Privatizing the Commons and Economic Degradation

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May 2005.

Abstract

We develop a dynamic model of the exploitation of an environmental resource with endogenous property rights. We are able to explain both the evolution of property rights and environmental quality. In some circumstances the time path of environmental quality is U-shaped and resembles an Environmental Kuznets Curve. However this patern derives from changes in the property rights regime, not from changes in income.

Keywords: Commons, Property Rights, Environmental Resource.
J*EL Classification: Q20, Q50.*

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Acknowledgements: Birdyshaw wishes to acknowledge research support provided by the University of Alaska, Fairbanks.

1 Introduction

Since the early 1990s, an abundance of empirical evidence has emerged to indicate an inverse U-shaped relationship between environmental degradation and economic growth. Evidence of this pattern, called an environmental Kuznets curve (EKC) due to its similarity to the relationship found between income and income inequality by Kuznets (1955), has been found for a diverse set of environmental indicators. However, relatively less work has been done to develop the underlying theory of the EKC. Understanding the economic theory that drives the EKC is critical since it allows researchers to obtain better empirical estimates, and leads to more efficient public policy. In this paper, we contribute to the theoretical development of the EKC hypothesis by focusing on the ownership structure in markets for renewable resources. Specifically, we develop a model showing that the evolution of private property rights, over a previously open-access resource, may be sufficient to generate a U-shaped pattern for environmental quality.

Most researchers trace the origins of this literature to Grossman and Krueger (1992, 1995), who argued that increased trade resulting from the NAFTA could improve environmental quality; and Shafik and Bandyopadhyay (1992), who examined the relationship between income and environmental quality in a background paper for the 1992 World Development Report. Both studies found EKCs for sulfur dioxide, suspended particulate matter, and various measures of water quality. Subsequently, researchers have found EKCs for various forms of environmental degradation including carbon (Holtz-Eakin and Seldon, 1995; and Suri and Chapman, 1998), deforestation (Cropper and Griffiths, 1994; and Bhattari and Hammig, 2001), lead emissions (Hilton and Levinson, 1998), and hazardous waste exposure (Wang, et al, 1998).[1]

Existing theoretical explanations for the EKC focus on structural changes in economies, pollution abatement technology, and the income elasticity of demand for the environment. Panayotou (1993) suggests that this non-monotonicity occurs naturally as developing countries shift from largely agrarian to industrial, then to service based economies. As the composition of output changes, so too does the flow of environmental degradation. Andreoni and Levinson (2001) propose that an EKC may emerge as a result of increasing economies of scale in the abatement of pollution. In general, one would expect that the technologies necessary for the abatement of environmental damage, or recovery of damaged resources, will be more plentiful in countries with greater incomes. Others have stressed shifts in consumption choices over income and environmental quality (Lopez, 1994; Stokey, 1998). Low income countries with an abundance of environmental amenity find it desirable to transform resources into income. As income rises, and environmental quality declines, the marginal value of an income falls relative to the marginal value of the environment. Eventually, consumers choose to buy "cleaner" goods, or demand less polluting production processes.

2 Property Rights and Environmental Degradation

This paper shows that environmental quality, as measured by the productive capacity of a natural resource, will follow a U-shaped pattern as ownership of the resource gradually shifts from communal to private property. Intuitively, the resource initially suffers from a tragedy of the commons since the cost of each agent's usage is shared equally by the entire community. In order to avoid the commons problem, some agents are willing to incur a one-time fixed cost to privatize a portion of the resource. Gradually, as more of the resource shifts to private ownership, environmental quality begins to improve.

[1] However, recent empirical work has questioned the robustness of these results. See for example, Harbaugh, Levinson, and Wilson (2002), Stern and Common (2001) de Bruyn, van den Bergh, and Opschoor (1998), or Agras and Chapman (1999). These works underscore the importance of theoretically clarifying the determinants of environmental quality change.

Low income communities with an abundance of environmental amenities, and no well defined or enforced property rights, will tend to overexploit their resources due to the cost externalities associated with each individual's use. Over time, as resources become scarcer, the need to monitor and regulate resource extraction becomes more pressing. Also, as income grows, the technological and social conditions necessary to develop and enforce regulations emerge. While this regulation may take the form of centrally planned quota system, in many cases it is more efficient to find a decentralized solution through privatization. This paper focuses on the latter case, but either can result in more sustainable resource use.

It has been argued that the EKC is an empirical proxy for the evolution of property rights, and the incentive structure embedded within them (Yandle, Bhattarai, and Vijayaraghavan, 2004). Case studies of the pastoral regions of china (Longworth and Williamson, 1993) suggest that the absence of property rights have led to overgrazing. Similarly, Dennen (1976) provides evidence that Cattlemen's Associations improved the productivity of rangeland in the late 19^{th} century United States. During the Edo period in Japan, 1603 - 1867, economic growth brought the demand for lumber in conflict with the demand for arable land. Depleted forests threatened agricultural production by disrupting the flow of water to the low-lands, by flooding and river-silting, for example. As a result the Japanese developed several policies to combat the problem including the practice of Yamawari, by which communal land is divided among villagers as private property.[2] More recently, private property rights have been extended to wildlife in southern Africa in an attempt to preserve the Black Rhinoceros and other species.

