operate with its endowed productivity given at $t = 0$. While the Bellman system can be generalized in terms of time t , it is explicitly written in terms of firms' entry decisions at $t = 0$. This is to emphasize that I am looking at an irreversible entry decision by firms into either the formal or informal sector.

The value function in the formal sector $V^F(z_0)$ is strictly decreasing in the tax rate. Similarly, the value function for firms operating in the informal sector is strictly decreasing in the probability of getting caught, μ . In order for firms to operate in the formal sector, they must

¹³Note that $1 - \mu$ in the informal firms' problem acts as an additional discount factor.

¹⁴Typically, the firm's value could also be zero if it decided not to operate. Since entry and exit are exogenous in the model, that option is excluded.

have an initial productivity draw, z_{i0} , such that

$$V^F(z_0) > V^I(\bar{z}_1). \quad (3.11)$$

Let \hat{z}_1 be the least productive firm, based on its z_{i0} productivity and the size of the spillover, \bar{z}_1 , that enters the formal sector for $t = 1$. Note that without innovation, the firm's decision is trivial: if $\tau > \mu$ all firms enter the formal sector and if $\mu > \tau$ all firms enter the informal sector.¹⁵ Innovation, therefore, plays a crucial role in determining the sectoral choice of firms.

Firms in both sectors share a profit maximizing rule for hiring labor. Using (3.5) and (3.6) firms in both sectors demand for labor is

$$l_{it}^F = w_t^{-\rho} \left(\frac{\rho - 1}{\rho} \right)^\rho Y_t e^{z_{it}}, \quad (3.12)$$

and

$$l_{it}^I = w_t^{-\rho} \left(\frac{\rho - 1}{\rho} \right)^\rho Y_t e^{\bar{z}_t}, \quad (3.13)$$

respectively. Finally, Equation (3.10) implies that the first order condition governing formal firms' investment decisions is

$$c'(q) = \left(\frac{\beta(1 - \delta)}{1 - \tau} \right) (V^F(z_0 + \Delta z) - V^F(z_0 - \Delta z)). \quad (3.14)$$

Equilibrium

The economy operates with a fixed labor supply, L . Labor market clearing requires that

$$L = \sum_{i=1}^M l_{it}. \quad (3.15)$$

Substituting (3.12) and (3.13) for the appropriate mass of firms given \hat{z}_t , and simplifying, yields the equilibrium wage rate for both sectors:

$$w_t = \left(\frac{L}{Y_t Z_t} \right)^{-\frac{1}{\rho}} \left(\frac{\rho - 1}{\rho} \right), \quad (3.16)$$

¹⁵This result stems from truncating the firm's problem to a static profit maximization problem since its productivity cannot be improved through time.

where $Z_t = \sum_i e^{z_{it}}$ is a measure of aggregate TFP that includes both formal and informal firms.¹⁶ The equilibrium wage rate can be used to simplify the first order condition for the firm's labor decisions. Substituting (3.16) into (3.12) and (3.13) implies that

$$l_{it}^F = \left(\frac{e^{z_{it}}}{Z_t} \right) L, \quad (3.17)$$

and

$$l_{it}^I = \left(\frac{e^{\bar{z}_t}}{Z_t} \right) L. \quad (3.18)$$

The simple intuition of this condition is that firms that constitute a larger fraction of aggregate TFP, hire more labor.¹⁷ Finally, in equilibrium, aggregate output is

$$Y_t = Z_t^{\frac{1}{\rho-1}} L. \quad (3.19)$$

Given that L is constant, the expected growth rate of output is

$$g_Y = \frac{Z_{t+1}^{\frac{1}{\rho-1}} - Z_t^{\frac{1}{\rho-1}}}{Z_t^{\frac{1}{\rho-1}}}. \quad (3.20)$$

A complete derivation of this condition and specifics on how it is calculated is included in the Supplementary Materials.

Definition An equilibrium in this economy consists of a collection of aggregate quantities $\{C_t, Y_t, Z_t, T_t, D_t\}$, aggregate prices $\{w_t, P_t\}$, firm decisions $\{l_{it}, q_{it}, p_{it}, y_{it}\}$, productivity levels $\{z_t, \hat{z}_t, \bar{z}_t\}$, and initial conditions $\{A_0, z_0\}$ such that all firms maximize the discounted present value of their expected profits subject to a tax policy τ , the aggregate labor constraint is met, households maximize their utility subject to their budget constraints, and the labor and goods markets clear.

¹⁶Note that the wage rate differential that generated previous results, like Rauch (1991), is no longer present. This implies that the formal/informal dichotomy can be generated instead using restrictions on innovative activity between the two sectors.

¹⁷This condition results from the fact that initial labor demands were proportional to productivity e^z and were scaled by $\frac{1}{(\rho-1)}$ in firms' production functions.

TABLE 12. Innovation Cost Calibrations

Innovation Costs	b	Target	Avg. q	Actual Growth Rate
Low	3	All firms innovate	1.0000	0.67%
Moderate	5.5	No growth	0.5032	0.00%
High	50	No firms innovate	0.0006	-0.65%

Quantitative Application

This section outlines the parameters and variables utilized in the numerical simulations. The main question that I ask is how do innovation and sector choice affect aggregate outcomes? To do this, I vary tax rates in three different innovation environments: a low cost economy, a moderate cost economy, and a high cost economy. The functional form for innovation costs is He^{bq} as in Atkeson and Burstien (2010). I set $H = .001$ to pin down the level of costs and then calibrate the parameter b , to generate positive, zero growth, and negative growth environments.¹⁸ The innovation costs are calibrated using a baseline 20% tax rate. The lowest cost innovation environment is calibrated such that all firms in the formal sector choose to innovate. I use the highest costs that achieve that criteria. Further decreases in the costs of innovation marginally increase growth but only through more firms switching to the formal sector. Likewise, in the high cost environment, I find the lowest costs in which no firm decides to innovate. Further increases in costs decrease growth, but only on the margin of sector choice. Table 12 outlines the cost structures and growth rates used in the simulations.

Tables 13 and 14 provide information for other variables and values used in the simulations. All of the simulations report results for the first period in which firms operate, after making their sector choice. The elasticity of substitution, ρ , is set to 5 as in Atkeson and Burstien (2010).¹⁹ While this value is fairly standard, Hsieh and Klenow (2010) discuss how even higher values than $\rho = 5$ may be appropriate. The final set of results presents robustness results for other values ρ .

The model is calibrated so that each time period corresponds to a year. Firms anticipate a 10% chance of exit each year. I calibrate the step-size of innovation, Δz , to correspond to 2.7% growth in TFP for the mean firm in the initial draw. This estimate comes from Brandt et al.

¹⁸Because the only source of growth in the model is increases in TFP, the maximum growth rate is determined by the parameter Δz .

¹⁹Hsieh and Klenow (2009) use $\rho = 3$.

TABLE 13. Variables of the Model

Variable	Definition	Variable	Definition
ρ	Elasticity of substitution	$c(q)$	Cost function for innovation
δ	Probability of exit	q_{it}	Probability of successful innovation
τ	Tax rate on profits	\bar{z}_t	Spillover to informal sector
M	Number of firms	Γ	Distribution of z_0
w_t	Wage rate	β	Discount rate of firms
\hat{z}_t	Formal cut-off	Δz	Step-size of innovation
L	Labor Supply	μ	Probability of detection

TABLE 14. Parameter Values

Variable	Value	Source
ρ	5	Atkeson and Burstein (2010)
δ	0.10	Standard
μ	0.10	
τ	Varies	World Development Index 2012
M	10000	
L	100000	See below
β	0.96	
Γ	Uniform distribution	See below
q_{it}	Endogenously determined	
w_t	Endogenously determined	
Δz	0.027	Brandt et al. (2012)

(2012) who estimate the average TFP growth of manufacturing firms in China to be 2.7%. The implied discount rate, β , the exogenous death rate, δ , and the step-size of innovation meet the parameter restrictions to bound the net present value of firms. Specifically, $\beta(1 - \delta)e^{\Delta z} < 1$. If these parameter restrictions are not met, firms would be able to innovate faster than future variable profits are discounted.

The spillover is calibrated using Nataraj (2011).²⁰ She reports mean TFP for both the formal and informal sectors. Using these means and an estimate of the variance, I generate a normal distribution to fit the distribution of log TFP that she reports. I then calibrate the spillover such that the σ percentile of the formal sector generates the mean TFP for the informal sector. I estimate that $\sigma = 48.8$, that is, the 48.8th percentile of the formal sector matches the

²⁰Her data is for pre-reform India in 1989, but it is the only data set I am aware of containing TFP estimates for both formal and informal firms. Hsieh and Klenow (2010) report the distribution of plant size for both informal and formal firms in India, but do not provide firm-level productivity figures.

mean in the informal sector. The value of \bar{z}_t in the model is calculated as the TFP of the 48.8th percentile of firms operating in the formal sector in time period t .

The initial distribution for z is drawn from a uniform distribution centered on $z = 0$. As noted in the introduction, this does not translate into productivity being uniformly distributed. Observed productivity in the model is $e^{z/(\rho-1)}$, so that the distribution of productivity is exponentially distributed.²¹ Importantly, this distribution shares many of the same characteristics of the Pareto distribution, mainly the concentration of firms at the lower tail.²² Further discussion of the initial distribution of firms is left for Discussion section. The number of firms involved in the simulations is set large enough to avoid large fluctuations from utilizing too few firms.

The range of taxes on profits is informed by data from the World Bank's World Development Indicators for 2012. Specifically, it reports the "Total Tax Rate," which is the total tax rate that firms pay as a percentage of their profits. It includes taxes on profits, labor taxes, and other taxes like property and municipal taxes. For a vast majority of countries, these taxes range from 10% to 70% of profits. Figure 5 illustrates the range of tax rates and growth rates for a randomly selected subset of countries included in the data. There are, however, several outliers that have tax rates beyond 100%. These outliers seem to be driven by high "Other Tax Rates," likely reflecting political shocks that are beyond the scope of the current project (like one-time taxes on property).

In addition, the tax rate in the model is inclusive of corruption. It is not quite clear whether this inclusion should raise or lower the actual tax rate faced by formal sector firms. The 2005 World Bank Development report indicates that informal firms paid approximately 2 times more (as a share of sales) in bribes than formal sector firms. On the other hand, the incidence of paying bribes in the informal sector was only about 50% of that in the formal sector. Given these ambiguous factors, as well as the broad range of bribes and corruption documented in Olken and Pande (2012), I take a conservative approach and suggest that the presence of corruption may add

²¹This assumption is no different than assuming an exponential distribution for the initial productivity draw and allowing firm productivity to be simply z_{it} .

²²Unlike in the previous chapter, I do not trim the tails of the distribution. The initial distribution of productivity allows for large firms as seen in the data, but still mitigates individual firms from having first-order effects. The number of firms relative to the number of possible starting productivities is large such that there is usually more than one firm at any given level of productivity. This fits with firms' behavior in that they do not consider changes in aggregate variables stemming from their own behavior.

FIGURE 5. Total Tax Rate (% of Profits) vs. GDP Growth Rate (Annual %)



Source: World Development Indicators 2012. The scatter plot includes 81 countries that have available data on total corporate tax rates and GDP growth for 2012. Countries with total tax rates above 100% are dropped. I also randomly exclude half the sample for clarity and visibility.

from -5% to 5% to the range of taxes in the formal sector. In totality, I look at tax rates (τ) from 5% to 75% in the formal sector.

Finally, q is endogenously determined since firms choose the likelihood with which their research and development is successful. However, in Atkeson and Burstein (2010) the value of q is calibrated for large firms in order to keep the dynamics of those firms constant through time. In the current model, *all* firms make decisions regarding how much to innovate. In both the high cost and low cost environments, there is little to no heterogeneity in firms' choice of q .

Results

The results are presented in four sections. The first section explores and develops intuition for how the model operates. Specifically, it illustrates the distribution of productivity resulting from firm sectoral choice and the spillover of technology. The next set of results investigates how innovation affects the sector decisions of firms. It clearly shows that changes in tax rates have sizable impacts on the size of the informal sector. The third section looks at the aggregate effect of the changes in sector choice. I find that changes in the tax rate have significant impacts on the size of the informal economy, TFP growth, and tax revenue. The final section tests the robustness of the model to changes in the substitutability of intermediate firms' goods. All results are for $t = 1$, the first period in which firms realize their sector decisions and operate in their chosen sector. Therefore, changes in innovation costs and tax rates should be seen as changing this initial sector decision and not a reallocation within the first period.

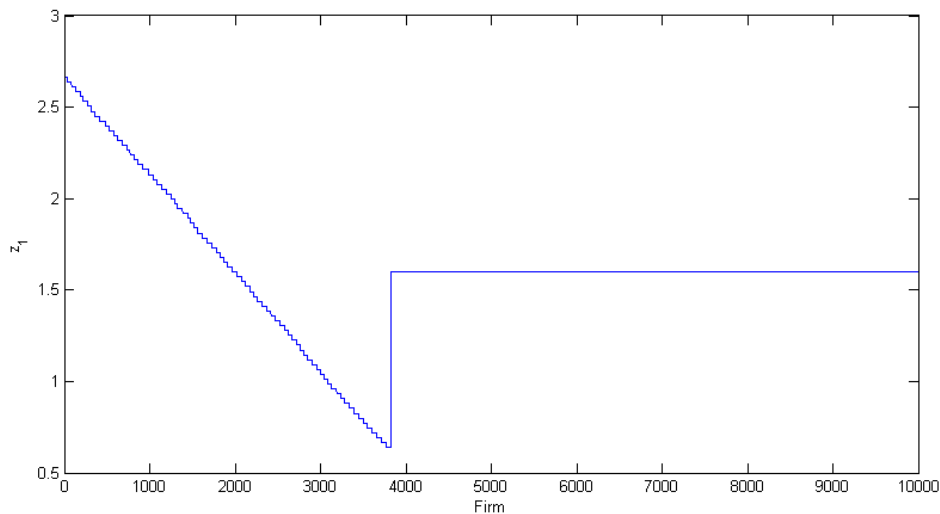
Developing Intuition

It is important to develop some intuition for how firms enter each sector. Importantly, firms that decide to enter the informal sector forgo the ability to innovate and instead are endowed with a productivity spillover from the formal sector, \bar{z}_t , that is derived from the formal sector. Figure 6 illustrates how this assumption operates. Ordering firms from highest productivity to lowest productivity within each sector, firms with higher productivity enter the formal sector. This can be seen on the left-hand side of Figure 6. The right-hand side of Figure 6 is populated by lower productivity firms that operate in the informal sector. Importantly, notice that the cut-off level of firms operating in the formal sector is lower than \bar{z} , implying that some firms in the

formal sector are less productive than some firms in the informal sector. This can be seen by the discrete jump in Figure 6. The firms that operate in the formal sector below \bar{z} anticipate the gains from innovation to offset the taxes they pay in the formal sector. Since these firms produce a unique intermediate good, they are not driven out of the market.

