

Eliciting Individual-Specific Discount Rates

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Eliciting Individual-Specific Discount Rates

Longstanding debate over the appropriate social discount rate for public projects stems from our lack of knowledge about how individual discount rates vary across people and across choice contexts. Using a sample of roughly 15,000 choices by over 2000 individuals, we estimate utility-theoretic models concerning private tradeoffs involving money over time that reveal individual-specific discount rates. We control for experimentally differentiated choice scenarios, sociodemographic heterogeneity, and elicitation formats, and complex forms of heteroscedasticity. Statistically significant heterogeneity in discount rates is quantified for both an exponential discounting model and a competing hyperbolic model, but neither specification clearly dominates. (D91, H4, C25, C35)

Whenever the benefits and costs of a non-tradable durable good or a public good have different time profiles, discounting is a necessary step in any assessment of that good's desirability. A pervasive feature of the existing social choice literature is the notion that we need one common discount rate for social decision-making. One single discount rate is a convenient assumption for many models. If capital markets were perfect, of course, then the market interest rate would accurately reflect everyone's intertemporal preferences. However, factors such as transactions costs and people's tendencies to define artificially separate budgets for different activities invalidate the assumption of perfect capital markets. This foils any expectation that individuals will adjust levels of present and future consumption to bring marginal rates of substitution into line with a single market interest rate.

Both Robert C. Lind (1990) and Kenneth J. Arrow et al.(1996) argue that discount rates should be based on how individuals trade off between present and future consumption, and that these rates are indeed likely to differ contextually. At the individual level, at least for non-tradeable goods, the discount rate is an artifact of preferences over current versus future consumption, just as willingness to pay for different commodities is an artifact of preferences over the contemporaneous consumption of different goods. There is no reason why

marginal rates of time preference should be any less individual, or less context-specific, than marginal rates of substitution between any pair of contemporaneous goods.² The quest for a single representative discount rate to use in making social choices stems from the problem that heterogeneity in the relevant individual discount rates is largely unquantified.

For individual i , where $i = 1, \dots, N$, let b_{it} represent *net* benefits in periods $t = 1, \dots, T$ and let $(b_{i1}, b_{i2}, \dots, b_{iT})$ be the time profile of net benefits.³ In the absence of information about individual-specific time-preferences, the present discounted value of aggregate net benefits of a particular durable or public good in each future period, $\sum_{i=1}^N b_{it}$, must be computed using some aggregate discount rate, r_a :

$$PDV_a = \sum_{t=1}^T (1 + r_a)^{-t} \left(\sum_{i=1}^N b_{it} \right) \quad (1)$$

In contrast, a formula that honors individual time preferences would use individual discount rates, r_i :

$$PDV_i = \sum_{i=1}^N \left(\sum_{t=1}^T (1 + r_i)^{-t} b_{it} \right) \quad (2)$$

In this case, the first step is to discount individual net benefits back to the present using a discount factor appropriate for that individual, $(1 + r_i)^{-t}$. The second step is to aggregate these individual discounted net benefits into a measure of social benefits. The practical problem for implementing this alternative measure is that we typically do not know much about the values of r_i , for an individual with particular attributes, that might apply in a particular choice context.

In this paper, we propose and demonstrate a strategy for the measurement of individual-specific discount rates via survey methods. We first lay out a formal random utility framework to accommodate the conceptual problem of consumer choice when the individual faces a time profile of costs in order to obtain some time

²The individual discount rate is equal to the marginal rate of time preference, minus one. The "pure" rate of time preference differs from the individual discount rate in that it is evaluated along the individual's intertemporal budget constraint at a point corresponding to equal amounts of current and future consumption (Emily C. Lawrance, 1991).

³We will not discriminate between benefits in terms of consumption and benefits in terms of utility.

profile of services of a durable or public good, as well as a second choice concerning the receipt of money with different time profiles. This general model involves two types of choices because we envision that this approach will have value in producing clean estimates of fitted individual discount rates that can be used simultaneously to explain variation in other individual decisions in a multi-equation system. For identification, as in any simultaneous equations model, it will be important to include exogenous determinants of individual discount rates that do not also explain the durable or public goods choice.

We then focus empirically on just the individual discount rate portion of the model. We query survey respondents concerning their preferred way to receive some lottery winnings—either as a stream of annual payments over some time horizon, or as a smaller lump sum in the current period. With these choice data, we can generate empirical estimates of exponential and simple hyperbolic discount functions and also describe the results of attempting to fit a more general hyperbolic form. Our models accommodate broad heterogeneity in preferences and complex forms of heteroscedasticity in the underlying indirect utility specification. We also assess the sensitivity of estimated discount rates and error variances to a number of alternative choice scenarios and elicitation formats.

The main idea we wish to promulgate, by framing our model in this way, is that human behavior with respect to related choices should be consistent. One utility function should underlie all of the choices made by any single individual. Any random utility model we use to take advantage of choices that highlight subjects' individual discount rates should also be able to accommodate these same individuals' choices with respect to durable goods or public goods. The random utility models used to capture each type of choice will have common preference parameters, so the different types of choices can be pooled and estimated jointly. The specific discount rate choice and the additional durable or public goods choice can be combined in one model to improve our chances of identifying a wider range of preference parameters. A discount rate is an attribute of individual preferences that is not usually separately identifiable within the context of a single durable or public goods choice unless some strong assumptions are made. However, if durable or public goods choice

data can be combined with other choices by the same individuals that expressly and exclusively involve tradeoffs of money over time, within a fully compatible utility specification, then there is some hope for separately identifying heterogeneity in discount rates and heterogeneity in preferences for durable or public goods.

I. Background

A. Contextual Differences in Empirical Discount Rates

Over the last twenty years or so, both economists and psychologists have explored factors that can affect individual discount rates. Shane Frederick et al. (2002) provide a thorough survey of theoretical and empirical research concerning time discounting and time preference. They tabulate over forty attempts at empirical estimation of discount rates according to type (experimental or field), good(s) (money, life-years, etc.), real or hypothetical, elicitation method (choice, matching, rating, or pricing), time range (from less than one day, to 57 years), and finally according to the range of implied discount rates and the associated discount factors. What is striking about this summary is the extraordinary variance across studies in empirically estimated private discount rates across different choice contexts, even without considering possible systematic differences across sociodemographic groups. This accumulating evidence strengthens the case for departing from the convention of using one representative discount rate in decision-making. Where there are substantial groupwise differences in discount rates, it may be very important to preserve these differences in net benefits estimation. It is also possible that differences in discount rates across contexts (long- or short-term tradeoffs, private or public tradeoffs) will be sufficiently large that just one menu of group-specific discount rates will be insufficient. All of this points to a need for new techniques to elicit reliable group- and context-specific discount rates.

B. Discounted Utility Anomalies and Hyperbolic Discounting

Frederick et al. (2002) identify economists' reliance on the expedient single-parameter discounted utility (DU) model suggested by Paul A. Samuelson (1937) as one of the impediments to progress in discount rate

research. They inventory the suite of DU-anomalous results that have induced a number of researchers to think about other representations of discounting behavior. Many researchers have now explored these anomalies (e.g. George F. Loewenstein and Richard H. Thaler (1989), Loewenstein and Jon Elster (1992), and Loewenstein and Drazen Prelec (1992)). Frederick et al. (2002) emphasize that individual intertemporal tradeoffs can reflect a whole host of different processes that play out at the individual level, not just "pure time preference." Among possible confounding factors, they enumerate consumption reallocation, intertemporal arbitrage, concave utility, uncertainty, inflation considerations, expectations of changing utility, and the collection of tendencies labeled as habit formation, anticipatory utility, and visceral influences.

Among alternative discounting formulas, a generalized hyperbolic discount function is discussed by Loewenstein and Prelec (1992):

$$\phi_g(t) = (1 + \gamma t)^{-\beta/\gamma}, \beta, \gamma > 0 \tag{3}$$

where the γ parameter dictates how far the function departs from constant (exponential) discounting. The $\phi_g(t)$ form appears to have been defined by Charles M. Harvey (1986), and derived axiomatically by Prelec (1989). In the limit, as γ goes to zero, the generalized hyperbolic function becomes the standard exponential discounting function,

$$\phi_e(t) = \exp(-\beta t) \tag{4}$$

Generalized hyperbolic functional forms seem to have had their genesis with a one-parameter special case, $\phi_m(t) = (1 + \gamma t)^{-1}$, proposed by Richard J. Herrnstein (1981) and explored further by James Mazur (1987). The $\phi_m(t)$ form involves the constraint $\beta = \gamma$. Harvey (1986) suggested an alternative case,

$$\phi_h(t) = (1 + t)^{-\beta} \tag{5}$$

embodying the constraint that $\gamma = 1$.