3 The Model

We assume an economy in which a population of n identical individuals may exploit a common property resource, also of size n. Define α_t as the quality of the resource, in terms of its productivity, at time t. In the absence of human exploitation, resource quality evolves according to the difference equation

$$\alpha_t = \alpha_{t-1} + \tau \left(\bar{\alpha} - \alpha_{t-1} \right) \tag{1}$$

where $\bar{\alpha}$ is the unexploited steady state level of resource quality, and $\tau \in (0,1)$ is a parameter that describes the speed of adjustment.

Individuals can choose either to exploit the resource as a commons, or, at a cost to be defined below, appropriate exactly one unit of the commons and exploit it as a private resource. Let $Z_t = \sum_{j=1}^{t} Z_j$ represent the total quantity of the commons that has been privatized prior to period t. Then, in any time period, the total quantity of common property available to the producers is given by $n - \sum_{j=1}^{t} Z_j$.

Each producer lives for one period and is endowed with a fixed quantity of time, T, that may be devoted to exploiting the resource, l_{it}, or leisure, $T - l_{it}$. Time devoted to exploitation of the resource is transformed into a consumption good, c_{it}, according to the function

$$c_{it} = \alpha_t l_{it} \left(\frac{1}{H} \right)^{\mu} \tag{2}$$

where H represents the rate of exploitation and the parameter μ is assumed to lie within the interval $(0,1)$. This implies that production of the consumption good increases, at a decreasing rate as the inverse of the rate of exploitation increases. In other words, a reduction in the rate of exploitation increases production.

[2] See Totman (1984).

The consumption good can be given quite a broad interpretation, as an example it might represent exploiting land for grazing cattle, or, alternatively, it could involve the consumption associated with the observation of native flora and fauna.

Producer i receives utility from consumption and leisure in period t according to the function

$$u_{it} = (c_{it})^{\gamma} (T - l_{it})^{\eta}. \tag{3}$$

We make the standard assumptions that $\gamma \in (0,1)$ and $\eta \in (0,1)$ so that the utility function possesses diminishing marginal returns.

For common property, the rate of exploitation is given by the ratio of the sum of all labor applied to the commons to the total quantity of common property available. For private property, the rate of exploitation is given by the ratio of an individual producers labor effort to their privately held resources. Since private resources are fixed at one, the rate of exploitation can be written as

$$H = \begin{cases} \frac{\sum_i l_{it}^c}{n - \sum_{j=1}^t Z_j} & \text{if the resource is exploited as commons} \\ l_{it}^r & \text{if the resource is privatized} \end{cases} \tag{4}$$

where the superscripts c and r indicate common and private property respectively.

Exploitation reduces resource quality according to a damage function, $d(H)$, with $d'(H) > 0$. Hence introducing this argument into expression (1) we see that resource quality evolves according to

$$\alpha_t = \alpha_{t-1} + \tau(\overline{\alpha} - \alpha_{t-1}) - d(H). \tag{5}$$

or, as will prove algebraically convenient later,

$$\alpha_t = \Omega\alpha_{t-1} + K, \tag{6}$$

where $\Omega = 1 - \tau$ and $K = \tau\bar{\alpha} - d(H)$. The dynamics of resource quality thus depend on the damage function, $d(H)$, which itself depends on the rate at which the resource is exploited, H. Below we demonstrate that exploitation rates differ systematically across property right regimes, hence changes in property right regimes impact the dynamics of resource quality in a way that will shortly be made precise.

3.1 Property Rights Regimes

Each unit of the resource may be exploited as either common or private property. We assume that, if an individual chooses, they may convert one unit of common resources to private at a cost in terms of time given by

$$p_t = p(Z_t) \tag{7}$$

where $Z_t = \sum_{j=1}^t Z_j$. Assume further that $p'(Z_t) > 0$, and $p''(Z_t) > 0$, so that the cost function is increasing and convex in the amount of resources privatized. This cost function can be interpreted as representing both time spent establishing a private claim on the resource and/or the expense of installing an exclusion mechanism. That successive units of the resource are more costly to privatize may be due to difficulties associated with geography or terrain as, for example, if the resource is forest or range land. Alternatively, increasing costs might simply reflect the impact of increased demand on the price of purchasing and installing an exclusion technology, such as with fencing range land or preventing access to a lakeshore.

We assume that resources previously privatized require only maintenance of property rights, and that this is less expensive than undertaking new privatizations. The cost of property rights maintenance has only quantitative implications for our analysis; therefore we assume this cost is zero.

In choosing whether or not to privatize each individual needs to compare their equilibrium utilities under the two property rights regimes. This requires that we first solve the individuals' utility maximization problems under these two regimes.