The effect of this assumption is to create a large mass of firms at the lower end of the distribution of productivity, as can be seen in Figure 7. Since firms hire labor proportionally to e^z , Figure 7 also approximately shows the distribution of firm size. For now, it is important to note that there will be a large number of relatively small firms. Further discussion of alternative distributions for initial productivity and the desirability of this distribution is left for the Discussion section.

FIGURE 6. Firm Productivity

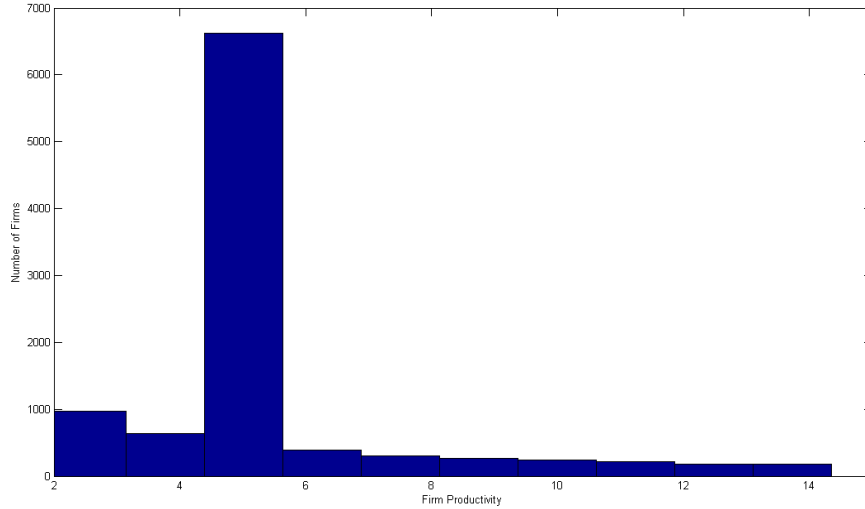


The figure is generated using 10,000 firms that face a 20% tax rate on profits in the formal sector. Innovation costs are moderate ($b = 5.5$) and $\rho = 5$. Note that observed productivity in the economy is actually $e^{\frac{z}{\rho-1}}$, which means that $z = (\rho - 1)\log(\text{observed TFP})$. In the figure above, 61.3% of firms operate in the informal sector.

Informality and Innovation

In order to discuss impacts on aggregate variables, it is first necessary to illustrate the role that innovation has on firm sector choices. If innovation does not have a significant impact on firm choice, the complexity added through integrating innovation within a model of monopolistic competition is likely not an improvement over earlier static models of sector choice. Figure 8 plots

FIGURE 7. Distribution of Firm TFP



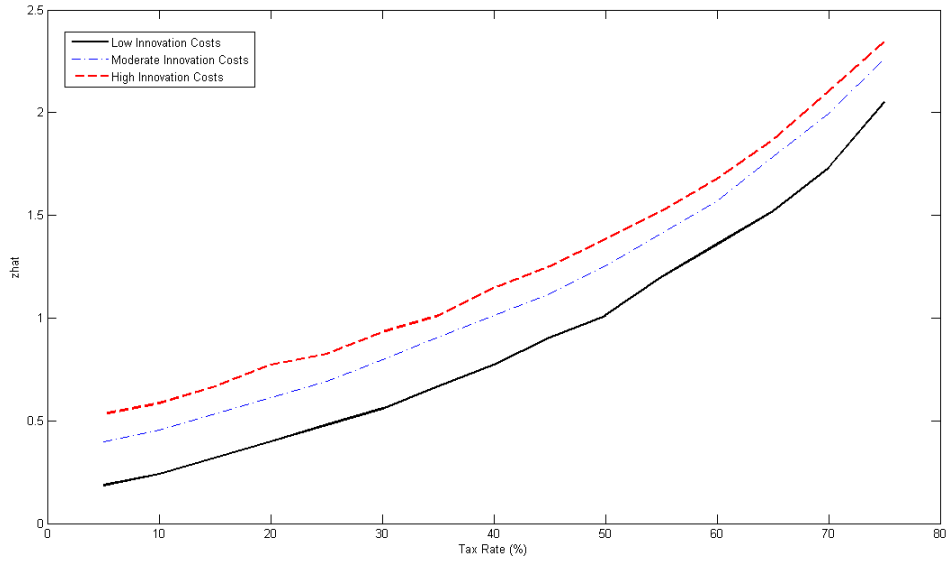
The figure is generated using the same parameter values as in Figure 6. Firm TFP is $e^{z_{it}}$.

the cut-off value of \hat{z} for firms to enter the formal sector. Three different costs for innovation are implemented so that firms pursue different amounts of research and development as described in Table 12.

The value of \hat{z} with high innovation costs is always higher than \hat{z} under low costs to innovation. Since \hat{z} represents the lowest value of productivity that firms would need to enter the formal sector, lower values of \hat{z} correspond to additional firms entering the formal sector. Lowering the cost of innovation raises the valuation of firms in the formal sector who can pursue innovation to increase their future productivity and profits.

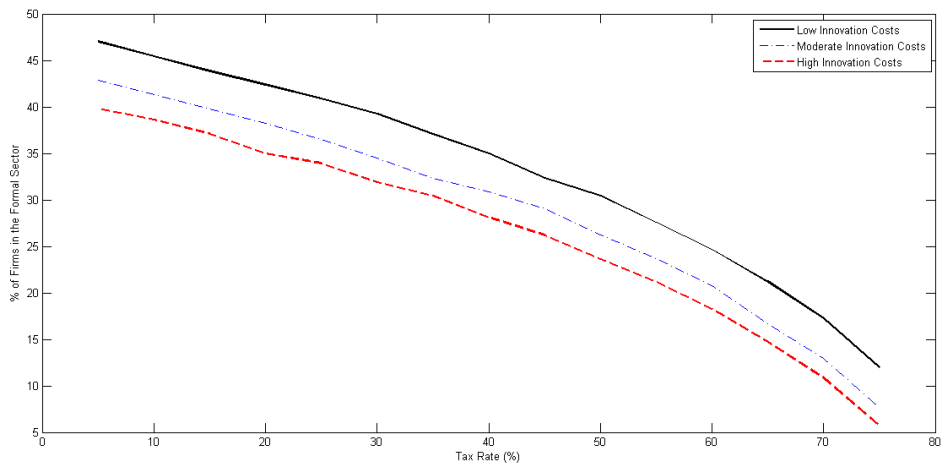
These results are corroborated in Figure 9. Reducing the cost of innovation increases formal sector participation at every tax rate. More firms opt to improve their production processes rather than simply produce using the productivity spillover from the formal sector. These changes in sector size over the different tax rates are quite sizable. In the moderate cost innovation environment, increasing the tax rate from 5% to 75% implies a roughly 82% reduction in the size of the formal sector. Smaller changes in the tax rate have important effects too. For instance, a cut in taxes from 50% to 40% would imply a 17.6% increase in the size of the formal sector. This process is augmented by the gradual increase in the size of the spillover as firms migrate

FIGURE 8. Cut-off Value of Initial Productivity for Entry into the Formal Sector



Innovation costs are as referenced in Table 12. The tax rate corresponds to τ in the model.

FIGURE 9. Percentage of Firms Operating in the Formal Sector

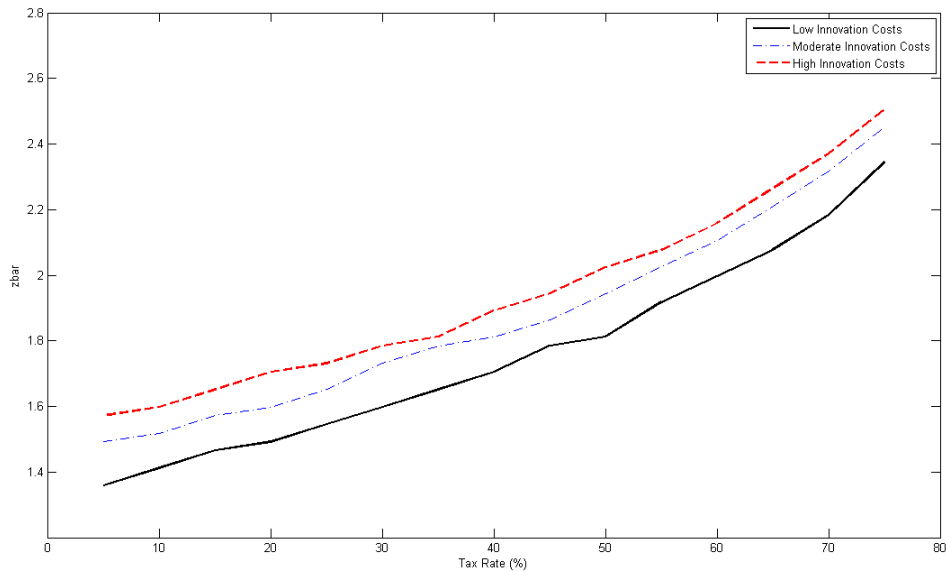


The tax rate corresponds to τ from the model.

to the informal sector. Notice that at higher levels of τ , firms opt into the informal sector at an increasing marginal rate.

Suppose that taxes are increased just enough to persuade one additional firm to be informal rather than formal. This marginal change affects the size of the spillover from the formal sector. As the lowest productivity firm in the formal sector switches to the informal sector, it raises the average productivity in the formal sector, increasing the spillover, \bar{z}_t , to the informal sector. This process is illustrated in Figure 10.

FIGURE 10. Productivity Spillover to Informal Firms

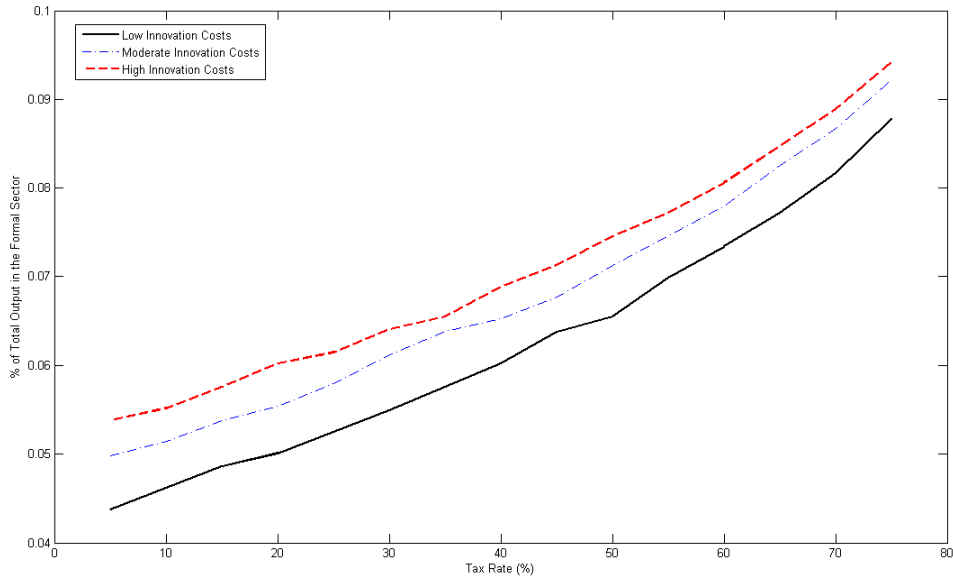


An immediate effect of the technology spillover is that higher costs of innovation lead to higher productivity levels in the informal sector. This feature has two intuitive interpretations. First, it reflects the fact that in economies with higher costs of innovation, more productive entrepreneurs may eschew innovation and instead evade taxes in the informal sector. Second, it also implies that higher innovation costs lead to a smaller dispersion of productivity. While it may be counter-intuitive that the informal sector ought to be more advanced in a society with higher innovation costs, this effect is temporary. Firms in the low cost economy innovate with $q = 1$, while firms in the moderate cost economy innovate with $q = .5023$. Since greater values of q correspond to greater innovation and TFP growth, the lower the innovation cost today, the higher the productivity of formal sector firms in the future.

Informality and Aggregate Effects

As the tax rate increases, the ratio of output in the informal sector to total output increases, as can be seen in Figure 11. In absolute terms, however, the amount of output that is produced in the informal sector is relatively small when compared to data on the size of informal sectors in developing countries, as seen in Table 11. Informal firms, with their smaller productivity levels of \bar{z} , operate on a smaller scale than their formal counterparts. They are, however, more numerous under all of the specifications I evaluate. Further discussion of this result is left for Section 3.5.

FIGURE 11. Percentage of Output Produced in the Informal Sector

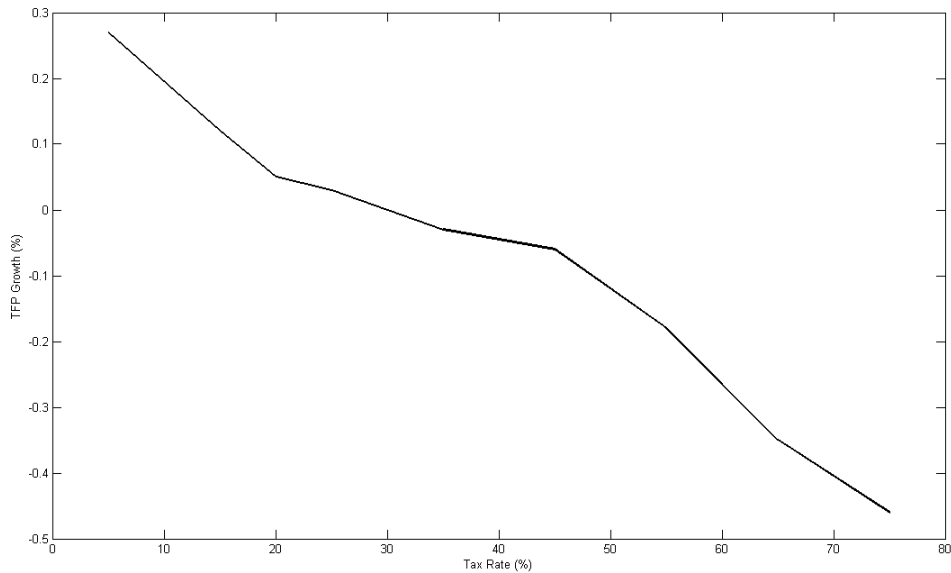


The figure above illustrates the percentage of total output produced in the informal sector across innovation and tax rates.