C. Weitzman's Expert Survey Sample

In a recent paper in this *Review*, Martin L. Weitzman (2001) arrives at a functional form for discount rates identical to that in equation (3), but via a different route.⁴ From a survey of the opinions of over 2,000 professional Ph.D.-level economists, he determines that the social discount rates advocated by these experts range from -3 percent to +27 percent, with a sample mean of about 4 percent and a standard deviation of about 3 percent. He notes that the empirical marginal distribution appears to compare favorably, in terms of its shape, to a gamma probability density function. The key empirical insight is that individual expert opinions about discount rates vary rather substantially. Still, Weitzman's goal is to develop a model to produce a single social discount rate for policy evaluation that nevertheless accommodates heterogeneous opinions of experts concerning the intertemporal tradeoffs we should be willing to make.⁵

Weitzman acknowledges that "...the average panel member knows about, and typically does not feel acutely uncomfortable with, the approximation of constant exponential discounting. The primary disagreement among panel members is over the appropriate value of the as-if-constant discount rate." (p. 264). Weitzman notes that the amount of "uncertainty" about discount rates in the sample generates a sliding-scale effective discount-rate schedule, whose decline over time is significant enough to recommend that it be incorporated into discounting of long-term projects.⁶ We believe that this "uncertainty" might be better described as "heterogeneity." Had Weitzman collected the characteristics of each of these economists, he might have fit a regression-type model to explain the differences in their subjective social discount rates. However, he only differentiates between 50 "leading" economists and the rest of us who answered his survey. His point estimate for the mean preferred social discount rate in the "leading" sample differs from that for the other group, being slightly larger, but he undertakes no formal hypothesis testing, nor does he seek to

⁴Weitzman does not, himself, identify the similarity between his specification and that of Loewenstein and Prelec (1992).

⁵For details concerning the relationship between Weitzman's approach and the Loewenstein and Prelec (1992) model, see Appendix I.

⁶Richard G. Newell and William A. Pizer (2000) delve further into the consequences of uncertain discount rates. When the expectation is taken with respect to a density function for "the" discount rate, the expected discounted value of future benefits will be greater than the discounted value computed using the expected discount rate, since discounted values are a convex function of the discount rate.

use any more than just this implicit dummy variable to look for systematic differences in discount rates.

D. Empirical Estimation of Individual Discount Rates

A seminal paper by Jerry Hausman (1979) uses observed household choices among consumer durables (air conditioners) with higher and lower capital and operating costs to infer discount rates. In one model, these rates are allowed to vary with income levels.⁷

An experimental setting with numerous "matching"-type questions per subject is used by Kris N. Kirby and Nino N. Maraković (1995). This strategy allows estimation of individual-specific $\phi_e(t)$ and $\phi_h(t)$ discount functions, but does not extend to explaining heterogeneity in terms of any observable individual characteristics. John A. Cairns and Marjon M. van der Pol (1997) also ask matching questions (of roughly 500 survey subjects) concerning both "short run" and "long run" choices. They find evidence favoring the non-constant discounting models over the conventional constant discounting model.⁸ The possibility of casting discount rates as systematic varying parameters is noted, but they indicate that their data were not collected to make these distinctions.

Survey "choice"-type questions (the family of methods employed in the present study) appear to have been first used to infer marginal rates of time preference by Magnus Johannesson and Per-Olov Johansson (1997), albeit in a health context, and they do not explore how this rate itself varies with sociodemographics. Intertemporal preferences for health are also elicited via survey in van der Pol and Cairns (2001). Implied discount rates for two samples of about 400 respondents vary according to whether own health or others' health is being considered, but these discount rates are not systematically differentiated by sociodemographics.

In another vein, Harrell W. Chesson and W. Kip Viscusi (2000) address discounting jointly with uncertainty, estimating implicit rates of time preference with respect to deferred gambles. They find that estimated discount rates decrease with the time horizon of the gamble, a result that is consistent with the

⁷Dermot Gately (1980) uses a similar approach with refrigerators and finds even higher implied discount rates (although he does not report any systematic heterogeneity in discount rates).

⁸They base their assessments on the sums of squared deviations between the actual empirical discount factors conveyed by respondents, and the fitted discount factors implied by each model, where these factors are determined by line-searches.

predictions of Loewenstein and Elster (1992) concerning time horizon effects. However, they acknowledge that “the combined tasks of discounting and probability assessment exceed the cognitive capabilities of many survey subjects.”

The only extant large-sample empirical estimates of discount rates in a revealed preference context are offered by John T. Warner and Saul Pleeter (2001, in this *Review*). In an ambitious study, they analyze the decisions of many thousands of US military personnel concerning a choice between a lump-sum separation benefit or an annuity, relying on a reduced-form model for the latent individual discount rate based on the discrete choices of individual subjects between their two payment alternatives.⁹ Statistically significant heterogeneity in the implicit exponential discount rates is confirmed, but there is little formal attention paid in the paper to the distinction between exponential and hyperbolic discounting models. The huge cross-sectional samples also raise the usual questions about heteroscedasticity, but its presence in the model is not assessed.¹⁰ In the present paper, we consider alternative discounting models and we explicitly model the variances of the errors, which are related to choice consistency across subgroups. (See Joffre Swait and Jordan Louviere, 1993; J.R. DeShazo and German Fermo, 2002).

Two very recent examples should also be mentioned: L. Robin Keller and Elisabetta Strazzera (2002) estimate both $\phi_e(t)$ and $\phi_h(t)$, but use an existing data set from Thaler (1981) to generate simulated data for their analyses. They mention the possibility of, but do not pursue, systematically varying individual discount rates. Survey data concerning choices among alternative climate change mitigation programs in the context of forest loss prevention are used by David F. Layton and Richard A. Levine (2002). In conjunction with Weitzman’s social discount rate distribution for professional economists, they calculate posterior and

⁹Such models have actually been in wide use for many years in the environmental non-market valuation literature, since they are suitable for the analysis of referendum contingent valuation data. These models admit for structural choice modeling, but these authors do not pursue a formal random utility framework for their analysis.

¹⁰Warner and Pleeter are careful to control for self-selection into their observed sample from the overall population of military personnel (not all of whom were eligible for the payment choice). But they still cannot compensate for non-random selection into the military in the first place, which may render these samples very different from the general population in terms of unobservables. We still have no real idea whether the estimated discount rates for their two samples are any more representative of rates in the general population (of similar ages and education levels) than are discount rates estimated from other special samples of respondents.

prior odds in favor of several discount rate intervals. This approach accommodates heterogeneity in discount rates but does not parameterize the relationship between the sizes of discount rates and other individual characteristics.

II. Formal Random Utility Choice Framework

Our formal model is intended to accommodate two choices by individuals. One is a choice concerning a non-tradable durable good or a long-lived public good. The second choice (upon which we will focus in this paper) is a stylized choice concerning whether the individual would prefer to take lottery winnings as (a.) a series of annual payments, or (b.) a smaller lump sum now. In the most general specifications of the random utility model presumed to underlie both types of choices, we will assume that current utility v_i depends linearly upon current net income, y_i (as a proxy for the consumption of all other goods), and the current flow of services from the durable or public good, g_i . In the case of homogeneous preferences, current utility can be expressed as:

$$v_i = \mu y_i + \delta g_i + u_i \tag{6}$$

where u_i is an error term. Of course, a linear specification for utility allows one to ignore the usual problem of departures between the discounting of utility and the discounting of consumption.

Current utility, however, is not the sole determinant of choices in cases where the individual faces different time profiles for future costs and future benefits. Taking advantage of the linearity in our specification, assume that the stream of future benefits from the flow of services of the durable or public good can be converted into a present discounted value G_i . For the money-denominated argument of utility, the durable good choice also implies a change in the present discounted value of future net income, Y_i , through both the one-time initial capital cost, C_i , and a time profile of operating costs over future periods, which in the simplest case could be assumed to be a constant per-period cost of c_i .

In this exposition, we will use just a binary choice concerning whether to purchase one specific model of a durable good, or, analogously, to vote in favor of the provision of a particular long-lived public good.¹¹ In binary choice models, the difference in utility levels across the two alternatives in the stated choice scenario, ΔV_i , is presumed to drive the individual's choice. In our linear model, this difference in discounted utilities will depend upon the difference in discounted net income levels, ΔY_i , and the difference in discounted net durable- or public-good benefits, ΔG_i . For completeness, the discounted error term would also need to be distinguished: u_i^* .