3.2 Individuals that Exploit the Commons

We assume that there is a static commons problem in that individuals who exploit the commons maximize

$$\max_{\{c_{it}^c, l_{it}^c\}} (c_{it}^c)^\gamma (T - l_{it}^c)^\eta \tag{8}$$

subject to $c_{it}^c = \alpha_t^c l_{it}^c \left(\frac{1}{H}\right)^\mu$ taking as given the rate of exploitation. Denoting the lagrange multiplier by λ, the first order conditions for this problem are given by

$$\gamma (c_{it}^c)^{\gamma-1} (T - l_{it}^c)^\eta + \lambda = 0, \tag{9}$$

$$-\eta (c_{it}^c)^\gamma (T - l_{it}^c)^{\eta-1} - \lambda \alpha_t^c \left(\frac{1}{H}\right)^\mu = 0. \tag{10}$$

Using the symmetry of the model, we can write the rate of exploitation of the commons as

$$H^c = \frac{\sum_i l_{it}^c}{n - Z_t} = \frac{\hat{n} l_{it}^c}{n - Z_t} \tag{11}$$

where $\widehat{n}$ denotes the number of producers who choose to exploit the common property resource. However, since each producer who privatizes receives exactly one unit of the resource, the number of producers left exploiting the commons must be exactly $\widehat{n} = n - Z_t$. Therefore we can write the rate of exploitation as $H = l_{it}^c$.

Making this substitution, and using standard methods, it can be shown that the optimal labor supply is given by

$$l_{it}^c = \frac{\gamma}{\gamma + \eta} T. \tag{12}$$

Therefore, the rate of exploitation is simply $H^c = \frac{\gamma}{\gamma+\eta} T$. Substituting this expression into (2) gives the individual producer's optimal demand for the consumption good:

$$c_{it}^c = \alpha_t^c \left(\frac{\gamma}{\gamma + \eta} T\right)^{1-\mu}. \tag{13}$$

The producer's indirect utility function is then given by

$$v_{it}^c (\alpha_t^c, T) = (\alpha_t^c)^\gamma \left(\frac{\gamma}{\gamma + \eta}\right)^{\gamma(1-\mu)} \left(\frac{\eta}{\gamma + \eta}\right)^\eta T^{\gamma(1-\mu)+\eta}. \tag{14}$$

3.3 Individuals that Exploit Privatized Resources

Individuals that exploit private resources fully account for the effects of their own exploitation on the productivity of the resource and maximize

$$\max_{\left\{c_{it}^{p}, l_{it}^{p}\right\}} \left(c_{it}^{r}\right)^{\gamma} \left(T - l_{it}^{p}\right)^{\eta} \tag{15}$$

subject to $c_{it}^{r} = \alpha_{t}^{r} l_{it}^{r} \left(\frac{1}{l_{it}^{r}}\right)^{\mu}$. The first order conditions for this problem are given by

$$\gamma \left(c_{it}^{r}\right)^{\gamma-1} \left(T - l_{it}^{r}\right)^{\eta} + \lambda = 0 \tag{16}$$

$$-\eta \left(c_{it}^{r}\right)^{\gamma} \left(T - l_{it}^{r}\right)^{\eta-1} - \lambda \left(1-\mu\right) \alpha_{t}^{r} \left(l_{it}^{r}\right)^{-\mu} = 0. \tag{17}$$

Again, usual methods give

$$l_{it}^{r} = \left(\frac{\gamma\left(1-\mu\right)}{\gamma\left(1-\mu\right)+\eta}\right) T, \tag{18}$$

$$c_{it}^{r} = \alpha_{it}^{r} \left[\left(\frac{\gamma\left(1-\mu\right)}{\gamma\left(1-\mu\right)+\eta}\right) T\right]^{1-\mu}. \tag{19}$$

The indirect utility function for producers exploiting private resources is therefore

$$v_{it}^{r}\left(\alpha_{it}^{r}, T\right) = \left(\alpha_{it}^{r}\right)^{\gamma} \left(\frac{\gamma\left(1-\mu\right)}{\gamma\left(1-\mu\right)+\eta}\right)^{\gamma(1-\mu)} \left(\frac{\eta}{\gamma\left(1-\mu\right)+\eta}\right)^{\eta} T^{\gamma(1-\mu)+\eta}. \tag{20}$$

Before continuing, note that the environmental damage to the commons is greater than the environmental damage incurred by private resources. Specifically, since $\mu \in (0,1)$,

$$H^{c} = \left(\frac{\gamma}{\gamma+\eta}\right) T > \left(\frac{\gamma\left(1-\mu\right)}{\gamma\left(1-\mu\right)+\eta}\right) T = H^{r}, \tag{21}$$

and $d\left(H^{c}\right) > d\left(H^{r}\right)$. This is a direct consequence of the static commons problem. Individual resource exploiters are quite aware that they face this problem and that it impacts the utility levels they enjoy. The incentive to privatize the resource stems directly from the desire to avoid the deleterious effects of the commons problem.

3.4 The Decision to Privatize

If an individual chooses to privatize a unit of resource, they incur a time cost of $p\left(Z_t\right)$, hence we rewrite the indirect utility function (18) to obtain the value of privatization:

$$v_{it}^{r}\left(\alpha_{t}^{c}, T - p\left(Z_t\right)\right) = \left(\alpha_{t}^{c}\right)^{\gamma} \left(\frac{\gamma\left(1-\mu\right)}{\gamma\left(1-\mu\right)+\eta}\right)^{\gamma(1-\mu)} \left(\frac{\eta}{\gamma\left(1-\mu\right)+\eta}\right)^{\eta} \left(T - p\left(Z_t\right)\right)^{\gamma(1-\mu)+\eta}. \tag{22}$$

Notice that we have replaced α_{it}^{r} with α_{it}^{c}. In the period when a producer privatizes a unit of resource, it is taken from the commons and therefore enjoys the current commons level of quality. So, in the initial period of privatization, resource quality is identical between commons and private property. The decision to privatize is driven, not by a desire to benefit from a higher immediate resource quality, but to avoid the static commons problem.