In certain innovation environments, the tax rate also changes the rate of TFP growth. This process can be seen in Figure 12. The mechanism for decreasing TFP growth is the innovation rate of firms. As taxes increase, the future value of innovation decreases, and firms invest in innovation less. However, in both the high and low cost environments, the costs of innovation dominate changes in the tax rate such that the innovation rate does not change, and hence the TFP growth rate does not change. In those settings, as τ increases, firms flee the formal sector,

leaving fewer innovators, as seen in Figure 9. This process also drives up the spillover to the informal sector. The combination of these effects, without any accompanying change in innovation rates among formal sector firms leads to no change in TFP growth. Fortunately, firms in the real world are not likely at either of those bounds and do change their innovation activities in response to changes in corporate tax rates.

FIGURE 12. Growth in Aggregate TFP



Changes in the aggregate TFP growth rate are calculated for moderate innovation costs as outlined in Table 12.

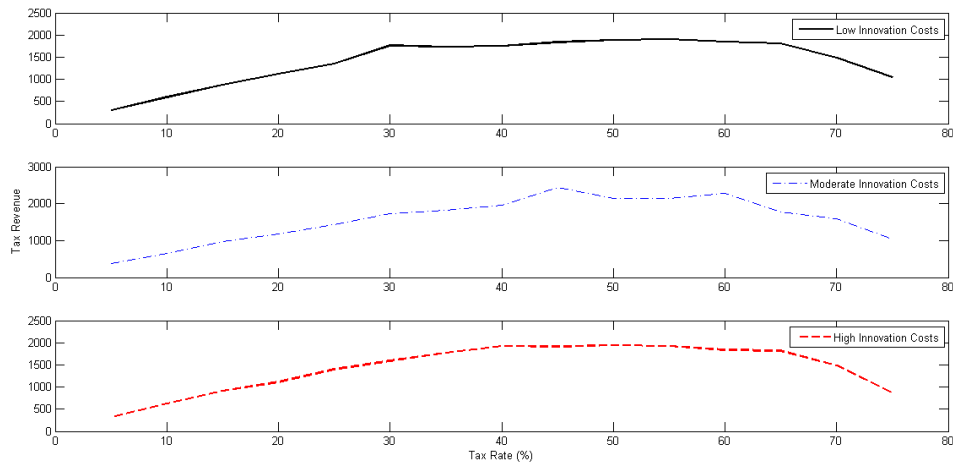
Despite the small level of output in the formal sector, the impact of taxation plays a visible role in tax revenues. As firms leave the formal sector in the face of higher taxes, the tax base decreases. Figure 13 illustrates a Laffer curve relationship, where marginal tax revenue is decreasing with the tax rate. Significantly, for tax rates of 30% to 40% tax revenue stagnates and possibly decreases, depending on innovation costs.

How should these results be interpreted with regard to optimal policy? The model is able to suggest the effects of changes in tax policy with regard to formal sector participation, TFP growth, and tax revenue. These effects should be considered local to an existing tax policy. For instance, an economy operating with a tax rate of 20%, considering a 10% increase in taxes, should expect to see formal sector participation and TFP growth to fall, but tax revenues to

increase. Given that government does not play a direct productivity enhancing role here, the model cannot determine an *optimal* policy, but rather the possible trade-offs that a policy maker faces. In addition, it is important to realize that these results assume that only tax policy changes, holding fixed all other parameters, including innovation costs. It is likely the case that lower tax rates may affect the ability of the government to enforce property rights, raising innovation costs. This would work against the current results as a tax cut would also raise innovation costs, lowering the incentives for firms to participate in the formal sector (essentially combining to the two dimension in Figure 9).

The results of the model illustrate that including innovation costs into firms' decisions has an important impact on their sector decision. This decision, however, has small implications for tax revenue due to the small amount of output produced by informal forms relative to formal firms. The next section discusses several elements in the model that are principal drivers of these results and the anticipated effects of parameter changes.

FIGURE 13. Total Tax Revenue across Tax Rates.

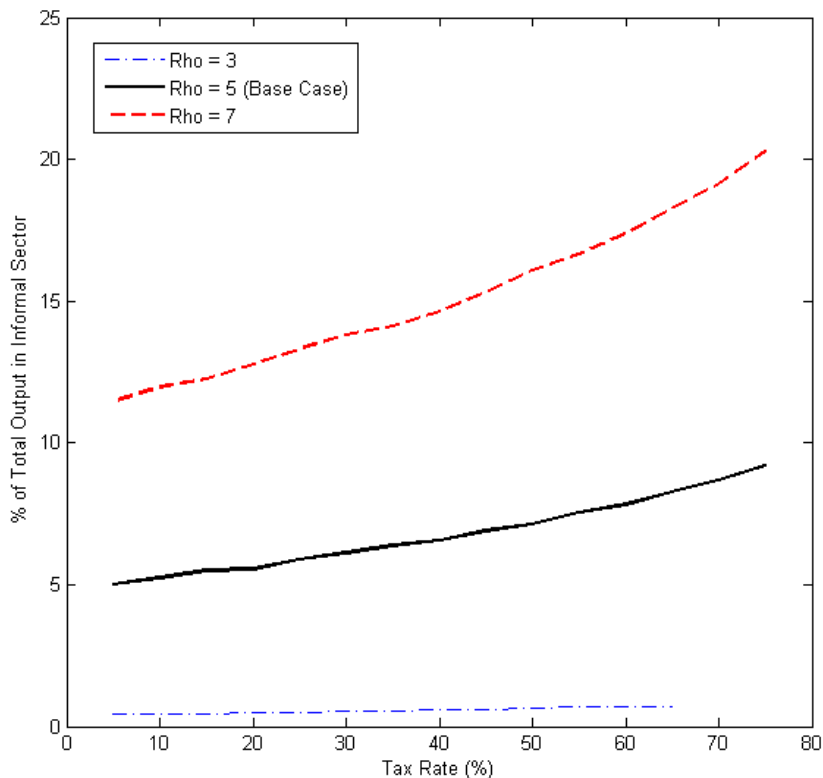


Robustness Checks

Below I investigate the effects of changing the value of ρ in the simulations. The parameter ρ governs the substitutability of different intermediate goods in the production of the final good. As ρ increases, the substitutability between goods increases, decreasing market power and the

incentives for firms to innovate in the formal sector. This result is illustrated in Figure 14.²³ Notice that the percentage of total output produced in the informal sector increases with ρ . In the case of $\rho = 7$, the informal economy has a much larger impact, constituting 11.5% to 20.2% of the economy.

FIGURE 14. Percent of Total Output Produced by Informal Firms

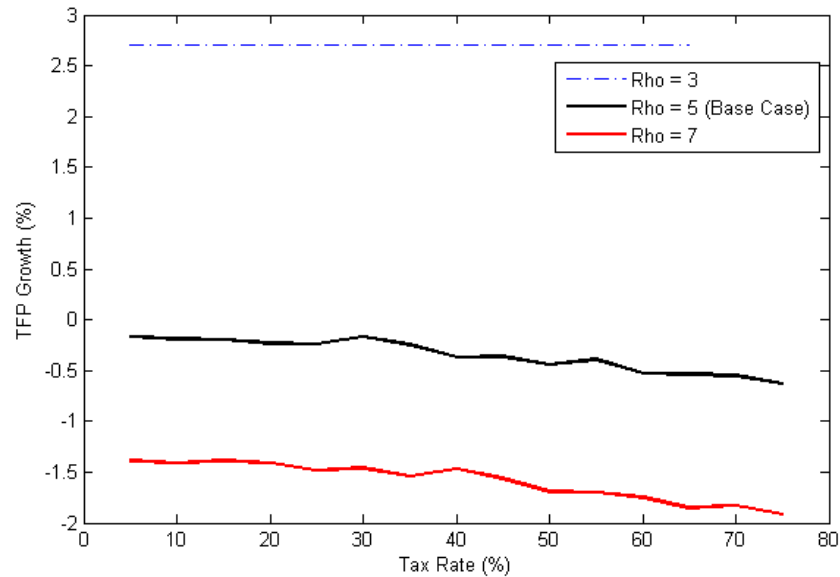


Changes in ρ also affect TFP growth, as can be seen in Figure 15. The case of $\rho = 3$ is unique. In that case, changes in the tax rate are dominated by incentives to innovate. Mainly that given the costs, lowering ρ increases the profitability of innovating, as static profits are proportional to productivity in the model. This is a very similar set of circumstances that explained why TFP growth did not vary for $\rho = 5$ in the high and low cost innovation environments. Raising the value of ρ lowers the level of TFP growth as firms are less profitable

²³The curve for $\rho = 3$ does not appear for levels of $\tau \geq .65$ in either Figure 14 or Figure 15. For higher tax rates, the simulation does not converge. The high tax rate and low level of ρ create a situation in which all firms want to innovate, driving up the value of \bar{z} . This in turn drives a large portion of firms into the informal sector, driving \bar{z} back down. This process continues indefinitely.

because their market power diminishes. Overall, changes in the value of ρ have predictable effects resulting from static profits decreasing as ρ increases. This translates into decreasing the profitability of innovation.

FIGURE 15. TFP Growth across Values of ρ



Discussion

Below, I focus on three areas of particular concern or interest: the parameterization of the initial distribution of productivity, the formulation of the technology spillover to the informal sector, and the assumption of firms making a single entry decision. These areas seem to be responsible for both the desirable qualities of the model, as well as some of its potential shortcomings.

The parameterization of initial productivity in the model is a particularly nebulous issue. In most developing countries, the distribution of productivity generates a firm-size distribution with a large number of small firms as documented in Tybout (2000). In the model, firm-size is linked to firm productivity such that less productive firms are also smaller. In this sense, the current parameterization captures the ubiquity of small firms that is documented by Tybout (2000). Choosing a realistic initial distribution hinges on two competing concerns. First, the dispersion of productivity is responsible for determining how relevant the informal sector is. A

smaller dispersion of productivity implies that firms in the informal sector will produce a greater percentage of total output. Drawing the initial productivity from a very narrow range of z would increase the significance of informal firms and possibly amplify declines in tax revenue as tax rates increased, accentuating the Laffer curve in Figure 13. A greater dispersion of productivity may also introduce further problems as the decisions of informal firms may affect aggregate outcomes, implying the need to extend the model to incorporate these very large firms.

On the other hand, the desire to calibrate the distribution to match data regarding the size of informal economies must be tempered by a realistic assessment of how productive formal sector firms are compared to informal sector firms. As the 2013 World Development Report points out, few countries collect reliable data on the informal sector, and there are few reliable estimates of the true ratio of productivity between the sectors. Hsieh and Klenow (2009) establish a lower bound by reporting that the ratio log TFP for the 90th percentile to the 10th percentile is 5.0 for India and 4.9 for China. These numbers represent a lower bound since both small and informal firms are excluded from their data sources.

As a means for comparison, in a simulation with moderate innovation costs and a tax rate of 20%, the ratio of the highest productivity firm to the productivity of firms in the informal sector (based on \bar{z}) is 1.67, and the ratio of the highest TFP to lowest TFP is 4.17. Especially with regard to the second statistic (which is most similar to that of Hsieh and Klenow (2009)), the initial productivity draw seems entirely plausible. The first statistic is more difficult to compare since, to my knowledge, there is no analog in the literature.²⁴ The World Bank Enterprise Survey (WBES) has collected data on both formal and informal sector firms, but those data sets are usually collected in different years.

It is important to note that as reported in Nataraj (2011) the model does generate an overlap in productivity between the formal and informal sector, by design. This element is not seen in early models of sectoral choice. In those models, such as Rauch (1991), there is a strict dualism where firms in the informal sector are always less productive than the least productive formal firms.

²⁴This measure is tied to the size of the spillover from the formal to informal sector. The calibration of the spillover is extrapolated from Nataraj (2011) as described in Section 3.3. However, even her database of firms is based on 1989 data.

Closely related to the choice of initial distribution of firm productivity is how productivity spills over to the informal sector. Decreasing the size of the spillover, for instance, by endowing firms in the informal sector with the lowest quartile of productivity in the formal sector, would decrease the incentives to switch sectors when taxes increase. At the same time, such a change would increase the ratio of productivities between the sectors by lowering productivity in the informal sector.

A larger question: does such a spillover make sense? The nature of the spillover is intended to capture the fact that firms in the informal sector tend to adopt changes without investing as much in innovation. In this sense, their productivity improves over time, but is not necessarily at the technological frontier. For instance, evidence from the World Enterprise Survey indicates that smaller (highly correlated with informal) firms are less likely to utilize e-mail. This indicates that some firms choose to adopt new technologies that are already commonplace in the formal sector. Informal sector firms did not invent e-mail or revolutionize its applications, but they adopt its usage to improve their productivity when they see its widespread usage in the formal sector.

The answer to how realistic the specification of the spillover is ultimately hinges on country-specific context. For instance, informal production in some countries may resemble relatively simple home production. This production may occur in rural areas that are not in close proximity to dense manufacturing areas. On the other hand, in places like India, some data indicates that there is a complementarity between production in the formal and informal sectors. Sundaram et al. (2012) document a strong positive correlation between factor movements in the formal sector and informal sector. They conclude that there is likely a strong complementarity between the sectors. In this case, spillovers may be larger than specified in the model.