For conventional exponential discounting with individual discount rate r_i , in discrete time, the exponential discount factor is $\phi_e(t) = (1 + r_i)^{-t}$. If earned income in all relevant future periods up to T_i would be the same across the two choices, its level would net out of the linear utility difference in the following formula for ΔV_i :

$$\Delta V_i = \mu_i \left\{ \left[y_i - C_i - c_i \sum_{t=0}^{T_i-1} (1 + r_i)^{-t} \right] - [y_i] \right\} + \delta_i \Delta G_i + \varepsilon_i \quad (7)$$

where ε_i is the difference in the u_i^* error terms associated with discounted utility levels under the two alternatives.¹² As usual, the scale of the error term and the parameters cannot be separately identified and must be normalized to unity for some subgroup.

When such a durable- or public-goods choice is used alone, it is often difficult to separately identify all three of the parameters μ_i , δ_i and r_i . However, we can use the same basic utility difference function to capture a second choice, concerning how to take lottery winnings, which involves no difference in the net present value of the services of a durable goods. Here, L_i is the optional current-period lump sum lottery disbursement, to be compared against a sequence of T_i annual payments in the amount p_i , starting today.

¹¹It would be straightforward to generalize our model to accommodate not just the choice of whether to buy a durable good, but also a choice between several durable goods. This could lead to a multinomial logit choice model for the durable good decision.

¹²The properties of the underlying current-period errors, u_i , may be assumed to be such that the transformed and differenced ε_i errors in any equation for ΔV_i conform to the logistic distribution underlying an ordered logit model.

$$\Delta V_i = \mu_i \left\{ [y_i + L_i] - \left[y_i + p_i \sum_{t=0}^{T_i-1} (1 + r_i)^{-t} \right] \right\} + \delta_i[0] + \varepsilon_i \quad (8)$$

From this type of choice used alone, it is not possible to identify both μ_i and δ_i , but greater resolution can be obtained for r_i . The key insight is that pooling the types of choices in equations (7) and (8) may allow all parameters to be readily identified. If the same discount rate applies to both increases and decreases in future net income, the durable/public goods choice can serve to identify the marginal rates of substitution between money and the services of the durable/public good, whereas the lottery winnings choice can more specifically identify the individual discount rate.

Concentrating on just the lottery winnings portion of the joint choice model, simple hyperbolic discounting (in the sense of Harvey (1986), using $\phi_h(t)$) would lead to a utility difference function, ΔV_i , that looks a little different:

$$\Delta V_i = \mu_i \left\{ [y_i + L_i] - \left[y_i + p_i \sum_{t=0}^{T_i-1} (1 + t)^{-\beta_i} \right] \right\} + \delta_i[0] + \varepsilon_i \quad (9)$$

where β_i is the hyperbolic discounting parameter. For generalized hyperbolic discounting, ΔV_i for the lottery winnings choice will involve an additional parameter, γ , as part of the $\phi_h(t)$ discount factor:

$$\Delta V_i = \mu_i \left\{ [y_i + L_i] - \left[y_i + p_i \sum_{t=0}^{T_i-1} (1 + \gamma t)^{-(\beta_i/\gamma)} \right] \right\} + \delta_i[0] + \varepsilon_i \quad (10)$$

The full specifications described above are appropriate models to handle the *joint* estimation of (a) a policy choice when the individual faces different costs at different times in order to obtain a change in the present value of a flow of services from public goods ΔG_i , and (b) an auxiliary financial time-wise tradeoff such as our lottery winnings choice. Our empirical illustration, however, focuses on just the subsidiary problem of identifying individual-specific discount rates (either r_i or β_i , and possibly γ), so some of the

generality of the theoretical specification is foreclosed.¹³

In our estimating data, the only thing that differs across the alternatives posed in the stated preference choice scenario about lottery winnings is the present discounted value of net income, so $\Delta G_i = 0$ for all choices. There is no information upon which to estimate δ_i , the marginal indirect utility from the public good. As a result, μ_i , the marginal indirect utility of net discounted income cannot be identified either, since it merely affects the scale of net utility, which is irrelevant to choices. The “difference” upon which the choice is based can be conceived as merely a difference in net discounted income, rather than a difference in utilities. Our estimating specifications therefore constrain $\mu_i = 1$, and the simplified versions of our random utility models for the lottery winnings choice become:

$$\text{Exponential:} \quad \Delta V_i \propto L_i - p_i \sum_{t=0}^{T_i-1} (1 + r_i)^{-t} + \varepsilon'_i \quad (11)$$

$$\text{Simple hyperbolic:} \quad \Delta V_i \propto L_i - p_i \sum_{t=0}^{T_i-1} (1 + t)^{-\beta_i} + \varepsilon'_i \quad (12)$$

$$\text{Generalized hyperbolic:} \quad \Delta V_i \propto L_i - p_i \sum_{t=0}^{T_i-1} (1 + \gamma t)^{-(\beta_i/\gamma)} + \varepsilon'_i \quad (13)$$

These measures of the indirect utility differences, ΔV_i , are the building blocks for our random utility econometric models. Appendix II describes in detail the construction of an appropriate log-likelihood function for our data. The function is a sum of component ordered-logit log-likelihoods for choices with two,

¹³In principle, it is possible to allow the underlying indirect utility function to be non-linear with respect to net income. If the magnitudes of these costs are sufficiently large to represent a substantial portion of the individual’s income, it should be possible to separately identify individual-specific degrees of risk aversion (nonlinearity in income in the indirect utility function) as distinct from individual-specific discount rates. For our data, there appears to be insufficient variability in net incomes to permit a distinction between discounting and risk aversion if risk aversion in preferences is permitted. Also, since the stream of future payments and the current period lump sum that define our specific choice scenario are represented as being certain, it is not too surprising that there is insufficient information in the answers to the lottery choice question to accurately identify the individual’s degree of risk aversion. Chesson and Viscusi (2000) also have difficulty introducing risk aversion as a property of preferences that is distinct from discounting. Their hypothetical choice scenarios compound uncertainty and discounting, however, and they end up estimating implicit discount rates with the acknowledgement that failing to allow for risk aversion will bias these estimates downward.

three, four, and five possible responses pooled over independent samples. We impose non-negativity for the estimated discount rates by specifying them as $r_i = \exp(r'Z_i^r)$ and $\beta_i = \exp(\beta'Z_i^\beta)$. The vectors of explanatory variables, Z_i^r and Z_i^β , need not be identical, but we will drop their superscripts.

There is also the matter of heteroscedasticity. In any stated preference context, there is always a concern that the quality of the choice information elicited in a hypothetical choice scenario is dependent upon how seriously the respondent takes the choice exercise, upon their prior experience in similar choice situations, and on any constraints that prevent them from considering their choices sufficiently carefully. These factors can be labeled as "inclination," "ability," and "opportunity." It is typically important to acknowledge the existence of different subgroups of respondents who may exhibit systematically greater or lesser dispersion in the error term in the underlying random utility choice model. Utility-difference error variances have been argued to affect "choice consistency" (DeShazo and Fermo, 2002).

III. Survey Sample

Our data are derived from an Web-based (internet) survey with over 2000 participants from a wide variety of classes at universities throughout the US and Canada. It can be viewed as a national and international extension of the typical "classroom survey," but there is no pretense that the sample for this study represents the US and Canadian populations, or even the population of college students in these countries. There are significant disparities across institutions in access to web-based resources, across classes in the salience of the larger survey topic (global climate policy), and in the opportunity costs of students' time spent in completing the survey.¹⁴ The module of the survey that was designed specifically to elicit individual discount rates asks the respondent to imagine they have just won a lottery. They are asked to choose between taking their winnings as a series of T annual installments, starting "today," or as a smaller overall lump sum payable

¹⁴For social choice problems involving very long time horizons, it can be argued that the preferences of today's young people deserve particular attention, since they will be the surviving (net) beneficiaries of whatever policies are adopted in the near term. And while survey research is inevitably vulnerable to criticism based upon its hypotheticality, we at least pursue the issue of "construct validity" very aggressively in this paper. It is crucial, for example, to assess whether there is systematic variation in the error terms in one's model and to determine whether the nature of this variation (which cannot be avoided) is plausible.

immediately. Table 1 outlines our available variables and provides descriptive statistics for our sample.

Across respondents, the dynamically loaded survey page randomly varies the size of the annual installments among \$300, \$600, \$1200, \$2400, \$3600, and \$4800, and the number of these installments (the time horizon) among 20, 30, and 40 years. The sizes of the annual installments were intended to reflect increased monthly costs of \$25, \$50, \$100, \$300, and \$400 for a public good, and the time horizon captures some of the more-expensive and longer-term environmental programs, such as climate change mitigation. Each respondent is presented with an ordered list of lump sums and asked to indicate whether (and sometimes, to what extent) they would prefer each lump sum to the single pattern of annual installments.