Individuals will choose to privatize as long as $v_{it}^r(\alpha_t^c, T - p(Z_t)) \geq v_{it}^c(\alpha_t^c, T)$. The equilibrium level of privatization, $\overline{Z}$, can therefore be found as the solution to

$$\begin{aligned} & (\alpha_t^c)^\gamma \left(\frac{\gamma}{\gamma+\eta}\right)^{\gamma(1-\mu)} \left(\frac{\eta}{\gamma+\eta}\right)^\eta T^{\gamma(1-\mu)+\eta} \\ = \; & (\alpha_{it}^c)^\gamma \left(\frac{\gamma(1-\mu)}{\gamma(1-\mu)+\eta}\right)^{\gamma(1-\mu)} \left(\frac{\eta}{\gamma(1-\mu)+\eta}\right)^\eta (T - p(Z_t))^{\gamma(1-\mu)+\eta}, \end{aligned} \tag{23}$$

which can be reduced to

$$p\left(\overline{Z}_t\right) = T\left[1 - \left(\frac{(1-\mu)\gamma+\eta}{\gamma+\eta}\right)\left(\frac{1}{1-\mu}\right)^{\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}}\right]. \tag{24}$$

This gives an implicit solution for $\overline{Z}_t$. Notice that $\overline{Z}_t$ is both time and resource quality invariant, and is determined by each individual trading off the time cost of privatization against the gains from avoiding the commons problem. Assuming that $p(Z_t)$ is bijective, we can write $\bar{Z} = p^{-1}(\cdot)$, which is constant. For simplicity, we also assume that $n = t^{**}\bar{Z}$, so that all resources will be privatized in time $t = t^{**}$. This also implies that no producer will be left without resources when the commons is completely privatized. This assumption has no substantive consequences for our results.

4 The Evolution of Total Resource Quality

Environmental quality, E_t, is measured by the productive capacity of the resource. For common property resources, environmental quality would therefore be $\alpha_t^c(n - Z_t)$. Thus, environmental quality in this model reflects all in situ benefits the resource may yield, but does not account for any nonuse value. To study the evolution of the resource we need to examine two dynamic regimes, the time period up to full privatization of the resource, $0 < t < t^{**}$, and the time period after full privatization, $t^{**} \leq t$.

4.1 The Evolution of Total Resource Quality with a Mix of Common and Private Ownership.

For $0 < t < t^{**}$ we may successively substitute (6) into itself to obtain the following expression for environmental quality

$$\begin{aligned} E_{t<t^{**}} &= n\Omega^t\alpha_0^c + \left(n - t\bar{Z}\right)K^c\sum_{j=1}^{t} j\Omega^{j-1} + \bar{Z}K^c\sum_{j=1}^{t}(t-j)\,\Omega^{t-j} + \bar{Z}K^r\sum_{j=1}^{t} j\Omega^{t-j} + \bar{Z}\bar{d}\sum_{j=1}^{t}\Omega^{j-1} \\ &= n\Omega^t\alpha_0^c + K^c\frac{1}{1-\Omega}\left[n\left(1-\Omega^t\right) - t\bar{Z} + \bar{Z}\frac{\Omega}{1-\Omega}\left(1-\Omega^t\right)\right] \\ &\quad + \bar{Z}K^r\left(\frac{1}{1-\Omega}\right)\left(t - \Omega\left(\frac{1-\Omega^t}{1-\Omega}\right)\right) + \bar{Z}\bar{d}\left(\frac{1-\Omega^t}{1-\Omega}\right). \end{aligned} \tag{25}$$

Where $K^c \equiv \tau\overline{\alpha} - d\left[\left(\frac{\gamma}{\gamma+\eta}\right)T\right]$, $K^r \equiv \tau\overline{\alpha} - d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)T\right]$ and $\overline{d} \equiv d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)T\right] - d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)\left(T - p\left(\overline{Z}_t\right)\right)\right]$. The evolution of environmental quality over the period $0 < t < t^{**}$ can then be understood by differencing this equation to obtain

$$\Delta E \quad | \quad _{0<t<t^{**}} \equiv E_t - E_{t-1} = n\alpha_0^c \Omega^{t-1} (\Omega - 1) + K^c \left(\frac{1}{1-\Omega} \right) \left[n\Omega^{t-1} (1-\Omega) - \left(1-\Omega^t\right) \bar{Z} \right]$$
$$+\bar{Z} K^r \left(\frac{1-\Omega^t}{1-\Omega} \right) + \bar{Z}\bar{d}\Omega^{t-1}. \tag{26}$$

We immediately note that in this regime the evolution of resource quality has some simple properties that we gather together in the following lemma.