Part of the complementarity that Sundaram et al. (2012) document is the ability of the informal sector to absorb excess labor and provide employment for workers who cannot find work in the formal sector. One reason the model does not accurately reflect the size of informal economies is that labor is the sole factor of production. Informal firms tend to be more labor intensive, while formal sector firms tend to be more capital intensive. Introducing a complementarity between the two sectors may increase the size of the informal sector so that it better reflects the available data.

Finally, it is worth discussing the validity of looking at firms' sector decisions, assuming that they stay in that sector rather than switching in a later period. While this assumption is made principally to isolate the role of innovation on firms' decisions, there are reasons to suggest that there are barriers to switching sectors. Nataraj (2011) reports that few firms in India switch from formal to informal despite having fewer employees than is necessary to be required to register as a formal firm. Additionally, barring a bad series of innovation shocks, the incentives for firms that reach high enough productivity levels to have them opt into the formal sector, would still seek to stay there. On the informal side, there is substantial data to suggest that there are large barriers to entry for the formal sector that are not explicitly modeled here. These costs would be incurred in addition to a higher tax rate, and they may deter firms from switching sectors.

Conclusion

This paper investigates how firms interact with government institutions to determine whether to operate formally or informally. Not only are firms' decisions shown to be significantly affected by taxes and innovation costs, but their sectoral choices also have important impacts on aggregate variables such as TFP. Considering the importance of TFP in determining cross-country income differences, understanding how firms' sector decisions affect TFP growth is a positive step in understanding the process of development.

By modeling how firms interact with institutions, make innovation decisions, and decide which sector to operate in, I am able to generate relevant policy implications. Specifically, governments discourage TFP growth through taxation by pushing firms into the informal sector and lowering the innovation rates of formal sector firms. Secondly, institutions that lower the costs of innovation are better for enticing firms to operate formally. Finally, the model indicates that some developing countries operate at tax levels beyond the level that maximizes revenue and at the detriment of participation in the formal sector. Ultimately, this research underscores the role of institutions and government policy in shaping the incentives of individuals and firms. Given the right incentives, these individuals and firms are able to generate income and innovations to aid in the process of development.

Supplementary Materials

This section outlines the derivation, calculation, and assumptions for calculating aggregate output growth. Let the aggregate growth rate be designated

$$g_Y = \frac{Y_{t+1} - Y_t}{Y_t}. \quad (3.21)$$

Substituting (3.19) in for Y_t and its equivalent for Y_{t+1} , yields (3.20). Recall that

$$Z_t = \sum_i e^{z_{it}}, \quad (3.22)$$

for all firms, both formal and informal. Aggregate productivity can be split into formal and informal sectors as

$$Z_t = \sum_{\hat{z}}^{z_{max}} e^{z_{it}} + \sum_{z_{min}}^{\hat{z}} e^{\bar{z}_t}, \quad (3.23)$$

with a slight abuse of notation with the indexing on the sums. The formal sector is composed of all firms with draws of $z \in [\hat{z}, z_{max}]$. Recall that \bar{z}_t is equal to the σ percentile of productivity in the formal sector. Let ω be the number of firms that participate in the formal sector. Similarly, let ξ be the number of firms that operate in the informal sector. Aggregate TFP at time t then is

$$Z_t = \sum_{\hat{z}}^{z_{max}} e^{z_{it}} + \xi e^{\bar{z}_t} \quad (3.24)$$

A similar expression can be derived for the expected value of Z_{t+1} . Given the process of innovation outlined in Section 3.2, firm's expected productivity in the formal sector is

$$E_t e^{z_{it+1}} = q_{it} e^{z_{it} + \Delta z} + (1 - q_{it}) e^{z_{it} - \Delta z}. \quad (3.25)$$

Thus, $E_t Z_{t+1}$ can be written similarly to (3.24):

$$E_t Z_{t+1} = \sum_{\hat{z}}^{z_{max}} E_t e^{z_{it+1}} + \xi E_t e^{\bar{z}_{t+1}}, \quad (3.26)$$

subject to (3.25). Notice that this result utilizes the fact that firms make a decision to enter a given sector under assumption that they will stay in that sector. Under this assumption, the

distribution of firms into each sector, mainly the parameters ω and ξ are fixed. Combining equations (3.20), (3.24), and (3.26) allows for the calculation of expected output growth. Expected output growth is a function of the distribution of firms productivity, z_t , the cut-off value \hat{z} that determines ω and ξ , and firms' choices for innovation, q_{it} .

CHAPTER IV

INFORMALITY, TAX POLICY, AND THE PROVISION OF PUBLIC GOODS

Introduction

The decision of firms to be informal is most often motivated by the desire to avoid formal sector regulations like minimum wage laws and taxation. Seminal works on explaining informality, like Rauch (1991), focus on the negative role of government on formal sector participation. The fact remains that some government policies serve a positive role in aiding firm production and promoting formal sector participation. In order to understand the policy implications of taxation on the provision of public goods and formal sector participation, a more complete formulation of government is necessary.

For this chapter I ask: how does the trade-off between taxation and public good provision determine formal sector participation, and what are the tax revenue implications resulting from these policies? There exists a natural trade-off in terms of formal sector participation that currently does not exist in the literature on informality. Mainly, formal sector firms benefit from the provision of public goods even while they face regulation and taxation. This trade-off, in addition to the ability to innovate which I investigate in the previous chapter, provides very direct policy implications with regard to optimal levels of public expenditure and taxation.

I construct a general equilibrium model where firms choose whether to participate in the formal or informal sector. These choices are influenced by two key elements of government policy: the provision of public goods and the levying of taxes. I calibrate the model and conduct numerical experiments to investigate the revenue-maximizing tax rate. A base case simulation indicates that tax rates of approximately 47% maximize government revenue, while a series of robustness checks indicate a range of revenue-maximizing rates from 39% to 48% for reasonable parameter values. These results are most notable for the avenue through which tax rate changes affect total revenue. It differs from the typical supply-side mechanism investigated elsewhere in the literature and is unique to the context of developing countries. This avenue relies on the decisions of firms to operate in the formal sector and pay taxes.

This line of research is best seen as the intersection of three branches of literature: the literature on informality, the literature on the role of public goods in the macroeconomy, and

the literature on optimal tax policy. The literature on informal firms has generally studied the negative effects of government institutions on formal sector participation. The seminal work in the area, Rauch (1991), analyzes a static model where firms seek to avoid an enforceable minimum wage law in the formal sector. Only the most productive firms enter the formal sector so that they are able to hire their profit-maximizing amount of labor. Empirical work has also emphasized the negative role of institutions on formal sector participation. Dabla-Norris et al. (2008) document that informality increases with tax burden and decreases with the quality of legal institutions. Their work echoes previous research by Loayza (1996) that showed informal economies were larger in countries with greater tax burdens and labor-market regulations and smaller in countries with stronger institutions.

Macroeconomic models have regularly incorporated public goods into both exogenous and endogenous growth models.¹ These models typically emphasize the role of public goods in augmenting firm production. Other work has evaluated the provision of public goods in a broader institutional context, which is similar to the goals of this chapter. Acemoglu (2005) considers a model where a political elite controls the provision of public goods and only provides them if it increases its utility. These elites also control tax rates. He shows that weak states have fewer incentives to provide public goods. In a similar vein, Chakraborty and Dabla-Norris (2011) analyze a model where the quality of public goods is affected by a corrupt bureaucracy. Importantly, these works have not considered the role of informal firms in this context.

There is a considerable literature on both optimal tax policy and revenue-maximizing tax rates. Optimal tax policy has usually focused on the most efficient tax schedule to raise a given level of revenue. Starting from the foundational work of Mirrlees (1971), papers on optimal tax policy have varied widely in their findings and focus. Optimal tax policy has been found to depend on everything from underlying utility specifications to the type of tax (what factor of production is taxed).² Many of the lessons from this literature are synthesized in Mankiw et al. (2009). The optimal tax literature tends to use more advanced tax schedules than those evaluated in this chapter.

¹See Barro and Sala-i-Martin (2004) chapters 3.1 and 4.4, respectively, for standard examples.

²See Chamley (1986), Judd (1999), and Saez (2001) for a sampling of the literature and a discussion of prominent issues.

It is important to note that the optimal tax policy literature has a different policy focus than the current work. This chapter seeks to find the tax rate that maximizes total tax revenue. This exercise amounts to calculating the peak of the Laffer Curve. Estimates in the literature for this peak vary, but there does seem to be limited consensus for a peak near 70% in developed countries.³ The mechanism in developed countries may differ substantially from the developing world. Most estimates of the Laffer Curve rely on a supply-side mechanism whereby individuals cutback their labor supply in response to taxation, decreasing output and, eventually, tax revenue. However, the prevalence of informal economies in the developing world provide an additional avenue for this effect that has not been specifically addressed in the literature.

This chapter emphasizes the differential usage of public goods between the formal and informal sector. In most contexts, informal firms seek to hide their economic activity. This requires, in many instances, that they cannot utilize public goods to the same extent as formal sector firms. Informal firms have less access to law enforcement and legal institutions to settle disputes. This channel, however, is already captured in the previous chapter through the enforceability of the tax code.

Beyond this law enforcement channel, formal firms also enjoy access to preferential credit, social welfare programs for their workers, and skill training programs. Most significantly, formal sector firms face no restriction on the use of electricity outside of capacity and cost constraints. Informal sector firms face a higher probability of being discovered by utilizing electricity, which is a fundamental input into most manufacturing processes. In the case of India, the role of electricity in defining informality is actually codified. Firms of fewer than 10 people are considered informal if they utilize electricity. If not, firms can remain informal while employing up to 20 workers according to the Indian Factories Act of 1948. At an even more basic level, the use of road systems and other basic infrastructure may exclude informal firms through required use of licenses.

Given the practical limitations of informal firms utilizing public goods, calibrating and simulating a model that incorporates a public good is a necessary step in understanding the policy implication of taxation in the formal sector. The trade-off between taxes and the usage of public goods adds an interesting element to entry into the formal sector, fills a gap in the literature, and

³See for instance Stuart (1981) for the Laffer Curve in Sweden.

generates a more complete understanding of the role of government policy in determining formal sector participation and its effects on tax revenue. The Laffer Curve generated in this chapter relies on a new channel that has not previously been investigated in the literature.

The remainder of this chapter proceeds as follows. The next two sections outline the model and provide underlying intuition. The following two sections conduct a quantitative exercise and evaluate the results. I then discuss areas of difficulty in the current model and avenues for future research. The final section concludes.

Model

Aggregate Economy

The aggregate economy consists of a final good produced in a perfectly competitive market. The final good is a CES aggregate of M intermediate goods produced by monopolistically competitive firms:

$$Y_t = \left(\sum_{i=1}^M y_{it}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (4.1)$$

where y_{it} is the output of intermediate good producer, i , and $\rho > 1$ is the elasticity of substitution between goods. Final good producers choose their output to maximize profits subject to the prices of inputs, p_{it} , and the price of the final good, P_t , which is normalized to 1. Standard profit maximization on the part of final good producers determines the demand curve faced by intermediate good producers:

$$p_{it} = \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}}, \quad i = 1, \dots, M. \quad (4.2)$$

Intermediate Good Producers

Intermediate good producers supply unique inputs into the production process of the final good. Each firm produces only one type of good. Their output is sold in monopolistically competitive markets, where each firm's market power is determined by the parameter ρ . Intermediate good producers must make decisions about which sector to operate in (formal or informal), the price of their product, how much to invest in innovation, and how much labor to hire. They make these decisions subject to (4.1), (4.2), and their initial productivity draw,

z_{i0} . Firms from both sectors, formal and informal, compete in the same market subject to the conditions above.

The Formal Sector

Firms in the formal sector benefit from unfettered access to a public good and the ability to innovate, but must pay taxes which are perfectly enforceable. Firms in the formal sector (F) produce output using constant returns to scale technology:

$$y_{it}^F = z_{gt}^F e^{z_{it}/(\rho-1)} l_{it}, \quad (4.3)$$

where z_{it} is a firm specific productivity parameter and l_{it} is the amount of labor. Throughout, production and innovation decisions are made for $t \geq 1$ given a $t = 0$ initial draw of productivity. Further description of the determination of z_{gt}^F , a formal sector firm's productivity enhancement from the public good, is reserved for Section 4.2. Firm productivity in (4.3) is scaled by $\frac{1}{(\rho-1)}$ as in Atkeson and Burstien (2010). Conveniently, this scaling makes labor decisions and static profits proportional to firm specific productivity, $e^{z_{it}}$.

Firms in the formal sector earn static profits of

$$\Pi_t^F(z) = \max_{p_{it}, l_{it}} (1 - \tau) (p_{it} y_{it} - w_t l_{it}), \quad (4.4)$$

where τ is the tax rate on profits. Given the static level of profits in (4.4) and subject to (4.2) and (4.3), firms in the formal sector set their price as a constant mark-up over marginal cost:

$$p_{it}^F = \frac{\rho}{\rho - 1} \left(\frac{w_t}{z_{gt}^F e^{z_{it}/(\rho-1)}} \right). \quad (4.5)$$

Finally, firms in the formal sector hire labor according to a static first order condition:

$$l_{it}^F = w_t^{-\rho} \left(\frac{\rho - 1}{\rho} \right)^\rho Y_t e^{z_{it}} (z_{gt}^F)^{\rho-1}. \quad (4.6)$$

Firms in the formal sector have the ability to improve their future productivity through innovation. A formal sector firm that invests $e^z c(q)$, where $c(q)$ is the cost function of innovation, has a probability q of having productivity of $e^{z+\Delta z}$ and probability of $1 - q$ of having productivity

of $e^{z-\Delta z}$ in the next period. The cost function, $c(q)$ is increasing and convex in q . Firms choose a value of $q \in \{0, 1\}$, which determines the probability that an innovation is successful. Note that the cost of innovation scales with the productivity level of the firm. This implies innovation for more advanced firms is more costly than innovation among less advanced firms. The firm-level innovation process is similar to Atkeson and Burstein (2012).