A selection of sociodemographic characteristics was elicited after the various choices in the survey had been recorded, including age brackets, gender, educational attainment, field of study, whether courses have been taken in economics, work status, subjective conservatism, and family income bracket. Some less conventional variables were also collected. To proxy for individual capital market constraints, we ask for an estimate of the largest sum of money the individual believes they could qualify to borrow, without collateral. The survey software also keeps track of timing as respondents progress through the survey.¹⁵

Finally, preference elicitation formats were randomized across respondents. Some saw a long list of thirteen lump sums, some saw seven, five, or only three, although the range in lump sums was identical across these four different designs. The objective exponential discount rates implicit in the list of lump sums presented to respondents who saw all thirteen lump sums were integer values between 1 percent and 10 percent, as well as 12 percent, 15 percent and 20 percent.¹⁶ Across respondents, lump sums were either increasing or decreasing in size from the top to the bottom in the list. Finally, the format of the actual choice with respect to each lump sum involved different numbers of response options (just two, for "Yes/No", up to five levels including "Definitely Yes", "Probably Yes", "Not Sure", "Probably No", and "Definitely No").

¹⁵The online survey was programmed by the authors.

¹⁶Early versions of the survey included 30 percent and 50 percent implicit rates, but the very small corresponding lump sums were overwhelmingly rejected. Since their presence in the menu of possibilities added little information, these lump sums were dropped in later editions of the survey.

The horizontal ordering (Yes to No, No to Yes) of these answer options was also randomized. By using an assortment of elicitation formats, we can assess the impact of any one format selection on the discount rates that we infer from respondents' choices. In this paper, empirical results concerning elicitation formats will be treated as tangential to the main question and will be described in the appendices.

IV. Empirical Results

There are three types of variables in play: those that describe the randomly assigned context of the choice from which individual discount rates are inferred, those that measure individual-specific characteristics, and those that describe the particular variant of the randomized design of the elicitation format.

A. *Heteroscedastic Exponential and Hyperbolic Parameter Estimates*

Table 2 details the parameter point estimates and asymptotic t-test statistics¹⁷ for heteroscedastic variants of the two special cases of the generalized hyperbolic discounting model: conventional exponential discounting, $\phi_e(t)$, and simple hyperbolic discounting, $\phi_h(t)$, in the sense of Harvey (1986).¹⁸

The maximized values of the log-likelihood functions for these two models are negligibly different. With only one real exception, the variables that account for contextual and individual heterogeneity bear coefficients of the same sign that are statistically significant at approximately the same levels across the two specifications. Thus it is possible to review most of these results generically.

The choice scenarios presented to individuals were randomized in terms of the size of the annual payments that were being proposed, and the number of future years over which these payments were being offered, so

¹⁷The final column of Table 2 displays the R^2 values for auxiliary regressions where each regressor in the group is employed successively as the dependent variable in a model that uses the others in that particular index as explanatory variables (a good indicator for multicollinearity). These statistics are provided only for those variables that were not randomized

¹⁸Estimation was accomplished using Matlab 6.1.0.450, Release 12.1. Since respondents were asked to react to multiple different lump sums, there are 3595 choices corresponding to 2-level answers, 3749 corresponding to 3-level answers, 3499 corresponding to 4-level answers, and 4065 corresponding to 5-level answers. We will not, in this paper, pursue the panel aspects of the data set. The sets of lump sum payment alternatives presented to each individual are assigned randomly across individuals and the offered amounts are entirely exogenous. Any unobserved heterogeneity bias has been minimized by the degree of randomization that is present in the model. Given the nonlinearity of the model, however, there may be some gains from panel methods stemming from the monotonic ordering of the lump sums, despite the randomized assignment of their sizes and the direction of this ordering.

there can be no correlations among them or with other variables. In our sample, the larger the amount of money that is at stake, the higher is the apparent discount rate for the individual. These results contrast with the experimental results discussed by Thaler (1981), Uri Benzion et al. (1989) and Loewenstein and Thaler (1989). The existing literature suggests that people may be more willing to wait a year for “\$150 then versus \$100 now” than they are to wait a year for “\$15 then versus \$10 now.” Hersh Shefrin and Thaler (1988) suggest that large future amounts may be viewed merely as foregone savings interest, whereas smaller amounts may be viewed as foregone consumption, which may be more highly valued.

Our results concerning the time horizon constitute the only appreciable difference between the exponential discounting model and the simple hyperbolic discounting specification. In the exponential discounting specification, the longer the time horizon over which the lottery winnings are to be paid, the lower is the apparent discount rate (controlling for age). This result is consistent with the anomalies observed elsewhere in the discounting literature. In the hyperbolic discounting specification, however, a longer time horizon leads to a statistically significantly *larger* implied discount rate. Neither model, therefore, produces discount rates which are independent of the time horizon in question.

Warner and Pleeter (2001) find that individual discount rates decline with age and education and they report similar findings by Harry J. Gilman (1976) and Matthew Black (1984) in earlier military studies. In our sample, discount rates appear to be *larger* for individuals in this sample who are older (controlling for educational attainment). Older students tend to be present in the sample because they did not pursue college education at the same time as their peers. A higher discount rate may have led them to forego college when they were younger because the greater future earnings were more heavily discounted.¹⁹ Age is also negatively correlated, although not perfectly, with remaining life expectancy, which is not a factor included in our tabulated results. The rate at which the logarithm of fitted individual discount rates increases with age (or, becomes lower with greater with life expectancy) is greater for males than for females. At 18.9 years

¹⁹Of course, it is not possible in a cross-sectional sample to determine whether this is an age effect or a cohort effect or some combination of the two.

of age, females tend to have discount rates that are roughly equal to those of similar males, but beyond that age, discount rates for females rise more slowly with age than those of males. (Warner and Pleeter identify statistically significant gender effects only in their sample of enlisted personnel, where males display higher discount rates.) Our estimated age effects also conflict with the speculation of Chesson and Viscusi (2000) that the young may be inclined to discount the future more heavily, leading to “temporal myopia” with respect to longer-term prospects.²⁰ However, we do find that the greater the individual’s educational attainment, the lower their discount rate, which suggests that more highly educated individuals are more willing to postpone income, which is intuitively plausible. Students with greater educational attainment will have self-selected to be in college longer, foregoing current earnings, which may imply that they have lower discount rates.

Curiously, discount rates in our study are larger for individuals with higher family incomes, suggesting that "impatience" is greater when the subject stems from a higher-income background. This contrasts with the reported findings of Gilman and Black, and the findings of Hausman (1979), where the discount rate decreases with increased income. Hausman posits that discount rates should vary with income class “owing to the progression of the income tax which causes intertemporal marginal rates of substitution to differ.” He argues that the discount rate should decrease as income rises, even with perfect capital markets, since the marginal tax rate rises with income while the services of many consumer durables are untaxed. Higher discount rates for the poor are attributed to “uncertainty of their income streams and their lack of savings.” In our data, however, higher family income may be correlated with many other omitted variables. Wealth, for example, is not measured in our study.

Similarly perplexing results emerge for our capital access variable. The “capital access” question in our survey was worded as follows: “The largest amount of money that I believe I could currently qualify to

²⁰We did make some effort to distinguish life expectancy from age. Our survey elicits life expectancy as a separate implicit variable by questioning respondents about whether they expected to be alive at each of each of a number of decadal intervals in the future. Crude proxies to control for life expectancy did not make any significant improvement to our models, nor did they lead to any appreciable change in the apparent effects of age on discount rates. In such a predominantly young sample, mortality risks of any kind may not yet be registering.

borrow from a bank, credit union, trust company or family member (without collateral) is:" The options, and percents of the sample selecting each option, were \$0 (5 percent), \$100 (4 percent), \$100 (31 percent), \$10,000 (44 percent), \$50,000 (10 percent), \$100,000 (2 percent), more than \$100,000 (4 percent).²¹ From a population of mostly college students, where 40 percent of the sample reports a level of family income lower than \$50,000, there is some question as to whether every respondent fully understood the idea of "without collateral." It is conceivable that family sources for particularly affluent students could provide capital access at the level indicated by some of them. Although the simple correlation between the family income brackets and the capital access variable was only 0.18, more than half of the respondents who indicated the highest category for capital access also indicate the highest category for family income.

In Hausman's revealed preference context, individual discount rates are derived from consumer's choices about spending money, whereas here, they are derived from individuals' choices about how to receive money. In the present case, capital market constraints that might be binding on purchase decisions may have much less of an effect on choices about the time pattern of receipt of money. If, compared to other people, a respondent was aware that they could borrow money at a lower effective interest rate, they should be less inclined to take the immediate lump sum and more inclined to favor the program of deferred payments, implying a smaller discount rate. Such logic does not seem to apply here. A larger discount rate might be associated with a greater perceived "need" for money in the present, or greater "impatience" about receiving money. One is left to speculate upon the distribution of desires for immediate gratification, or variations in the sense of entitlement, across college students from different socioeconomic backgrounds.