Lemma 1 *If full privatization takes "long enough" then the time path of resource quality is "U" shaped.*
(i) As $t \to 1 \Rightarrow \Delta E\mid_{0<t<t^{**}} \to A < 0$ *if* $\alpha_0^c \geq \overline{\alpha}$.
(ii) As $t \to t^{**} \to \infty \Rightarrow \Delta E\mid_{0<t<t^{**}} \to B > 0$
(iii) $\exists$ *a unique* $t^* > 0$ *for which* $\Delta E\mid_{0<t<t^{**}} = 0$ *if* $\alpha_0^c \geq \overline{\alpha}$.

Proof. Appendix. ■

Provided that the initial quality of the resource is at or above its unexploited steady state level, $\alpha_0^c \geq \overline{\alpha}$, its quality must be degraded in the first period of exploitation. This follows simply from the fact that both the common and private ownership steady states lie below the unexploited steady state and convergence is monotonic in both property rights regimes. If full privatization takes "long enough" this implies that the rate at which the resource is transformed from common to private ownership is slow. Units of the resource under common ownership converge to a lower steady state of environmental quality than those under private ownership. If the privatization process is slow, then many units of the resource degrade to a level of environmental quality below the privatized steady state; when these units are eventually privatized their quality must rise. This yields the U shaped time path for total environmental quality.

4.2 The Evolution of Total Resource Quality with Full Private Ownership.

Once all of the resource has been privatized the resource quality dynamic depends only on the relative rates of damage and recovery and not, by definition, on changes in ownership regime. Writing environmental quality in t^{**} as $E_{t^{**}}$ we have from (25)

$$\begin{aligned} E_{t^{**}} &= \bar{Z} \left[t^{**}\Omega^{t^{**}} \alpha_0^c + K^c \sum_{j=1}^{t^{**}-1} (t-j)\,\Omega^j + K^r \sum_{j=1}^{t^{**}} j\Omega^{t^{**}-j} + \bar{d} \sum_{j=1}^{t^{**}} \Omega^{j-1} \right] \\ &= t^{**}\bar{Z} \left[\Omega^{t^{**}} \alpha_0^c + \frac{K^c}{t^{**}(1-\Omega)} \left(\frac{\Omega\left(1-\Omega^{t^{**}}\right)}{1-\Omega} - t^{**}\Omega^{t^{**}} \right) \right. \\ &\quad \left. + \frac{K^r}{t^{**}(1-\Omega)} \left(t^{**} - \frac{\Omega\left(1-\Omega^{t^{**}}\right)}{1-\Omega} \right) + \frac{\bar{d}}{t^{**}} \left(\frac{1-\Omega^t}{1-\Omega} \right) \right]. \end{aligned} \tag{27}$$

Since $t^{**}\bar{Z} = n$ is the fixed size of the resource, the bracketed term is private resource quality. Denote this by $\alpha_{t^{**}}^r$. Then in each subsequent period, environmental quality is given by

$$E_{t^{**}+m} = n \left[\Omega^m \alpha_{t^{**}}^r + K^r \sum_{j=0}^{m-1} \Omega^j \right] = n \left[\Omega^m \alpha_{t^{**}}^r + K^r \left(\frac{1-\Omega^m}{1-\Omega} \right) \right]. \tag{28}$$

The change in environmental quality is now determined solely by the change in the quality parameter. That is,

$$\Delta E \quad |_{\ t^{**}\leq t} = n\left(\alpha^r_{t^{**}+m+1} - \alpha^r_{t^{**}+m}\right) = n\left(\Omega^{m+1}\alpha^r_{t^{**}} + K^r\frac{1-\Omega^{m+1}}{1-\Omega} - \Omega^m\alpha^r_{t^{**}} - K^r\frac{1-\Omega^m}{1-\Omega}\right)$$
$$= n\left(\alpha^r_{t^{**}}\left(\Omega^{m+1}-\Omega^m\right) - \frac{K^r}{(1-\Omega)}\left(\Omega^{m+1}-\Omega^m\right)\right). \tag{29}$$

This immediately leads to a second lemma.

Lemma 2 *With full privatized ownership*
(i) The system converges monotonically to the steady state $nK^r\left(\frac{1}{1-\Omega}\right)$.
(ii) Convergence is from above with declining environmental quality if $\alpha^r_{t^{**}} > \frac{K^r}{(1-\Omega)}$
(iii) Convergence is from below with increasing environmental quality if $\alpha^r_{t^{**}} < \frac{K^r}{(1-\Omega)}$.

Proof. Appendix. ■

The intuition here is quite straightforward. Once all the resource has been privatized the dynamics become quite simple, the level of exploitation of each unit of the resource occurs at a constant rate giving a constant level of degradation. If at the time the resource is fully privatized its environmental quality differs from the fully privatized steady state, then it converges monotonically to this level from above or below as appropriate.