The Informal Sector

Firms in the informal sector completely avoid taxation, cannot innovate, have less access to the public good, and face a constant probability of being discovered, fined, and closed. The productivity level of firms in the informal sector is endogenously determined by the productivity distribution of firms in the formal sector. Let the productivity level shared by all informal sector firms be denoted \bar{z}_t . This spillover is maintained in the model to empirically discipline the relative productivity of the two sectors.⁴ Firms in the informal sector operate with this productivity parameter in place of the ability to innovate and improve their initial $t = 0$ productivity draw.⁵ Since this parameter is time-dependent, entrepreneurs making their sectoral choice decisions must forecast the value of $\{\bar{z}_t\}_{t=1}^{\infty}$.

Under these conditions, firms that decide to enter the informal sector (I) earn operating profits of

$$\Pi_t^I(\bar{z}_t) = \max_{p_{it}, l_{it}} (1 - \mu)(p_{it}y_{it} - w_t l_{it}), \quad (4.7)$$

where output is the same as the formal sector with the exception of the specification for the public good and the firm-specific productivity parameter:

$$y_{it}^I = z_{gt}^I e^{\bar{z}_t/(\rho-1)} l_{it}. \quad (4.8)$$

The parameter μ in (4.7) captures the probability that an informal sector firm is caught operating informally. Upon being caught, the firm is fined its entire profits for that period and is forced to

⁴An alternative strategy would be to use the productivity enhancement from the public good to account for these differences. However, investigating changes in the parameters would then necessarily require a departure from the data.

⁵This assumption is mainly for simplification. Informal firms could innovate, likely with higher innovation costs, and there would still be incentives to participate in the formal sector and receive a greater productivity boost from the public good.

close. This parameter reflects the finding in Dabla-Norris et al. (2008) that informality decreases with the strength of legal institutions. Informal firms charge prices

$$p_{it}^I = \frac{\rho}{\rho - 1} \left(\frac{w_t}{z_{gt}^I e^{\bar{z}_t / (\rho - 1)}} \right). \quad (4.9)$$

Their first order condition for labor is

$$l_{it}^I = w_t^{-\rho} \left(\frac{\rho - 1}{\rho} \right)^\rho Y_t e^{\bar{z}_t} (z_{gt}^I)^{(\rho - 1)}. \quad (4.10)$$

Government

Government in the model is intended to capture several important types of policies in developing countries. As in the previous chapter, the government collects tax revenue, T_t , through levying a tax rate, τ , on the profits of firms in the formal sector. It is also charged with enforcing the tax code in the informal sector where it conducts audits that discover informal firms with probability μ . The tax rates and probability of being caught in the informal sector are known to all firms.

Importantly, the government also provides a public good that augments production in both sectors, albeit to different degrees. The public good enters formal firms' production function as

$$z_{gt}^F = g_t^\lambda, \quad (4.11)$$

where

$$g_t = \left(\frac{\epsilon T_t}{Y_t} \right), \quad (4.12)$$

and $\epsilon \in [0, 1]$ and $\lambda \in [0, 1]$. Informal sectors receive a smaller productivity enhancement such that

$$z_{gt}^I = \gamma g_t^\lambda, \quad (4.13)$$

where $\gamma \in [0, 1]$ captures the excludability of informal firms from some public goods. This specification for the public good accounts for congestion as in Barro and Sala-i-Martin (1992). For the same level of government expenditures, a higher level of aggregate output lowers the value of the public good for firms. The parameter λ is intended to capture decreasing returns to the

public good. The parameter ϵ governs the rate which tax revenue is converted into a productive public good. The parameter, ϵ , is necessary to reflect the fact that not all tax revenue is spent in ways that augment firm production. For instance, transfer payments, military expenditures, and revenue lost to corruption likely do not enhance firm production. The public good can be thought of as the stock of available capital at time t , where public capital depreciates completely each period as in Acemoglu (2005). I prefer to use the terminology “public good” rather than “public capital” to account for productive government spending like state-backed financing and education. Firms perfectly forecast the level of g_t in solving their dynamic optimization problems.⁶

Sectoral Choice Problem

Given the static profit streams outlined above, and the evolution of productivity for formal sector firms, the present value of expected profits for all firms given an initial productivity draw of z_0 satisfies a Bellman equation:⁷

$$V(z_0) = \max[V^I(\bar{z}_1), V^F(z_0)] \quad (4.14)$$

$$V^I(\bar{z}_1) = \max \sum_{t=0}^{\infty} (\beta(1-\mu)(1-\delta))^t \Pi_t^I(\bar{z}_t) \quad (4.15)$$

$$V^F(z_0) = \max_{q \in [0,1]} \left\{ \Pi_1^F(z_0) - (1-\tau)e^{z_0}c(q) + \beta(1-\delta) (qV^F(z_0 + \Delta z) + (1-q)V^F(z_0 - \Delta z)) \right\}. \quad (4.16)$$

Equation (4.15) is the value function for an informal firm. Notice that since it forgoes the ability to improve its productivity, its expected profits are a summation of profits given the value of the spillover, \bar{z}_t . The parameter δ represents the exogenous exit rate of firms. The value function for formal firms, in Equation (4.16), indicates that firms must make decisions about whether to innovate and improve their initial draw of productivity. The Bellman system is written explicitly in

⁶This forecasting problem, given the assumption of complete depreciation, is trivial. Entrepreneurs already forecast the breakdown of firms between the formal and informal sector. This breakdown, in combination with the initial distribution of productivity, determines aggregate output and total tax revenue. Since firms know the parameters, ϵ and λ , they also know the value of g_t each period without any additional forecasting.

⁷Typically, the firm’s value could also be zero if it decided not to operate. Since there is no endogenous exit here, that option is excluded. Including endogenous exit would obfuscate the mechanism for tax revenue changes by including an additional supply-size effect.

terms of $t = 0$ to emphasize that firms are making a decision about which sector to enter, which is irrevocable.

Given these value functions, a firm will decide to operate in the formal sector if

$$V^F(z_0) > V^I(\bar{z}_t). \quad (4.17)$$

Let \hat{z}_1 be the least productive firm, in terms of z_0 productivity, that enters the formal sector.

Finally, the value function for formal firms permits the derivation of the first order condition which governs formal firms' investment in innovation:

$$c'(q) = \left(\frac{\beta(1-\delta)}{1-\tau} \right) (V^F(z_0 + \Delta z) - V^F(z_0 - \Delta z)). \quad (4.18)$$

Equilibrium

The aggregate labor supply, L , is fixed, and labor is supplied inelastically. Labor market clearing requires that

$$L = \sum_i l_{it} \quad \forall t. \quad (4.19)$$

Substituting in firms' labor demands from each sector implies a market-clearing wage rate of

$$w_t = \left(\frac{\rho-1}{\rho} \right) \left(\frac{L}{Y_t Z_t} \right)^{-\frac{1}{\rho}}. \quad (4.20)$$

Define, with a slight abuse of notation,

$$\sum_{i \in formal} e^{z_{it}} = Z^F \quad (4.21a)$$

$$\sum_{i \in informal} e^{\bar{z}_t} = Z^I. \quad (4.21b)$$

Now, let

$$Z_t = (z_{gt}^F)^{\rho-1} Z^F + (z_{gt}^I)^{\rho-1} Z^I, \quad (4.22)$$

be public capital-augmented TFP in the economy. Utilizing (4.20) and (4.22) the first-order conditions for firms in both sectors can be re-written as

$$l_{it}^F = \frac{L}{Z_t} e^{z_{it}} (z_{gt}^F)^{\rho-1} \quad (4.23a)$$

$$l_{it}^I = \frac{L}{Z_t} e^{\bar{z}_{it}} (z_{gt}^I)^{\rho-1} \quad (4.23b)$$

Utilizing (4.22) and (4.23) aggregate output in the economy can be formulated simply as

$$Y_t = LZ_t^{\frac{1}{\rho-1}}, \quad (4.24)$$

which implies that aggregate growth is

$$g_Y = \frac{Z_{t+1}^{\frac{1}{\rho-1}} - Z_t^{\frac{1}{\rho-1}}}{Z_t^{\frac{1}{\rho-1}}}. \quad (4.25)$$

It is important to note that (4.24) is still implicitly defined. Since g_t depends on the level of aggregate output, Y_t enters on both sides of the equality.

Finally, utilizing (4.20) and (4.23), firms' operating profits can be re-formulated as

$$\Pi_t^F(z) = (1 - \tau) \frac{w_t^{1-\rho} (z_{gt}^F)^{\rho-1}}{(\rho - 1)^{1-\rho} \rho^\rho} Y_t e^{z_{it}} \quad (4.26a)$$

$$\Pi_t^I(\bar{z}_t) = (1 - \mu) \frac{w_t^{1-\rho} (z_{gt}^I)^{\rho-1}}{(\rho - 1)^{1-\rho} \rho^\rho} Y_t e^{\bar{z}_{it}} \quad (4.26b)$$

Equation (4.26) illustrates the convenience of scaling firm level productivity by $\frac{1}{\rho-1}$ in the firm's production function. Firms' operating profits in both sectors, as well as labor demand, are proportional to a firm's productivity. These expressions can be substituted into the Bellman system to simplify the firm's sectoral choice problem.

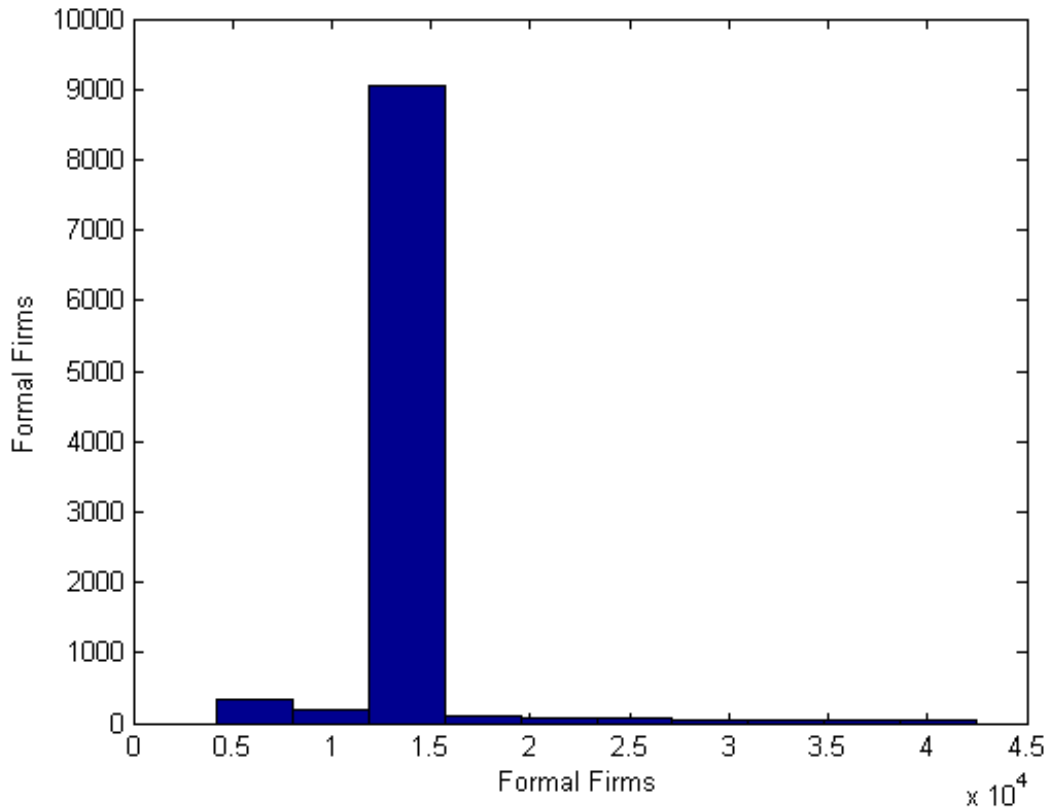
Model Intuition

The role of spillovers and free-riding in the model are of primary importance. Note that there are two separate mechanisms for spillovers. First, as in the previous chapter, firms in the formal sector provide a productivity spillover to the informal sector. This spillover is intended to capture elements of the data, mainly the overlap in productivity between the formal and informal

sector firms that is often neglected. Mechanically, this spillover augments firm incentives to move into the informal sector. As taxes increase, and firms increasingly choose to operate in the informal sector, the spillover captured by \bar{z}_t also increases.

This productivity spillover also ensures a firm-size distribution that qualitatively resembles that of many developing countries. Since all informal firms operate with the same level of productivity, \bar{z}_t , there is a large grouping of firms at the lower end of the productivity distribution. This effect can be seen in Figure 16. Ideally, this grouping of firms would be at the extreme left tail as seen in most developing countries.

FIGURE 16. Distribution of Firm Productivity



Parameter values are as described in Table 16. There are 10,000 total firms, and firm productivity is calculated as $e^{z_{it}}$ or $e^{\bar{z}_t}$ depending on whether the firm participates in the formal or informal sector.

The second spillover occurs through the public good. Informal firms are able to free-ride and have their productivity enhanced by utilizing the public good without paying taxes. A more productive formal sector implies greater tax revenue and a larger spillover to formal sector

firms. Due to the fact that the public good accounts for congestion, increases in informal sector productivity (through \bar{z}_t) decrease the efficiency of the public good through their increase in output.

Finally, it is important to discuss the role of innovation in the model. Innovation provides an incentive for some less productive firms to enter the formal sector in hopes that their investments in innovation will raise their future productivity above the level guaranteed in the informal sector. The step-size formulation of the innovation process, coupled with the large number of firms in the simulations, ensures that the results are approximations and will not be entirely smooth functions of the tax rate. For many levels of productivity there are several, if not many, firms that have the same parameter z_{it} . This implies that a marginal change in parameter values will incentivize many firms to change sectors rather than just a single marginal firm. While having a sufficiently large number of firms can mitigate this effect, the need to have a reasonable range of initial productivities ensures that the results cannot be completely smooth.⁸

Quantitative Exercise

The quantitative exercise seeks to find, and explain, the revenue-maximizing tax rate. This goal is similar to Acemoglu (2005) where the political elite set tax rates and determine the provision of public goods to maximize their rents. In this case, however, the provision of public goods is set at a fixed percentage of total revenue and subject to congestion. Table 15 provides a recap of the variables of the model. Table 16 lists the parameter values used in the quantitative exercise.