The nature of the individual's education also has some systematic effects on discount rates. In Warner and Pleeter's military sample, personnel in the Engineering and Scientist or Professional categories display lower discount rates than others, as did enlisted personnel in the top two "mental groups" with the highest test scores. Higher test scores are argued to reflect better capacity to understand the implications of intertemporal

²¹The open-ended category was arbitrarily coded as \$150,000 to produce a continuous variable to proxy for the individual's subjective access to capital markets.

choices. In our study, we distinguish those individuals who major in social science disciplines (which will include economics) and those who major in business. A separate dummy variable is included for those who have taken at least one course in economics (since one would hope that this status would convey a greater degree of financial savvy). This last variable distinguishes between non-economics social science majors and those with some exposure to economic thinking, as well as capturing economics courses taken as an elective by students from other disciplines. Business majors and those with economics training have statistically significantly lower discount rates, and non-economist social science majors have higher discount rates.

Being further to the right on a simple liberal-conservative spectrum may have a positive effect on discount rates in the exponential model, but the evidence is relatively weak. No significant effect is evident in the hyperbolic model. Whether the individual is part-time or full-time employed has no statistically significant bearing on discount rates.

See Appendix III for discussion of the various incidental parameters associated with these models (i.e. those concerning the characteristics of the elicitation format, heteroscedasticity, and the incidental ordered logit threshold parameters).

B. *More-General Models*

The maximized log-likelihood values for the simple exponential and hyperbolic discount rate models are extremely close.²² Thus, it may not be surprising that a generalized specification proves difficult to estimate. In principle, it is possible to generalize the γ parameter to a systematically varying parameter. However, such a generalization proves difficult in this application.²³

It is often problematic to estimate parameters that appear in ratio to each other, so we report upon a reparameterization of the generalized hyperbolic model adapted from the usual strategy used in estimation

²²For the entire set of 40 analogous estimated parameters in these two models, the correlation between the point estimates is 0.9963. A simple regression of the 40 hyperbolic model point estimates on the corresponding exponential estimates implies proportionality. This regression has an intercept no different from zero and a slope of 0.74 (with t-ratio in excess of 71). Limiting the regression to just the 22 discount rate slope coefficients, the hyperbolic coefficients average 0.84 times the exponential coefficients (t-ratio = 206).

²³A Lagrange multiplier (LM) test, evaluated at the restricted specification that is the simple hyperbolic model, might prove useful in other cases where there appear to be greater distinctions between the exponential and the simple hyperbolic models.

of Tobit models. Instead of estimating γ and a systematically varying version of β_i , we can instead define $\beta_i^* Z_i = \beta_i Z_i / \gamma$ and $\gamma^* = 1/\gamma$. The alternative version of the generalized hyperbolic discounting model, in our simplified form, then becomes:

$$\Delta V_i \propto L_i - p_i \sum_{t=0}^{T_i-1} [1 + \gamma^* t]^{-\beta_i^* Z_i} + \varepsilon_i \quad (14)$$

It is most convenient to approach the generalization by starting from the hyperbolic specification, since the generalization amounts to freeing up the restriction that $\gamma = 1$. But even with reparameterization, it is not possible to attain convergence for the general specification with regressors with these data.²⁴

Thus, as do Cairns and Van der Pol (1997), we find it very difficult to discriminate between the simple exponential and simple hyperbolic discounting functions by using the generalized hyperbolic model that has each of these as special cases.

C. Heterogeneity in Fitted Individual Discount Rates

Frederick et al. (2002) review estimates of individual discount rates in the literature that range from zero to infinity. What do our models imply about the distributions of fitted empirical discount rates, both exponential and hyperbolic, for this sample? We estimate $r_i = \exp(r' Z_i)$ and $\beta_i = \exp(\beta' Z_i)$ by maximum likelihood methods, so each of the discount rate "indexes," $r' Z_i$ and $\beta' Z_i$, can be assumed to be asymptotically normally distributed. The process of exponentiation takes the symmetric normal distribution and converts it into a skewed lognormal distribution. To produce expected values for each individual fitted discount rate, therefore, it is first necessary to calculate the estimated variances of these linear combinations, and then to use these variances in calculating the expected values of each estimated discount rate: $E[r_i] = \exp(r' Z_i) \exp(\sigma_{r_i}^2/2)$ and $E[\beta_i] = \exp(\beta' Z_i) \exp(\sigma_{\beta_i}^2/2)$. For any particular vector of explanatory variables,

²⁴In recourse, we can drop back to a specification with constant discount rates, to determine whether the task of estimating a single common discount rate could be achieved for a generalized hyperbolic specification, in addition to a simple exponential or simple hyperbolic model. While the single β parameter of the simple exponential, or the single γ parameter of the simple hyperbolic model are readily estimated and strongly statistically significant in each model, neither is individually significant in the generalized hyperbolic specification and there is a negligible change in the log-likelihood.

an approximate 95 percent asymptotic confidence interval can be calculated by exponentiating the calculated values of $r'Z_i \pm 1.96\sqrt{\sigma_{r_i}^2}$.

The last line of Table 2 displays the marginal mean across the sample of the fitted point estimates for each type of discount rate. Table 3 itemizes some key percentiles of the marginal distributions of the fitted values of $E[r_i]$ and $E[\beta_i]$. Keep in mind that all of the variability in these point estimates across the sample is created by differences in the choice scenarios posed, differences in respondent attributes, or differences in the mode of elicitation of the choice. Our fitted individual exponential discount rates range between about 2 percent and 22 percent, while the hyperbolic discount rates vary between about 0.15 and 1.00. Considerable heterogeneity in fitted discount rates therefore exists, and to an extent that is likely to be economically significant, as well as statistically significant.²⁵

V. Conclusions

It has been well-documented that empirically estimated discount rates vary dramatically across samples and across the choice contexts and techniques used to elicit them (Frederick et al., 2002).²⁶ Now, within this one study, we have varied a wide array of the factors that have elsewhere differed across studies, allowing us to make systematic assessments of the effects on implied individual discount rates of the choice context, individual characteristics, and (tangentially) the format of the elicitation method.

The distribution of discount rates in our multi-university and international student sample are not likely to be representative of the distribution of discount rates in the general population because of self-selection of individuals into the college population. Nevertheless, insights about how discount rates vary, within this college population, are new. This information may be useful in helping policymakers concerned with tertiary education to understand the different choices made by different types of students.

²⁵Figures 1 and 2 in Appendix IV.C. depict the precision with which individual fitted discount rates are estimated. The 2062 fitted rates are first sorted according to size, then the 95% confidence bounds for each fitted value are displayed along with that fitted value, all three as a function of the fitted rate. Points are connected to aid visualization.

²⁶With at least forty empirical measures of discount rates, it is somewhat surprising that no researcher has yet attempted a regression meta-analysis that can shed light on how the characteristics of a particular study contribute to higher or lower estimated values of the discount rate.

The empirical results presented in this paper also constitute a specific illustration of a method that holds promise for helping us understand the determinants of heterogeneity in individual discount rates in a wide variety of other choice contexts. We have demonstrated how to quantify individual-specific (latent) discount rates for use as an additional heterogeneous individual characteristic that can help explain other choices. Considerably more general-population empirical work will be needed before we are able to quantify individual-specific discount rates with sufficient accuracy to warrant the calculation of the present discounted value of a stream of future net benefits by first discounting individually or for distinct groups, then aggregating. (We have argued that this is the conceptually correct approach, rather than first aggregating net benefits in each future period, then discounting using a single representative discount factor.) We do, however, advocate a shift in the direction of "discount first, aggregate second" as a research goal.

The direct elicitation of individual-specific discount rates will always be problematic because ordinary citizens cannot be expected to know what a discount rate is, nor can they be expected to have introspected very carefully about the magnitude of their own individual rate. As has been the case since some of the first empirical efforts of Hausman (1979), researchers will typically be forced to infer implicit discount rates from either real or stated choices. Only a sample like that of Weitzman (2001)—professional economists—could reasonably be expected to articulate a discount rate directly with any degree of reliability.²⁷

We find that private discount rates differ with age and/or life expectancy, gender, income, access to capital, and with exposure to economics training, among other things. Perhaps Weitzman's "leading economists" subsample displays slightly higher discount rates than the broader population of economists simply because the individuals on Weitzman's list are older and exclusively male. They are also likely to have higher family incomes and greater access to capital. But we must also consider whether Weitzman's sample of economists should have the prerogative of dictating the social discount rate to be used in policy-making, especially when it seems that those who have taken a course in economics exhibit statistically significantly lower discount

²⁷Weitzman was persistent in pressing his respondents to provide a point estimate of the relevant social discount rate for climate change policy.

rates. As in other studies of how economists are different, though, there remains the question of whether economic training leads to lower discount rates, or whether individuals with innately lower discount rates self-select to become economists.