4.3 Environmental Quality Time Paths

We now have all the components we require to characterize the possible time paths for environmental quality

Proposition 3 *Environmental quality will follow a U-shaped curve if (i)* $t^* < t^{**}$*, or if (ii)* $E_{t^{**}} < nK^r\left(\frac{1}{1-\Omega}\right)$*,but will decline monotonically otherwise.*

Proof. Follows immediately from lemmas 1 and 2. ■

The proposition can perhaps best be understood with the help of a diagram

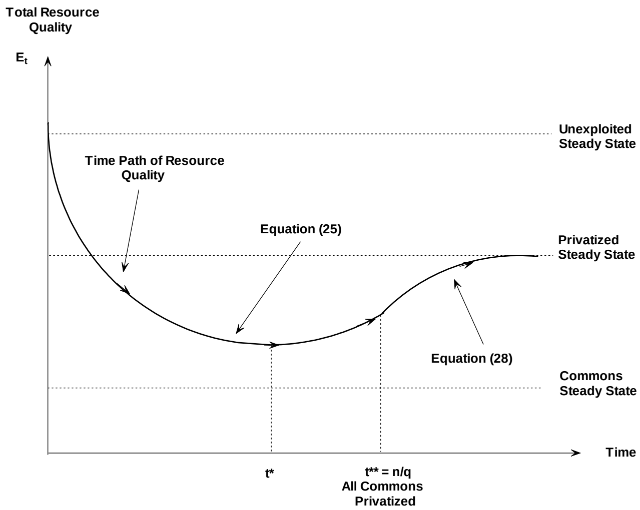

Figure 1a: Full Privatization after Environmenetal Recovery $t^{**} > t^{*}$.

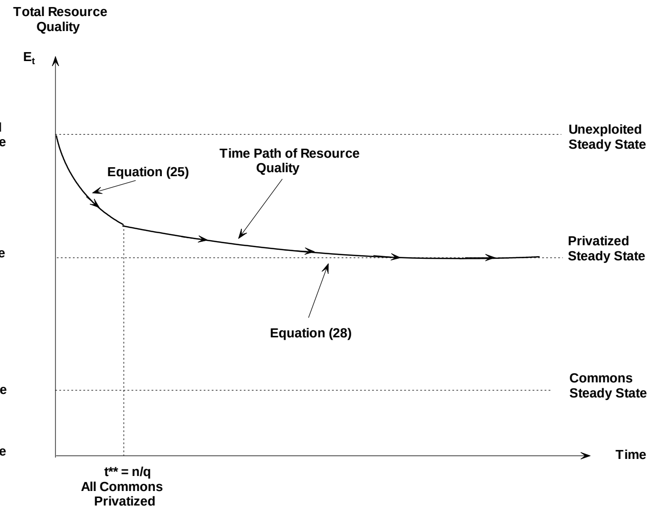

Figure 1b: Monotonic Environmental Degredation, $t^{*} \geq t^{**}$

Two possibilities present themselves. First, as in figure 1a, privatization proceeds slowly while the dynamics of environmental change are relatively fast. Hence we have the story described by lemma 1. Environmental quality rapidly approaches the commons steady state dipping below the full privatization steady state. However eventually more units of the resource are privatized, these rapidly recover and aggregate environmental quality rises to the fully privatized steady state. The second possibility is as described in figure 1b, privatization occurs rapidly relative to the rate of environmental degradation. The commons is fully privatized before the level of environmental quality drops below the fully privatized steady state. Hence environmental quality degrades monotonically.

5 The Severity of Environmental Damage and Speed of Environmental Recovery: Some Simulation Results.

Our preceding analysis demonstrates that as property rights evolve from a common to private property regime, environmental quality can follow either a U-shaped time path or one of monotonic degradation. In the case where the time path is U-shaped we wish to investigate what is crucial in determining how long it takes for environmental degradation to reverse itself. We also wish to investigate what is important in determining the severity of the environmental damage that must be experienced before an improvement occurs. Analysis of the model does not readily provide answers to these questions, however numerical simulations provide some interesting insights.

In order to analyze the factors determining the length of time that elapses before environmental quality begins to improve, note from the proof of Lemma 1, that prior to full privatization the derivative of environmental quality with respect to time can be written as

$$A\Omega^{t} + B\Omega^{t+1} + C \tag{30}$$

where $A = (1-\Omega)\alpha_0^c n \ln(\Omega) - \bar{d}\bar{Z}\ln(\Omega) - K^c n \ln(\Omega)$, $B = \frac{K^r \bar{Z} \ln(\Omega)}{(1-\Omega)} - \frac{K^c \bar{Z} \ln(\Omega)}{(1-\Omega)}$, and $C = \bar{Z}(K^r - K^c)$.

The last term, C, is positive since $K^r > K^c$. That the first term is negative is demonstrated in the proof of Lemma 1. The middle term, B, is negative since $K^r > K^c$ and $\ln(\Omega) < 0$. Since $\Omega < 1$, the first two terms disappear as $t \to \infty$. Therefore, as long as there is plenty of commons available, environmental quality will eventually begin to improve. Furthermore, it should be clear that the magnitude of Ω is pivotal in determining the speed with which environmental quality begins to improve: the lower is Ω, the faster the first two terms will approach zero. In other words, as τ, the natural speed of adjustment to the steady state, increases, environmental quality will improve more rapidly.

In addition, any change in the parameters that increases A or B, in absolute value, will prolong the degradation, while any change that increases C will result in earlier environmental improvement. For example, the greater is the ratio of initial environmental quality to the unexploited steady state, $\frac{\alpha_0^c}{\bar{\alpha}}$, the longer it will take for environmental quality to begin to improve. This occurs since, as $\frac{\alpha_0^c}{\bar{\alpha}}$ increases, environmental quality must fall farther to reach its new, exploited steady state.