Table 16 specifically outlines the values used in the base case, and the effects of important parameters, particularly ϵ , γ , and λ are investigated further. Initially, I choose a conservative value of $\epsilon = 0.2$, which would align with a narrow view of the productivity enhancement of public expenditures. Only 20% of total tax revenue is converted into a public good that enhances firm productivity. The remainder of tax revenues may leave in the form of transfer payments or be lost to government corruption and general inefficiency as in Chakraborty and Dabla-Norris (2011). For the value of γ , I start conservatively, estimating that informal firms enjoy 80% of the

⁸This same issue occasionally causes computational issues. Ironically, for very precise convergence criteria there is often cycling between two different allocations of formal and informal firms. The choice of one of these allocations over the other adds further noise to the graphs.

TABLE 15. Variables of the Model

Variable	Definition	Variable	Definition
g	Public good	γ	Differential usage parameter
ϵ	Tax transformation rate	λ	Returns to scale for pub. good
ρ	Elasticity of substitution	$c(q)$	Cost function for innovation
δ	Probability of exit	q_{it}	Probability of successful innovation
τ	Tax rate on profits	\bar{z}_t	Spillover to informal sector
M	Number of firms	Γ	Distribution of z_0
w_t	Wage rate	β	Discount rate of firms
\hat{z}_t	Formal cut-off	Δz	Step-size of innovation
L	Labor supply	μ	Probability of detection

productivity enhancement from the public good compared to formal sector firms. The parameters ϵ and γ are particularly nebulous to choose given the lack of clear data (especially given the role of the informal sector) and differing interpretations as discussed in Section 4.6. I begin with a preliminary value of $\lambda = 0.2$ and investigate the range of λ as in Chakraborty and Dabla-Norris (2011).

TABLE 16. Selected Parameter Values (Base Case)

Variable	Value
ϵ	0.20
λ	0.20
γ	0.80
ρ	5
δ	0.10
μ	0.10
τ	0.20
M	10000
L	100000
β	0.96
Γ	Uniform distribution
Δz	0.027

The elasticity of substitution is set to 5 as in Atkeson and Burstien (2012). I use a standard rate of firm exit of 10% annually and allow for a 10% chance that firms are caught in the informal sector. The 20% initial tax rate listed in Table 16 is used to calibrate the cost function of innovation such that in equilibrium the average innovation rate is $q = \frac{1}{2}$ for formal sector firms which implies that firms on average do not grow. The initial uniform distribution for the parameters z_{i0} ensure that firm productivity is exponential and qualitatively matches the

distribution of firm productivity in developing countries. Finally, the step size of innovation, Δz , is set such that the upper bound on firm-level productivity is 2.7%. This rate reflects the finding in Brandt et al. (2012) for firm-level productivity growth among Chinese firms. The model would generate average firm TFP growth of 2.7% only if all firms in the formal sector innovated with $q = 1$.

Results

The results are grouped into three categories. First, I explore the effects of changes in the tax rate on tax revenue and other model variables for the base case described in the previous section. Those results establish a basis to compare other parameter choices and priors. Next, I proceed to evaluate the anticipated effects of certain parameters of interest, mainly ϵ , γ , and λ . These parameters govern the effectiveness, in terms of productivity enhancement, and excludability of the public good. Given that the provision and productivity of public goods is likely very heterogenous across countries, these results help to inform potential country-specific applications of the model. Finally, I make a slight modification to the model to isolate the role of the public good in determining sectoral participation.

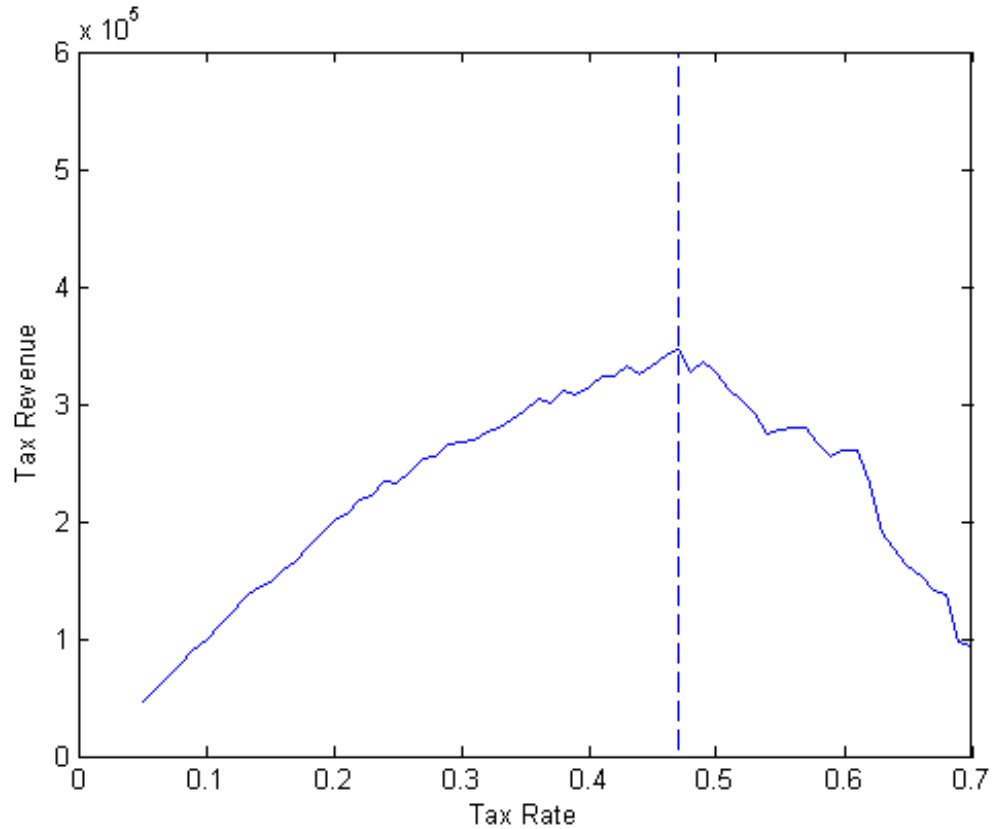
Each set of parameter values predicts which firms would choose to participate in the formal sector and pay taxes. Changes in parameters should be interpreted as a different economic environment rather than causing firms to physically shift from one sector to another. In each case, the results reflect firm decisions for $t = 1$ when firms make their first set of profit-maximizing decisions given their choice of sector. Across different parameter values, I find optimal tax rates that range from 39% to 48%.

Base Case

The base case simulation is helpful for providing a common set of results to compare to. Figure 17 illustrates the primary result of interest: the effect of changes in tax rates on total tax revenue. The Laffer Curve in Figure 17 shows a revenue-maximizing tax rate of 47%. Recall that this result assumes that 20% of tax revenue is converted into a productive public good that enhances firm productivity and that informal firms only receive 80% of the productivity

enhancement of formal sector firms. These parameter values are put into proper context in Section 4.6.

FIGURE 17. Tax Rates vs. Total Tax Revenue - Base Case

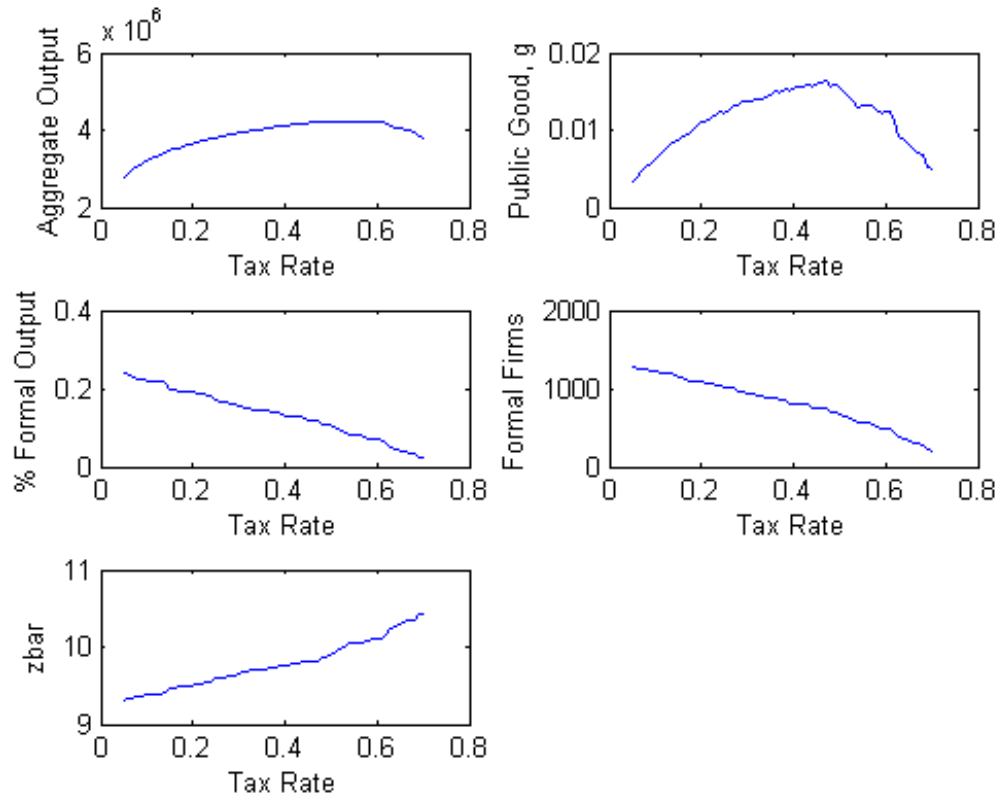


Simulations are run for each 1% tax rate from 5% to 70%. Parameter values are as defined for the base case in the previous section.

The panels in Figure 18 provide an insight into the mechanisms that generate Figure 17. In particular, the second panel in the second column illustrates that formal sector participation decreases as taxes increase, a result that was the focus of the previous chapter. The incentives for firms to enter the informal sector at higher tax rates are partially offset by increases in the public good, g , that is of greater benefit to the formal sector. The lowest panel captures the other source of productivity spillover, \bar{z}_t . As the formal sector shrinks, it consists of more productive firms on average. This implies that \bar{z}_t will grow as the marginal firm (i.e. least productive firm in the formal sector) decides to participate in the informal sector instead. Finally, notice that aggregate output continues to grow until a tax rate of about 60%. This indicates that under the base case

parameter values the productivity spillover has a larger impact than the public good. Beyond 60%, these effects begin to reverse.

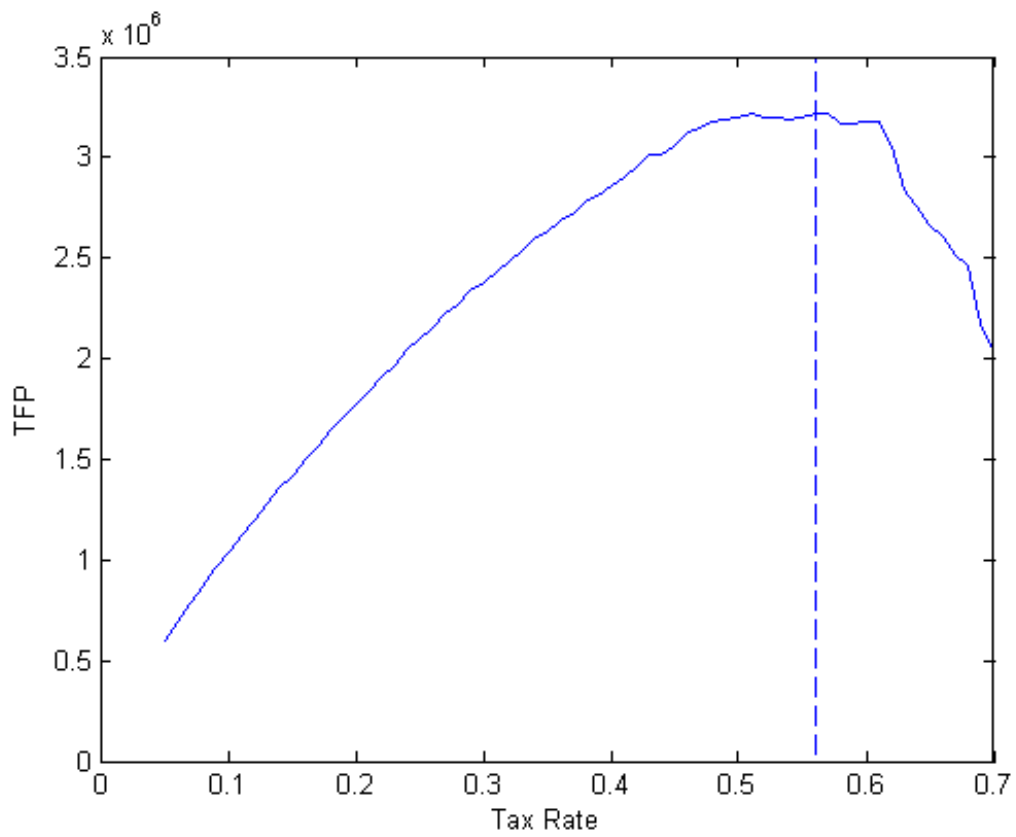
FIGURE 18. Variables of Interest - Base Case



The vertical axes, starting from top left are aggregate output (Y_t), the public good (g_t), the number of formal firms, the productivity spillover from the formal sector to the informal sector (\bar{z}_t), and the percentage of output produced in the formal sector. Simulations are run for each 1% tax rate from 5% to 70%. Parameter values are as defined in Section 4.4.

From the revenue-maximizing tax rate of 47%, a 10% decrease in taxes would decrease total revenue by 13.10% but also incentivize an increase in the size of the formal sector by 13.14%. On the other hand, an increase in tax rates from 47% would have no desirable increase in the size of the formal sector. A 10% tax increase would decrease tax rates by 19.54% and decrease the size of the formal sector by 27.08%. Given parameter uncertainty, and an underlying assumption that formality is preferable over informality, these results suggest that underestimating the revenue-maximizing tax rate is likely the more prudent policy. Not only do lower tax rates result in a larger formal sector, but the shape of the Laffer Curve indicates less severe consequences for total

FIGURE 19. Tax Rates vs. TFP - Base Case



Simulations are run for each 1% tax rate from 5% to 70%. Parameter values are as defined for the base case in the previous section.

tax revenue. This base case results are sensitive to changes in parameter values, particularly the excludability of the public good (as seen below). Therefore these results are best interpreted qualitatively, keeping in mind that they suggest an alternative channel for the Laffer curve than exists elsewhere in the literature.