Individual discount rate uncertainty, in addition to cross-sectional heterogeneity, remains an issue for further investigation. We have made some progress in this direction by controlling for heteroscedasticity in the error terms in the indirect utility functions used to infer individual discount rates. In our model, however, this heteroscedasticity is not a factor in the calculation of expected individual discount rates. Our expected discount rates are sensitive to the statistical precision with which the parameters of the discount rate index are estimated, but not to the amount of systematic noise in the choice process for that type of individual in that context.

What about the choice between exponential and hyperbolic discounting? It has proven impossible, in this application, to differentiate conclusively between a conventional (constant) exponential discount model with heterogeneity across individuals and a simple hyperbolic discount model with analogous heterogeneity. For this particular data set, the exponential model is narrowly better in terms of its ability to explain individual's stated choices. However, the difference in the log-likelihood function is less than one (for 14,908 total choices), and attempts to generalize the simple hyperbolic specification (with heterogeneity and heteroscedasticity) by freeing up a single additional shape parameter completely fail to converge. This outcome suggests that the standard exponential model and the alternative hyperbolic discounting model explain these observed choices about equally well. Even this large sample of choices does not provide sufficient information to discriminate much further.

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APPENDIX I

Gamma vs. Generalized Hyperbolic Discounting

Using the same notation as in Loewenstein and Prelec (1992), but adapting it to Weitzman's model, the formula for the individual (continuous-time) discount factor is:

$$\phi_i(t) = \exp(-x_i t) \quad (15)$$

Weitzman considers x_i to be a random variable. For each individual, x_i is a random draw from a gamma distribution with scale parameter d and shape parameter c , both strictly positive and assumed to be constant across the population:

$$f(x) = \frac{b^c}{\Gamma(c)} x^{c-1} \exp(-bx), \quad b, c > 0 \quad (16)$$

The mean of this gamma distribution is $\mu = c/b$ and the variance is $\sigma^2 = c/b^2$. In aggregating the diverse individual opinions about appropriate discount rates spanned by his sample, Weitzman proposes an aggregate $\phi(t)$, the “expected present discounted value of a dollar at time t ”, namely

$$\phi(t) \equiv \int_0^\infty \exp(-xt) f(x) dx. \quad (17)$$

Note that the expectation is the expected value of the continuous-time discounting function $\exp(-xt)$, rather than the expectation of discount rates x themselves. Weitzman further notes that the marginal or instantaneous effective discount rate at time t is defined to be

$$r(t) \equiv -\frac{\dot{\phi}(t)}{\phi(t)} \quad (18)$$

For the gamma probability density function, Weitzman shows that the expectation in (17) can be expressed

as

$$\phi(t) = \left(\frac{b}{b+t}\right)^c = \left(1 + \frac{1}{b}t\right)^{-c} \quad (19)$$

Alternatively, if the expectation is expressed in terms of the mean $\mu = c/b$ and the variance $\sigma^2 = c/b^2$ of the gamma distribution, it begins to resemble the Loewenstein and Prelec (1992) generalized hyperbolic discounting formula. In terms of μ and σ^2 , the formula is:

$$\phi(t) = \left(1 + \frac{\sigma^2}{\mu}t\right)^{-(\mu^2/\sigma^2)} \quad (20)$$

whereas, in terms of β and γ , it is:

$$\phi_g(t) = (1 + \gamma t)^{-\beta/\gamma} \quad (21)$$

If we let $\beta = \mu = c/b$ and $\gamma = \sigma^2/\mu = 1/b$, it is clear that the two formulations are identical. Weitzman's formula for the expected present discounted value of a dollar at time t equals the generalized hyperbolic discounting formula.

In the one-parameter hyperbolic discounting function of Harvey (1986), explored recently by Keller and Strazzera (2002), the restriction $\gamma = 1$ is imposed to yield a simplified discounting function:

$$\phi_a(t) = (1 + t)^{-\beta} \quad (22)$$

The γ parameter in the Loewenstein and Prelec (1992) formulation of the discounting formula corresponds to $(1/b)$ in Weitzman's parameterization of the gamma distribution. The gamma distribution collapses to an exponential distribution when $c = 1$.

The conventional continuous-time discounting function, $\phi(t) = \exp(-\beta t)$, is a limiting case of the gener-

alized hyperbolic discounting function when $\gamma \rightarrow 0$. This is easiest to see if one considers the instantaneous effective discount rate at time t for the generalized hyperbolic discount function $\phi(t)$:

$$r(t) = -\frac{\dot{\phi}(t)}{\phi(t)} = \frac{\beta}{1 + \gamma t} \quad (23)$$

Recall that the parameter γ corresponds to the variance of the underlying gamma distribution, divided by its mean. As γ approaches zero for any mean, β , of the distribution (note that $\beta > 0$ is required), this variance must approach zero and the probability density function $f(x)$ approaches a discrete mass with probability one at the mean β , whereupon the expectation in equation (17) converges to $\phi(t) = \exp(-\beta t)$, the familiar continuous-time exponential discounting formula. This is consistent with $r(t)$ in equation (23) approaching β .

APPENDIX II

Log-Likelihood Function

We assume $\varepsilon \sim \text{logistic}(0, \kappa_i)$ where the dispersion parameter κ may be distinct (proportional) across contexts or for different types of respondents. Implicitly in all logit-based random utility models, the linear “slope” coefficients (here, the μ and β vectors) can only be estimated up to a scale factor (i.e. relative to the implicit dispersion parameter of the error term), so researchers often proceed in terms of a normalized scale factor that is equal to one. If error variances differ across subsets of the data, then a factor of proportionality, relative to the error dispersion for the numeraire subset, may be estimated for other subsets. To ensure positive proportionality for the non-numeraire dispersion factors, they can be estimated as $\kappa_i = \exp(\kappa_i^*)$

The probability formulas that are relevant, for each different number of levels in the answer options presented to respondents in each of our split samples, can now be defined for the ordered logit models used in estimation. If each subsample were to be used independently, there would be $m - 1$ unknown threshold parameters to be estimated for each format. (We label our thresholds as α_{jk} , where j denotes the number of answer categories and k denotes the threshold number, counting from the bottom, starting with zero.) However, with the pooled data from all four variants, the boundary between “YES” and “NO” will be normalized to zero, which means that $\alpha_{20} = 0$ and $\alpha_{41} = 0$ in the 2-level and 4-level cases, respectively. The locations of the remaining thresholds are freely estimated (without symmetry restrictions) but it should be the case that $\alpha_{30} < 0$ and $\alpha_{31} > 0$ for the 3-level cases, and $\alpha_{40} < 0$ and $\alpha_{42} > 0$ for the 4-level cases, and $\alpha_{50} < 0$, $\alpha_{51} < 0$, and $\alpha_{52} > 0$, $\alpha_{53} > 0$ for the 5-level cases. In each of these cases, we will assess whether outcomes with the expected sign are captured within the interval estimate for each threshold parameter.

Note that we cannot allow different thresholds according to the number of response categories, while simultaneously allowing error variances to differ only by the number of response categories. This leaves either the thresholds, or the error variances, underidentified. We do allow our error terms to differ systematically with an array of other variables which are not strictly redundant with a set of dummy variables for the

number of response categories.