To see the impact of the various parameters on the length of time before environmental improvement occurs, consider the following simplified model. First, choose $p(Z_t) = Z_t$ and $d(H) = H$. Second, assume that agents' utility functions exhibit constant returns to scale, so that $\eta = 1 - \gamma$. We examine the impact of altering the exogenous factors by specifying a benchmark set of parameters, and simulating the model while allowing one parameter at a time to vary from the benchmark case. While this is not an exhaustive analysis, it does provide some insights into the impact these parameters have on the dynamics of the model.

For the benchmark case, we choose $\gamma = \eta = 0.5$, implying an equal budget share of consumption and leisure in the agents optimal choice. $\tau = 0.1$ is a sufficiently low speed of adjustment to allow for substantial environmental damage. The number of individuals must be large enough for a substantial commons problem to exist, we therefore choose $n = 100$. The ratio of initial to unexploited steady state environmental quality, $\frac{\alpha_0^c}{\bar{\alpha}}$, should be greater than or equal to one in order to conform to the conditions in our proof. We therefore choose this ratio to be one in the benchmark case and allow it to increase in the simulations.[3] Finally, T is arbitrarily set to 24 and $\mu = 0.582557$. This latter choice is made to ensure that the quantity of resources privatized in each time period is an integer, specifically, 2. This restriction is imposed to keep the benchmark case consistent with the analytical model, however, to reduce computational costs, it is not maintained throughout the simulations.

The table below shows the percentage change from the benchmark case of the time to environmental improvement, while changing various parameters.

Parameter	Range of Values	Percentage Change
τ	0.01 to 0.99	218.31% to -94.48%
η[4]	0.38 to 0.50	3.37% to 0.0%
	0.50 to 0.99	0.0% to 123.40%
μ	0.01 to 0.99	500.84% to -75.60%
$\frac{\alpha_0^c}{\bar{\alpha}}$	1.05 to 4.0	0.0% to 45.89%
n	50 to 199	-23.93% to 24.94%

(31)

As the table indicates, the length time to environmental improvement is most sensitive to changes in τ, η and μ. We noted above that higher values of τ imply environmental quality naturally adjusts faster to the

[3] It should be noted that this condition is sufficient for environmental quality to initially decline, however it is not necessary. For various parameterizations, this ratio may be less than one and initial environmental quality will still decline.

[4] Solutions of the model are not well-behaved for values of η less than 0.38 in this highly stylized version of the model.

steady value. As a result environmental quality recovers more quickly and the turning point occurs earlier. The assumption of constant returns to scale in utility implies that an increase in η, the budget share of leisure, is accompanied by an equiproportional decrease in the budget share of consumption. Therefore, increasing η causes consumers to devote more time to leisure and less to exploiting the resource and environmental quality declines much more slowly. As a result, the time to reach the minimum is considerably extended. The parameter μ captures the marginal loss in production due to resource exploitation. Low values of μ correspond to a *large* impact of exploitation, since production is a function of the inverse of the rate of exploitation. It follows that, for low values of μ, the resource is degraded rapidly and the recovery time is prolonged. Higher values of μ, imply a mild impact on production, and resource quality improves more rapidly.

Also of interest is the total amount of environmental damage before quality begins to improve. The table below indicates the percentage change in environmental quality from its initial to minimum points. As the table indicates, environmental quality is once again most sensitive to τ, η and μ. Higher values of τ correspond to a rapid rate of natural recovery, and so the maximum environmental loss is relatively mild as τ approaches one. As η approaches one, consumers devote more of their time to leisure and the maximum environmental loss is low. Similarly for μ. Higher values of μ correspond to a milder impact of exploitation on production. This allows producers to achieve their desired level of consumption with lower exploitation levels, and therefore the total environmental loss is decreased.

Parameter	Range of Values	Percentage Change
τ	0.01 to 0.99	100% to 11.95%
η	0.38 to 0.99	100% to 2.36%
μ	0.01 to 0.99	100% to 32.44%
$\frac{\alpha_0^c}{\bar{\alpha}}$	1.05 to 4.0	94.16% to 83.60%
n	50 to 199	90.81% to 100%

(32)

6 Conclusion

In this paper we have constructed a simple model that generates a U-shaped pattern for environmental quality solely as a result of the evolution of private property over a formerly common property resource. Initially a fixed set of producers jointly exploit a common property resource and resource quality declines. For a one-time, fixed cost producers can avoid the commons problem by privatizing a portion of the resource. In the equilibrium, we show that the commons will eventually be entirely privatized, and resource quality will improve as producers reduce the rate at which they exploit the now private resource.

Although the model is highly stylized, it does yield some important implications. First, it contributes to the underlying theory by demonstrating that an EKC may occur independent of consumer preferences and production technology. Instead, when faced with an externality that reduces the marginal product of their labor, producers find it optimal to eliminate the commons problem by privatizing the resource.

Second, while government is not formally modeled in this paper, the results imply that policies devoted to reducing the costs of establishing private property rights, and protecting those rights once established, can improve environmental quality. Improving access to legal institutions and law enforcement, even when not directly related to environmental stewardship, may improve environmental quality by providing incentives for more efficient behavior.