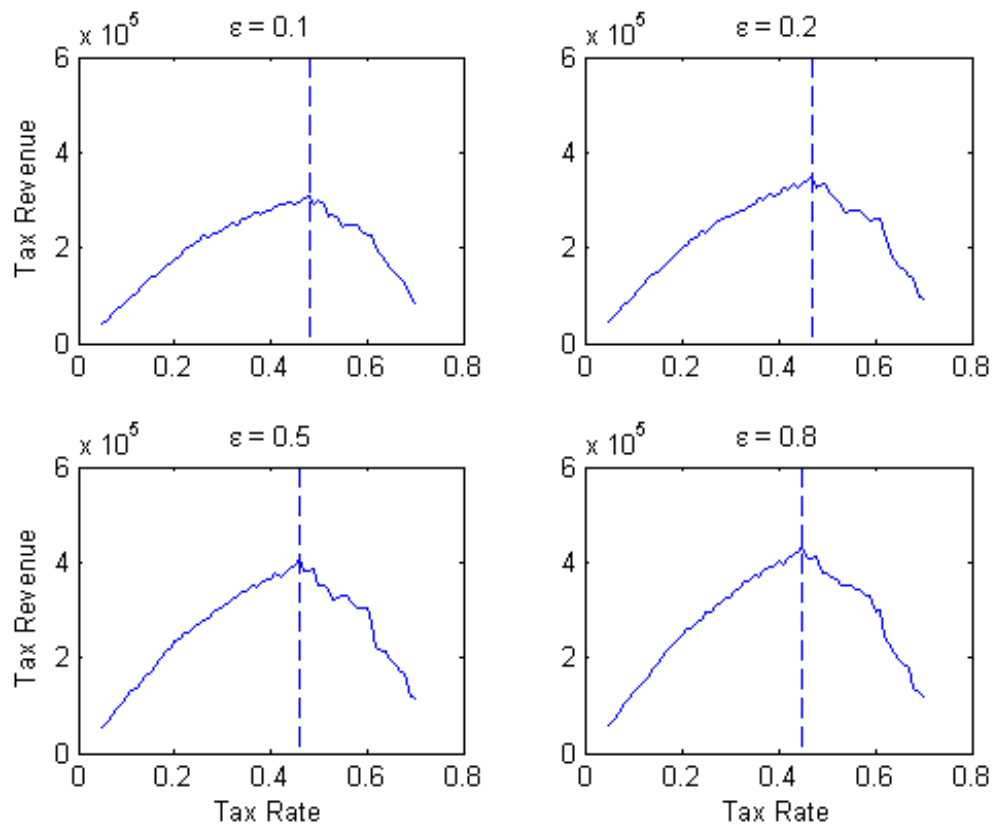
Notably, Figure 19 illustrates the TFP-maximizing tax rate (56%). This is the tax rate that would lead to the greatest level of aggregate TFP as defined in (4.22). Unlike in the previous chapter, the base case results can now be interpreted in terms of an optimal policy, with a caveat. Some type of objective function for the policy maker is necessary to evaluate whether to prioritize TFP level or revenue collection. Given this function, the model now captures the both the positive and negative effects of tax policies and suggests an optimal tax rate.

Changes in the Productivity of Public Expenditures

One of the most central parameters in the model is ϵ . Recall that ϵ governs the rate at which tax revenue is converted into the productivity enhancing public good. A higher level of ϵ , all else constant, increases the level of g and the total productivity of both formal and informal firms. Figure 20 illustrates the effect of ϵ on the Laffer Curve. Qualitatively, the general shape of the curves is remarkably consistent, and the revenue-maximizing tax rate only ranges from 48% for $\epsilon = 0.1$ to 45% for $\epsilon = 0.8$.

The increasing scale of the Laffer Curves in Figure 20 highlights the importance of ϵ in the model. As taxes are converted more efficiently to the public good, the total productivity of firms in both sectors increases. The increases in productivity in the formal sector leads to increased profitability and greater total tax revenue for a given tax rate. This effect is clear in Figure 20 despite the fact that formal sector participation decreases, which can be interpreted as a decrease in the tax base. For instance, changing ϵ from 0.2 to 0.5 at the revenue-maximizing tax rate from the base case implies a 6.88% increase in total tax revenue. The increase in ϵ increases the opportunity for the marginal firm to free-ride and enjoy limited access to the public good without paying taxes. Of course, the greater the excludability of the public good, represented by γ , the smaller the effect. This feature is investigated in the next section.

FIGURE 20. Tax Rates vs. Tax Revenue across Values of ϵ

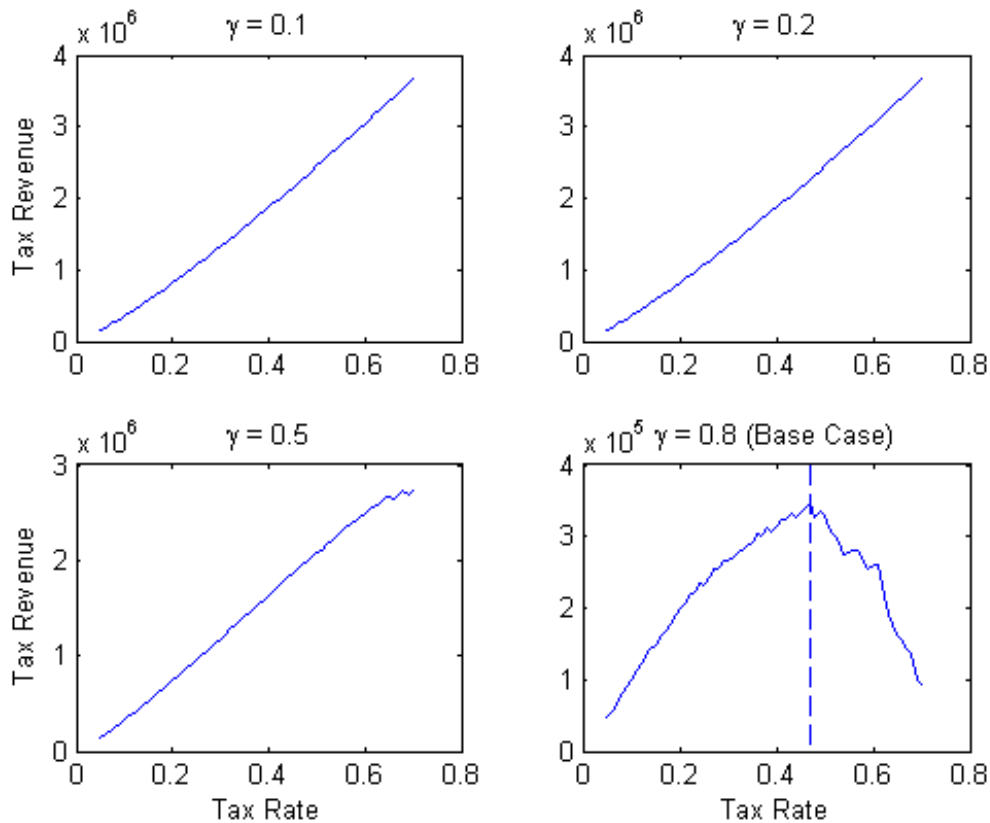


All other parameters except ϵ are the same as defined for the base case in the previous section. Simulations are conducted for each 1% change in the tax rate with the same initial draw of firm productivity parameters, z_0 .

Changes in the Informal Sector's Access to the Public Good

Figure 21 illustrates the impact of changes in the excludability of the public good from usage by informal firms. Higher values of γ imply more equal usage of public goods between the formal and informal sectors. Unlike the results for different values of ϵ , the panels in Figure 21 illustrate drastically different effects of tax rates on tax revenue across values of γ . The top two panels in Figure 21 represent very high levels of excludability. Intuitively, given the importance of the public good to firm production, the desirability of operating in the informal sector decreases precipitously as access to the public good is curtailed. This effect is strong enough to completely mitigate the effect of higher tax rates (at plausible tax levels).

FIGURE 21. Tax Rates vs. Tax Revenue across Values of γ



All other parameters except γ are the same as defined for the base case in the previous section. Simulations are conducted for each 1% change in the tax rate with the same initial draw of firm productivity parameters, z_0 .

By the time that γ reaches 0.5 the more traditional-looking Laffer Curve begins to emerge, albeit at a much higher tax rate. On a practical level, the ratio of public sector productivity, $\frac{1}{\gamma}$,

can be thought of as a “carrot” for participation in the formal sector. In economies where this ratio is larger, more firms will choose to participate in the formal sector at any given tax rate. Given a large enough carrot, tax rates as formulated in the model may not be enough to dissuade firms from participating in the formal sector.

Changes in the Returns to the Public Good

The final parameter of interest, λ , determines the returns to scale of the public good. The effects of a change in λ are fairly straightforward, as can be seen in Figure 22. All four panels illustrate traditional Laffer Curves with tax revenue-maximizing rates of 40% to 47%. Despite the similarities in shape and optimal tax rates, Figure 22 does illustrate important differences in scale. As λ increases the difference between the productivity of the public good in the formal sector and informal sector decreases. This incentivizes more firms to choose to enter the informal sector, a choice that is further augmented by the productivity spillover from the formal sector being comprised of fewer, more productive firms.

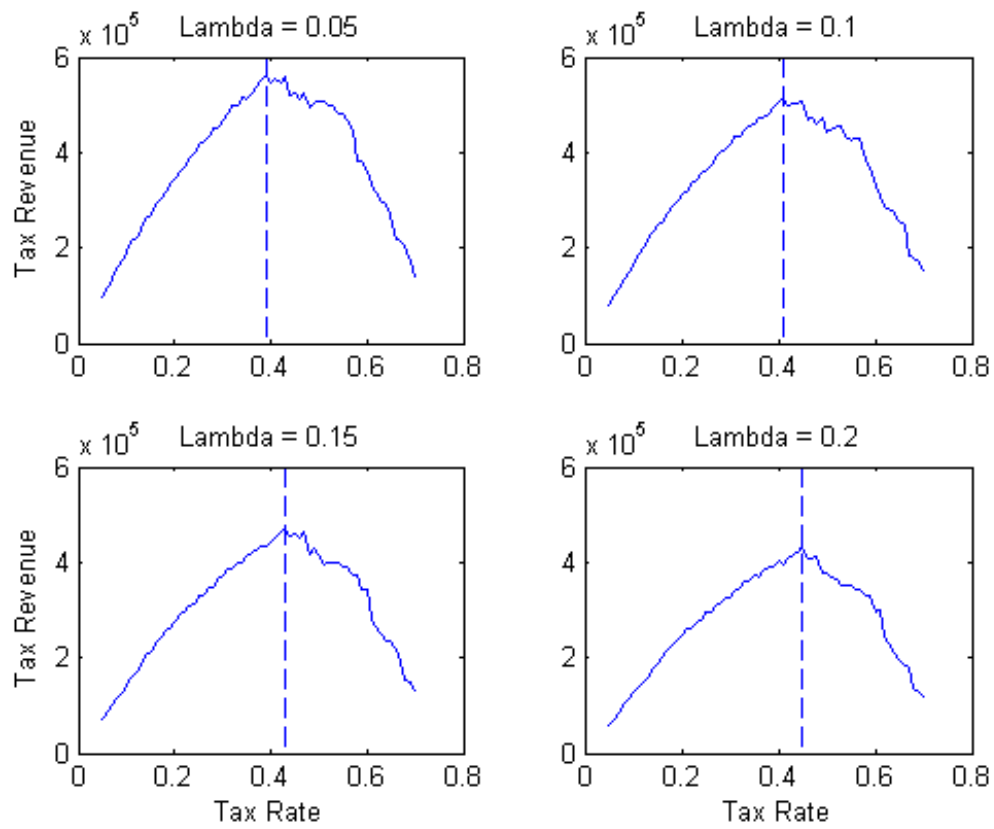
Constant Productivity Spillover

Isolating the role of the public good spillover can be informative in determining revenue-maximizing tax policy. To do this, I run the base case simulation with $\tau = 0.2$ and find a value of $\bar{z}_1 = 9.48$. I re-run each simulation in the base case for the normal range of tax rates while holding \bar{z}_1 constant at 9.48.⁹ Notice that this exercise requires a different interpretation than before. Assuming a constant spillover implies that there is some amount of technology spillover that will occur regardless of the size and productivity level of the formal sector. While a completely constant spillover may be a departure from reality, it does help to isolate the role of the public good.

Figure 23 illustrates the effect of holding the productivity spillover from the formal sector to the informal sector constant across tax rates. While the revenue-maximizing tax rate is nearly the same as the base case (48% instead of 47%), the amount of taxes collected are much higher, which results from greater formal sector participation. Additionally, the left-hand side of the Laffer Curve illustrates a stronger relationship between increases in tax rates and increases in tax

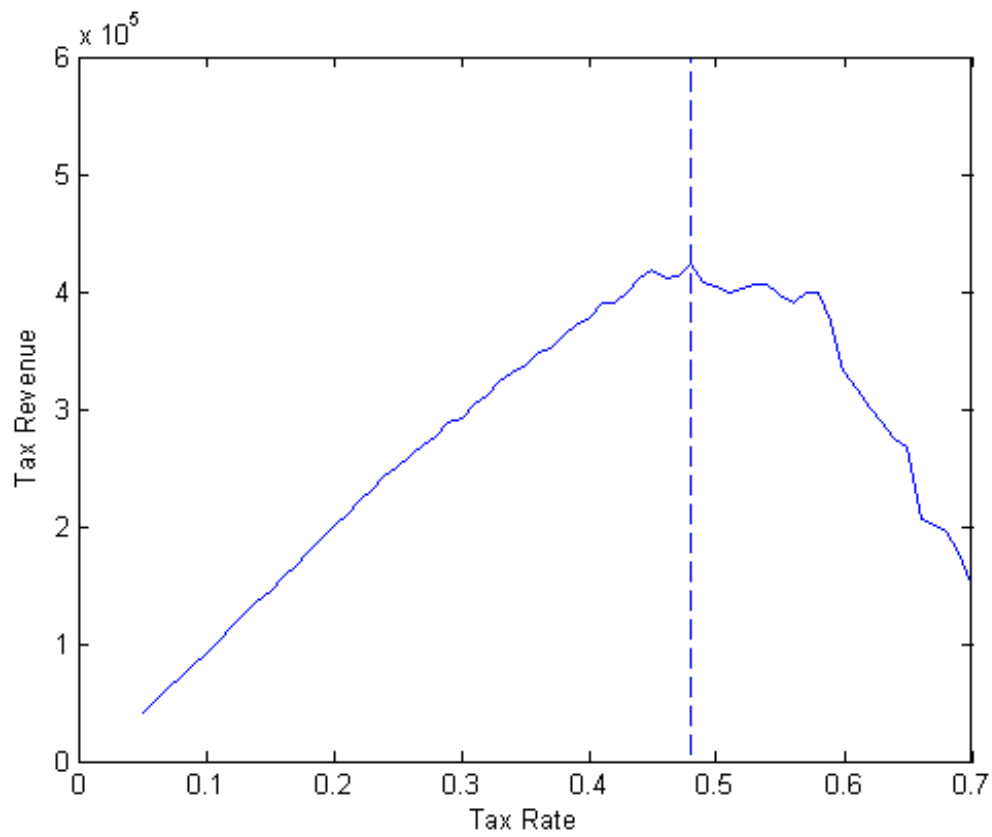
⁹Given this value, firms still forecast the expected evolution of \bar{z}_t for future periods.