The different options for the functional form of ΔV_i are itemized in the discussion of specifications in the body of this paper. The probability formulas for each type of response format are as follows:

For the 2-level (Yes / No) format:

$$\begin{aligned} P2Y_i &= \frac{1}{1 + \exp(\alpha_{20}/\kappa_i - \Delta V_i)} \\ P2N_i &= \frac{\exp(\alpha_{20}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{20}/\kappa_i - \Delta V_i)} \end{aligned} \quad (24)$$

For the 3-level (Yes / Not Sure / No) format:

$$\begin{aligned} P3Y_i &= \frac{1}{1 + \exp(\alpha_{31}/\kappa_i - \Delta V_i)} \\ P3NS_i &= \left(\frac{\exp(\alpha_{31}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{31}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{30}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{30}/\kappa_i - \Delta V_i)} \right) \\ P3N_i &= \frac{\exp(\alpha_{30}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{30}/\kappa_i - \Delta V_i)} \end{aligned} \quad (25)$$

For the 4-level (Definitely Yes / Probably Yes / Probably No / Definitely No) format:

$$\begin{aligned} P4DY_i &= \frac{1}{1 + \exp(\alpha_{42}/\kappa_i - \Delta V_i)} \\ P4PY_i &= \left(\frac{\exp(\alpha_{42}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{42}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{41}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{41}/\kappa_i - \Delta V_i)} \right) \\ P4PN_i &= \left(\frac{\exp(\alpha_{41}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{41}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{40}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{40}/\kappa_i - \Delta V_i)} \right) \\ P4DN_i &= \frac{\exp(\alpha_{40}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{40}/\kappa_i - \Delta V_i)} \end{aligned} \quad (26)$$

For the 5-level (Definitely Yes / Probably Yes / Not Sure / Probably No / Definitely No) format:

$$\begin{aligned}
P5DY_i &= \frac{1}{1 + \exp(\alpha_{53}/\kappa_i - \Delta V_i)} \\
P5PY_i &= \left(\frac{\exp(\alpha_{53}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{53}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{52}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{52}/\kappa_i - \Delta V_i)} \right) \\
P5NS_i &= \left(\frac{\exp(\alpha_{52}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{52}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{51}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{51}/\kappa_i - \Delta V_i)} \right) \\
P5PN_i &= \left(\frac{\exp(\alpha_{51}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{51}/\kappa_i - \Delta V_i)} \right) - \left(\frac{\exp(\alpha_{50}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{50}/\kappa_i - \Delta V_i)} \right) \\
P5DN_i &= \frac{\exp(\alpha_{50}/\kappa_i - \Delta V_i)}{1 + \exp(\alpha_{50}/\kappa_i - \Delta V_i)}
\end{aligned} \tag{27}$$

Each threshold parameter α_{jk} is normalized by the dispersion parameter, κ_i , for the error term in the relevant subsample of data.

The last necessary ingredient for the development of the log-likelihood function for this model is a set of indicators for choices. Indicators have the general format DnX_i . The value of n indicates how many answer levels were offered to the respondent ($n = 2, 3, 4, 5$), and X includes Y and N for Yes and No, with P for the modifier "probably" and D for "definitely". NS is the abbreviation for the "not sure" category. All indicators take a value of 1 if the designated response is selected, and are 0 otherwise.

All respondents provide either 3, 5, 7 or 13 responses to discounting questions. The different orderings and different formats of the answer options were randomized across split samples, so the log-likelihood formulas appropriate for each number of response options can simply be summed. The log-likelihood to be maximized by appropriate choices of the unknown parameters can now be written in its most compact form as follows:

$$\begin{aligned}
\text{Log}L = & \sum_{i=1}^{ND2} [D2Y_i \ln(P2Y_i)] + D2N_i \ln(P2N_i) \\
& + \sum_{i=1}^{ND3} [D3Y_i \ln(P3Y_i) + D3NS_i \ln(P3NS_i) + D3N_i \ln(P3N_i)] \\
& + \sum_{i=1}^{ND4} [D4DY_i \ln(P4DY_i) + D4PY_i \ln(P4PY_i) \\
& \qquad \qquad \qquad + D4PN_i \ln(P4PN_i) + D4DN_i \ln(P4DN_i)] \\
& + \sum_{i=1}^{ND5} [D5DY_i \ln(P5DY_i) + D5PY_i \ln(P5PY_i) \\
& \qquad \qquad \qquad + D5NS_i \ln(P5NS_i) + D5PN_i \ln(P5PN_i) \\
& \qquad \qquad \qquad + D5DN_i \ln(P5DN_i)]
\end{aligned} \tag{28}$$

APPENDIX III

Discussion of additional parameters

III.A. *Survey Design Effects (Elicitation Format)*

In this Appendix, we discuss the parameter estimates in the continuation of Table 2. All of the experimental “treatments” described in this section were randomly assigned across respondents. It appears to make no detectable difference to the implied discount rates whether the answer options are arranged horizontally from “Yes” to “No,” or the reverse.

It does seem to matter whether the lump sums are presented in increasing or decreasing order. This suggests "starting point effects." If the first lump sums in the individual’s list are smaller than later lump sums, the implied discount rate will be smaller (although this effect is just marginally significant at the 5 percent level in the exponential model, and significant at just greater than the 5 percent level in the hyperbolic specification). These are useful results to consider in the design of such instruments in the future.

There is little action in terms of the number of different lump sums that the respondent is asked to consider. (It is worth noting that homoscedastic models, not reported, suggested that discount rates might be sensitive to this factor. As always, one suspects a combination of “fatigue” and “learning” effects when the number of choice tasks increases.)

The final factor is the number of answer options provided (from 2-level choices to 5-level choices). The 3-level and 5-level choice formats each involve a “Not Sure” option, and discount rates implied in these two subsamples are statistically significantly lower, and by about the same amount, compared to the 2-level and 4-level formats where the respondent was forced to make a Yes/No distinction. Note that discount rates are lower for everyone when the choice format has a "Not Sure" option, not just for those individuals who may choose this option.

III. B. *Heteroscedasticity*

The ability of the individual to make a coherent informed decision about how they would prefer to take

their lottery winnings may depend to a certain extent on their familiarity with state-sponsored lotteries. Fortunately, we asked each respondent specifically if there was a state-sponsored lottery in their jurisdiction, and whether they could legally play. We also asked how many lottery tickets they purchased per year. These three variables are employed to shift the dispersion parameter, κ_i , (the inverse of the "scale factor" examined in some stated preference research (see Swait and Louviere, 1993, or DeShazo and Fermo (2002))). These lottery-related variables are intended to capture the respondent's familiarity with the usual choice about how to take one's winnings. Even if the individual has never won a lottery, just playing the lottery or hearing advertisements for their state lottery may invoke speculation about what they would do if they did win.

For our sample, the logarithm of the error dispersion in the random utility model is decreased statistically significantly if the respondent is more familiar with state lotteries due to the presence of a lottery in their jurisdiction. Intriguingly, the reduction in the error dispersion is twice as large if there is a lottery but the respondent cannot play the lottery than if they can. This status, however, is a proxy for age, so these results effectively show that exposure to a state lottery reduces error variances more for the youngest respondents. This seems plausible. We also find that the more often the respondent reports actually playing the lottery, the lower is the error dispersion.

The quality of choice information provided by a given respondent is also suspected to differ with the amount of care and attention devoted to the particular choice task. We can observe, for example, whether the individual managed to complete the entire online survey after having responded to the lottery winnings choice question. Respondents who eventually attained the penultimate "Debriefing" page of the survey were asked about the extent to which they had to rush to complete the survey. Possible answers were 0=no, 1=yes, a little, and 2=yes, a lot. Respondents who went on to complete the survey have systematically smaller error dispersions for their choices. For the "how rushed?" variable, though, being more rushed leads to a lower, rather than a higher error variance. One might initially think that a respondent who is rushing through the survey might be more inclined to just check one column, but low variance in observed choices

is not the same thing as low variance in the errors.²⁸

We also track the entry and exit times for each page of the online survey, which allows us to construct durations for each page. In a small fraction of cases, these durations are excessively long, suggesting that the respondent was diverted to some other task while the page was still open. We examined the marginal distribution of durations for the lottery winnings page and determined that durations up to approximately the 90th percentile of this distribution appeared to be valid, whereas somewhere beyond the 90th percentile, inconsistently large outlying values begin to be observed. We thus create a dummy variable to designate "good" durations on the lottery winnings page.

If a respondent's duration on the lottery winnings page seems reliable, the error dispersion is systematically smaller. For these reliable instances, we also allow the error dispersion to vary systematically with time spent on the page. Since different respondents faced different numbers of lump sums, and were therefore being asked to consider more choices on this page, we also control for the number of lump sums each person saw. The greater the amount of time the individual spent on the lottery winnings page, the lower the error dispersion. The number of lump sums being considered does not have any individually statistically significant effect on error dispersions.

One design element that appears to have a marginal effect on the magnitude of the discount rates elicited was whether the lump sum bids were presented as increasing or decreasing from top to bottom on the page. We include this variable also as a potential shifter of the dispersion term, but it has no statistically significant effect.

Finally, individuals' aptitudes for considering the lottery winnings tradeoffs may differ systematically with their familiarity with the idea of discounting. We therefore include a dummy variable for whether the individual has ever taken a course in economics. As expected, those who have taken such a course exhibit smaller error dispersions, suggesting a greater facility at making intertemporal tradeoffs in a time-constrained

²⁸Appendix IV describes a model that explores the factors which explain decisions to check all the same column (or to get the question backwards).

environment.