We have presented a simple model where producers consider only a static commons problem. An important extension of this paper would be to model agents who may realize a stream of benefits, or costs, and so may be effected by a dynamic as well as static commons problem. Intuitively we doubt that this would have significant qualitative implications, but the quantitative consequences might be large. Also, in our model, producers directly experience the negative impact of overexploitation. An economy where the costs of environmental degradation are borne, in part or wholly, by a group who do not directly benefit from production may provide a richer framework for further results. Finally, privatizing resources may alter considerably the distribution of wealth in an economy. The model could be extended to explore the social welfare effects when some agents are net losers in privatization.

7 Appendix

Proof of Lemma 1:. Taking the limits of (26)

$$\Delta E \quad | \quad _{0<t<t^{**}} \equiv E_t - E_{t-1} = n\alpha_0^c \Omega^{t-1}(\Omega - 1) + K^c\left(\frac{1}{1-\Omega}\right)\left[n\Omega^{t-1}(1-\Omega) - \left(1-\Omega^t\right)\bar{Z}\right]$$
$$+\bar{Z}K^r\left(\frac{1-\Omega^t}{1-\Omega}\right) + \bar{Z}\overline{d}\Omega^{t-1}.$$

as $t \to 1$ and $t \to \infty$ provides

$$\begin{aligned}\lim_{t\to 1}\Delta E \quad | \quad _{0<t<t^{**}} &\to n\alpha_0^c(\Omega - 1) + K^c\left(n - \bar{Z}\right) + \bar{Z}K^r + \bar{Z}\overline{d} \\ &= n\tau\left(\overline{\alpha} - \alpha_0^c\right) - \left(n - \bar{Z}\right)d\left[\left(\frac{\gamma}{\gamma+\eta}\right)T\right] - \bar{Z}d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)\left(T - p\left(\overline{Z}_t\right)\right)\right] \equiv A < 0\end{aligned}$$

and

$$\lim_{t\to\infty}\Delta E\mid_{0<t<t^{**}} \to \left(K^r - K^c\right)\bar{Z}\left(\frac{1}{1-\Omega}\right) \equiv B > 0$$

To show that the turning point exists and is unique requires solving (26) for $\Delta E\mid_{0<t<t^{**}} = 0$,which may be written as

$$n\alpha_0^c\Omega^t\left(\frac{\Omega-1}{\Omega}\right) + K^c\left(\frac{1}{1-\Omega}\right)\left[n\Omega^t\left(\frac{\Omega-1}{\Omega}\right) - \left(1-\Omega^t\right)\bar{Z}\right]$$
$$+\bar{Z}K^r\left(\frac{1-\Omega^t}{1-\Omega}\right) + \bar{Z}\overline{d}\left(\frac{\Omega^t}{\Omega}\right) = 0$$

It is now a simple matter to rearrange this expression and take logs to give

$$t^* = \frac{\ln\left[\frac{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right)}{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right) - n\alpha_0^c\left(\frac{\Omega-1}{\Omega}\right) + K^c\left(\frac{n}{\Omega}\right) + \overline{d}\left(\frac{\bar{Z}}{\Omega}\right)}\right]}{\ln\Omega}$$

since $\ln\Omega < 0$ we require $\ln\left[\frac{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right)}{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right) - n\alpha_0^c\left(\frac{\Omega-1}{\Omega}\right) + K^c\left(\frac{n}{\Omega}\right) + \overline{d}\left(\frac{\bar{Z}}{\Omega}\right)}\right] < 0$ or $\frac{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right)}{(K^r-K^c)\bar{Z}\left(\frac{1}{1-\Omega}\right) - n\alpha_0^c\left(\frac{\Omega-1}{\Omega}\right) + K^c\left(\frac{n}{\Omega}\right) + \overline{d}\left(\frac{\bar{Z}}{\Omega}\right)} < 1$. Since $K^r > K^c$ this reduces to $(1-\Omega)\,n\alpha_0^c > nK^c + \bar{Z}\overline{d}$ which may be reexpressed as

$$\tau n\alpha_0^c > n\left[\tau\overline{\alpha} - d\left[\left(\frac{\gamma}{\gamma+\eta}\right)T\right]\right] + \bar{Z}\left[d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)T\right] - d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)\left(T - p\left(\overline{Z}_t\right)\right)\right]\right]$$

Hence if $\alpha_0^c \geq \overline{\alpha}$ we know $n > \bar{Z}$ and the result follows from $d\left[\left(\frac{\gamma}{\gamma+\eta}\right)T\right] > d\left[\left(\frac{(1-\mu)\gamma}{(1-\mu)\gamma+\eta}\right)T\right]$ ■

Proof of Lemma 2:. Taking the limit of (29) as $t \to \infty$ we obtain

$$\lim_{t\to\infty}\Delta E\mid_{t^{**}\leq t} \to 0.$$

Further rearranging (29) provides

$$\Delta E\mid_{t^{**}\leq t} = n\left(\Omega^{m+1} - \Omega^m\right)\left(\alpha_{t^{**}}^r - \frac{K^r}{1-\Omega}\right).$$

Hence since $\left(\Omega^{m+1} - \Omega^{m}\right) < 0$ it follows that

$$\Delta E \mid_{t^{**} \leq t} \gtreqless 0 \text{ as } \alpha^{r}_{t^{**}} \lesseqgtr \frac{K^{r}}{1-\Omega}.$$

■

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