FIGURE 22. Tax Rates vs. Tax Revenue across Values of λ



All other parameters except λ are the same as defined for the base case in the previous section. Simulations are conducted for each 1% change in the tax rate with the same initial draw of firm productivity parameters, z_0 .

FIGURE 23. Tax Rate vs. Tax Revenue - Constant \bar{z}_1



Simulations are run for each 1% tax rate from 5% to 70%. Parameter values are as defined for the base case in the previous section

revenue. This result is driven by the fact that as firms leave the formal sector their only aggregate effect on the relative productivity of the two sectors is through the public good. Previously, when the marginal formal sector firm chose to participate in the informal sector instead of the formal sector, average productivity increased in the formal sector, increasing the size of \bar{z}_t utilized in the informal sector. Without the presence of the productivity spillover, the revenue-maximizing tax rate and TFP-maximizing tax rate are the same, suggesting a clear optimal policy.

Discussion

One of the principal difficulties in interpreting the applicability of the results are the parameters of ϵ and γ . These parameters are important in determining the shape of the Laffer Curve, but are hard to assign precise values. One source of uncertainty stems from the likely heterogeneity across countries. Different countries most certainly have different laws and regulations that exclude informal sector firms from utilizing specific public goods which would affect the value of γ . Countries also differ in the efficiency, which they provide public goods and whether the goods they supply actually augment the productivity of firms. Due to these difficulties, I have opted to explore the results over a range of value for γ and ϵ rather than focus on a specific country.

Additionally, the parameter ϵ comes with considerably different interpretations in the literature. One strand of the literature has focused on which public goods increase firm productivity and enhance growth. For instance, Aschauer (1989) argued that while non-military capital is important in determining productivity, military expenditures have little relation to productivity. Further “core” infrastructure projects were the most significant explanation for productivity, a result that was echoed in Easterly and Rebelo (1993). Other research like Barro (1990) has investigated excluding other expenditures like education spending.

Given this approach, I establish likely bounds for the value of ϵ . On the low end, suppose that only public construction expenditures augment firm productivity. Using U.S. data, this could imply an $\epsilon = .011$, after adjusting construction expenditures to represent a balanced budget. On the other end, excluding only military expenditures, which are rarely associated with greater firm productivity, would imply an $\epsilon = .8645$ for India and $\epsilon = .8832$ for all low income countries.¹⁰

¹⁰I use revenue, deficit, and military spending data from the World Bank’s Development Indicators. The data for U.S. public construction comes from the Federal Reserve Bank of St. Louis’ FRED database.

Another approach entirely would be to consider all public expenditures to be productive, but that spending occurs through an imperfect political, and possibly, corrupt process. This process would follow the work of Chakraborty and Dabla-Norris (2011) and could map the index proposed in Dabla-Norris et. al (2012) to values of ϵ . My base case choice of $\epsilon = 0.2$ is intended to be a conservative estimate for developing countries, recognizing that there is likely a large amount of unproductive government spending and spending that is lost to corruption and inefficiency. Given that higher levels of ϵ incentivize informality, my choice of a relatively low level of ϵ biases against my results.

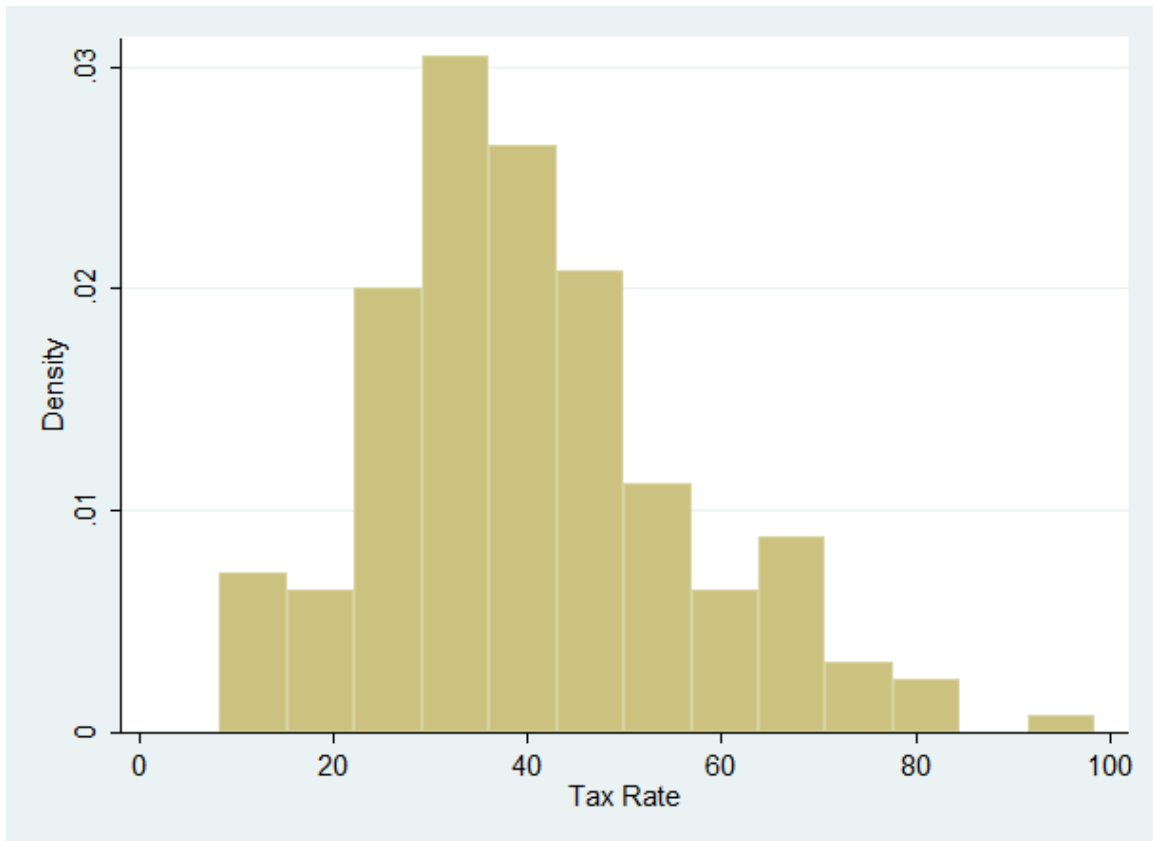
It is also important to recognize the relationship between ϵ and γ . The choice of which goods are productive (ϵ) may inform reasonable values of γ . Suppose, as a stark example, that the only productive public expenditures are investments in the electrical grid and state-backed loans for firms. While these goods would imply a very low value for ϵ they would also predict a very low value of γ since both goods would be easy to exclude from informal firms. Likewise, the larger the value of ϵ likely the larger value of γ as it becomes difficult to exclude informal firms from utilizing public goods.

Given the assumptions of the model, how applicable are the results to developing countries? Given the tax rates that are seen in developing countries, the results indicate that, even with parameter uncertainty, many countries exceed their revenue-maximizing tax rates. This fact is illustrated in Figure 24. The effect of tax rates on tax revenue in this chapter only captures the effect of firms deciding to enter the formal rather than the informal sector. Therefore, traditional supply-side decisions may further decrease the revenue-maximizing rate. Alternatively, firms may hide some proportion of total production, which may mitigate the effects of tax increases.

There are several reasons why the Laffer Curve in this chapter predicts a lower revenue-maximizing tax rate than elsewhere in the literature. First and foremost, is the developing world context. Rather than relying on individual's labor supply decisions, the Laffer Curve is generated by firms deciding to be informal. This mechanism can only occur when law enforcement is sufficiently weak, as in many developing countries.¹¹ If the probability of being caught operating informally is sufficiently high, this mechanism is essentially shut down. Secondly, these results view tax revenue as a static concept. As the previous chapter indicated, tax rates can have an

¹¹Recall that the base case model utilized a 10% chance that informal firms are caught, fined their entire profits for the period, and closed.

FIGURE 24. Distribution of Total Tax Rates



Tax data comes from the World Bank's Development Indicators. I use total tax rates for 2012 and drop tax rates that are beyond 100%.

important effect on innovation and economic growth. If a government sought instead to maximize its lifetime (or infinite) stream of tax revenues, this would suggest lower tax rates today in order to spur innovation and economic growth.

Conclusion

This chapter investigates how firm incentives to participate in the formal sector are influenced by government policy, particularly through taxation and the provision of public goods. In order to maximize revenue, governments must carefully balance the disincentives for formal sector participation posed by taxation with the need to raise revenue to provide a public good. Considering the deep literature on both revenue-maximizing taxation and the efficacy of public capital, this chapter makes a significant contribution by identifying an alternative mechanism for which increases in taxation can eventually decrease total tax revenue. This mechanism focuses on the sectoral decision of firms rather than the standard labor market decisions of individuals.

Through a series of calibrated exercises, I find revenue-maximizing tax rates from 39% to 48% across a range of plausible policy parameters. Given that the base findings are sensitive to parameter changes, the qualitative features of these findings are of greater importance. The range of rates is both notably lower than others in the literature have estimated and lower than the tax rates imposed in many developing countries. Moreover, the Laffer Curves that I find may be augmented by the traditional supply-side effects. Ultimately, the difference in findings relies on a difference in context. The mechanism I identify in this chapter is likely small in most developed countries. As the ability to prosecute participants in the informal sector increases, and informal firms can be excluded from the use of public capital, this channel dissipates. This research underscores the fact that tax policy and the provision of public goods must be evaluated based on country-specific context, taking into account the importance of informal economies in many developing countries.

CHAPTER V

CONCLUSION

The role of TFP in determining cross-country income differences has been a central focus of development economics. Understanding the growth in TFP requires the examination of firm-level decisions with respect to innovation. This dissertation has focused on how the incentives of firms to innovate and participate in the formal sector are influenced by changes in government economic policies. In doing so, it provides relevant policy prescriptions to stimulate economic growth in developing countries. Within this dissertation, I have outlined four main contributions to understanding the effects of government tax policies on firm incentives to innovate in developing countries.

The first contribution of this dissertation is to provide a general equilibrium framework to address both firm-level innovation and the decision to be formal or informal. Previous work on informality has empirically documented the incentives to participate in the informal economy, as in Dabla-Norris et al. (2008) and Loayza (1996), or modeled firm choice theoretically in response to formal sector regulation as in Rauch (1991). Importantly, none of this previous work included a dynamic decision-making process for firms. In this sense, they could not address how firm decisions about sectoral choice affected cross-country differences in TFP as identified by Hall and Jones (1999).

The second contribution is the result that size-based tax distortions are insufficient to generate the “missing middle” phenomenon identified by Tybout (2000). Chapter II illustrates that size-based distortions that impose greater taxes on medium-sized firms can only explain the upper tail of firm-size distributions in developing countries. Without additional frictions, optimal firm behavior suggests that a Tybout-style tax induces firms to be larger. This result is at odds with empirical data that shows a large number of small firms operating in developing countries.

The final two contributions explicitly answer questions with regard to government tax policy. Chapter III clearly showed how increases in tax rates have two important effects. The number of firms operating in the formal sector decreased as tax rates increased. Innovation in the economy also fell as formal sector profitability decreased and more firms chose to operate in the informal sector. Moreover, the results from Chapter III were economically significant. According

to the calibrated model, a tax rate increase from 50% to 60% implied a 20.9% reduction in the size of the formal sector, accompanied by a .07% decrease in annual growth. The policy significance of these findings is underscored by the fact that many developing countries regularly impose total tax rates in excess of 50%.

Finally, the results from Chapter IV show that many developing countries impose tax rates that not only increase the informal sector and detract from economic growth as shown in Chapter III, but also negatively affect total tax revenue. By adding the provision of a productive public good to the model of Chapter III, I am able to show that developing countries have a unique channel that can generate a Laffer Curve. This channel is unique in that it does not rely on typical supply-side effects such as the labor supply decision of individuals. Rather, it quantifies the impact of firms deciding to participate in the informal sector and free-riding on the productive public expenditures financed by the formal sector. Simulating this channel, with a variety of parameter values, I find revenue-maximizing tax rates from between 39% to 48%.

These contributions suggest several avenues for future research. On the empirical side, careful estimation of such key parameters as the excludability of informal firms from public goods would allow for more targeted policy conclusions. In addition, country-specific applications of the models in Chapter III and IV could provide valuable insights for policy makers, especially given the heterogeneity among many of the parameter values necessary for the models and the differing levels of the quality of institutions. Finally, while previous work has focused on tax policies in changing firm incentives, future work may evaluate other policies such as labor market regulations, copyright and patent laws (reduction in the costs of innovation), and law enforcement strategies. By continuing to investigate the impacts of specific policies on firm incentives and innovation, this research can hopefully identify best practices for governance in developing countries.

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