III.C. *Incidental parameters*

The ordered logit threshold parameters make up the remaining eight parameters estimated in each model. The expected signs of these thresholds should be (-, +, -, +, -, -, +, +). The only instances of incorrect signs are associated with insignificant point estimates, suggesting that correctly signed thresholds lie within the confidence intervals for each parameter.

APPENDIX IV

Digressions Concerning the Data

IV.A. *Age and Income Distributions*

The entire distributions of ages (Table IV.1) and incomes (Table IV.2) in the estimating sample may of interest to a critical reader.

IV.B. *Constant and "Reversed" Responses*

There was one discernible difficulty in our data. Rationality would imply that a respondent should provide answers that progress monotonically, from "No" answers for their willingness to accept the smallest lump-sum payments in lieu of the series of annual payments, through to "Yes" answers for the willingness to accept the highest lump-sum payments. In the most extreme cases, answers should be constant (as they were for about 29 percent of our sample (604 out of 2062). Individuals who react at all to the stimulus of different implicit discount rates embodied in their list of lump sum amounts should vary their answers. Non-monotonic progressions would appear to be irrational. 47 respondents who selected a non-monotonic progression of answers were deleted from the analysis.

Retained in our analysis were individuals who checked the same response for all lump-sum payments. Also retained were respondents who appear to have confused the direction of the comparison in the choice question. About 12 percent of our sample (namely 257 out of 2062 respondents) appeared to have interpreted our lottery winnings question backwards, as though it was asking whether they would prefer the series of annual payments, rather than the lump sum. Since their answers seem to be substantive, despite this reversal, we elect to keep them in the analysis after we have "corrected" the direction of their choices.²⁹

In order to better understand the characteristics of individuals who gave constant or "reversed" responses to the lottery winnings question (as opposed to "good" responses), we have estimated the multinomial logit

²⁹Table IV.3 contains additional detail concerning the characteristics of different types of respondents, and how these affect their fitted propensities to give constant choices or "reversed" choices.

model in Table IV.3. (Estimation was accomplished using Stata.) The omitted category is "good" responses. The coefficients show the sign (and significance) of each variable on the propensity for the respondent's answers to fall into each category. Only persistently significant (or nearly significant) parameters are retained. Complete specifications employing all the same variables as the fitted individual discount rates (not reported here) achieved maximized log-likelihood values of only -1179.1536 and -746.8533 respectively, so little is lost by moving to these more limited specifications.

Constant responses suggest that the individual was not inclined to spend much time reflecting on the choices being presented, unless their implicit discount rate is truly less than 1 percent or greater than the rate embodied in the smallest lump sum they were offered. Among the individual characteristics, respondents with higher family incomes are for some reason less likely to be unresponsive to the stimulus contained in the differing lump sums, checking just one column among the answer options instead of varying their choices in response to differences in the lump sum that is offered.. Likewise, younger respondents are less prone to check a single column as are social science majors, and (possibly) those who do not work either full or part time. This may reflect differences in the opportunity costs of time across different types of students. Higher family income may permit students to spend more time on course-related optional activities such as this survey, and less time on work, for example. It also seems clear that the more complex the elicitation format provided to the respondent, the less likely is he or she to check a single column of answers, rather than varying their selections.

Concerning reversed responses, some very different factors seem to predominate. Being female increases the chance that the selections will appear to be reversed, but this propensity diminishes with age. It is also lower if the respondent has taken a course in economics, or if they are employed part or full-time. Among the different formats, the only cases with significantly larger chances of reversed responses are those who were "treated" with the greatest number of lump sums.

IV. C. *Fitted Individual Discount Rates and Confidence Bounds*

Figure 1 (exponential discount rates) and Figure 2 (hyperbolic discount rates) depict the precision with which individual discount rates have been estimated.

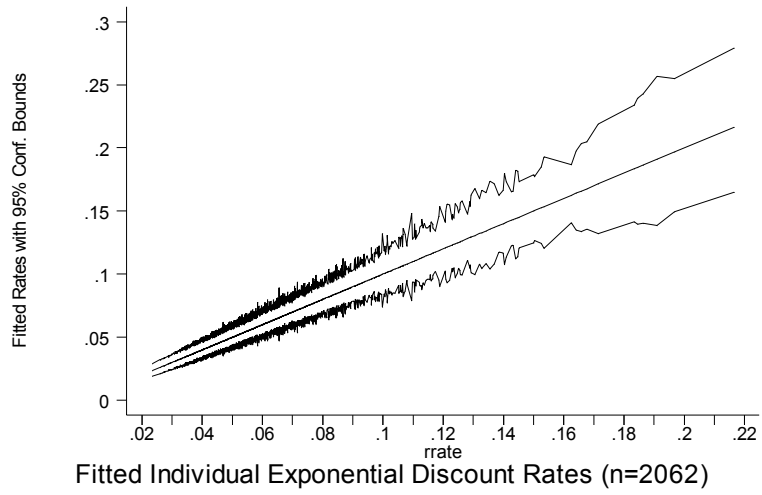


Figure 1:

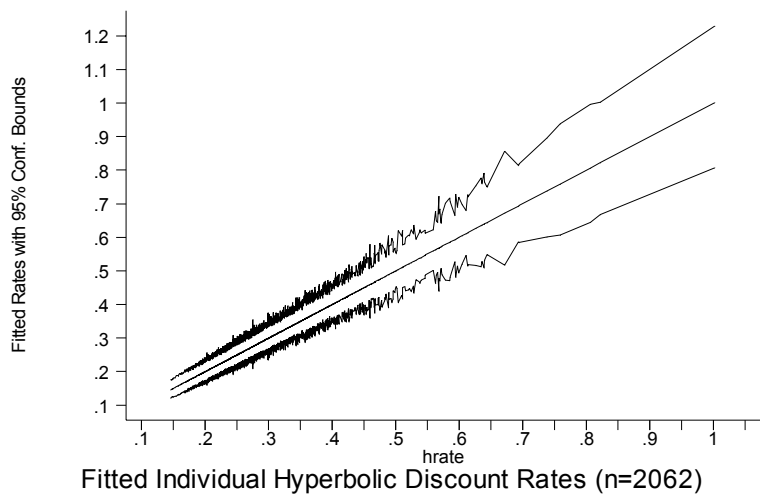


Figure 2:

Table 1
Descriptive Statistics (n = 2062)

Abbreviation	Description	Mean	Std.Dev.
<i>Nature of the choice scenario</i>			
annual amount	annual lottery payment (\$'000)	2.144	1.629
time horizon	number of annual payments	29.91	8.341
<i>Individual-specific characteristics</i>			
age	age (years; midpoints of 5-yr brackets)	22.24	5.502
female	= 1 if female, =0 if male	0.5024	
educ. attain.	0 = h/s or less, 4 = graduate degree	1.114	0.7008
family income	family income (\$'000, bracket midpts)	67.00	38.89
capital access	capital access (\$'000)	17.60	32.89
major = business	= 1 if major(ed) in business	0.3497	
major = soc.sci.	= 1 if major(ed) in social sciences	0.2978	
econ course	= 1 if economics course ever taken	0.8792	
conservatism	0 = liberal, 4 = conservative	1.742	1.095
work	= 1 if work full- or part-time	0.4360	
lottery (can play)	=1 if lottery available and can play	0.8337	
lottery (can't play)	=1 if lottery available but can't play	0.05723	
times played/yr	times lottery played per year	3.442	7.626
finished survey?	respondent finished entire survey	0.9326	
how rushed?	0=no rush, 1= somewhat, 2=very	0.3385	0.5070
"good" duration	"0-90th" percentile of durations on task	0.9011	
duration on task	duration (minutes) if "0-90th" percentile	1.048	0.6768
<i>Characteristics of the elicitation format (randomized)</i>			
"yes" on left	= 1 if "yes" on left	0.5145	
increasing sums	= 1 if lump sums increasing	0.4864	
5 lump sums	= 1 if five lump sums considered	0.2371	
7 lump sums	= 1 if seven lump sums considered	0.2570	
13 lump sums	= 1 if thirteen lump sums considered	0.2493	
# of lump sums	continuous number of lump sums	6.995	3.746
3-level answers	= 1 if 3-level response options	0.2473	
4-level answers	= 1 if 4-level response options	0.2342	
5-level answers	= 1 if 5-level response options	0.2716	

Table 2
Parameter Estimates, Heteroscedastic Exponential and
Hyperbolic Discounting Models (n=2062)

	Exponential		Hyperbolic		
	$r_i = \exp(r'Z_i)$		$\beta_i = \exp(\beta'Z_i)$		
Variables	Estimate	t-ratio	Estimate	t-ratio	Aux. R²
constant	-5.806	(-13.69)**	-4.152	(-12.00)**	
<i>Contextual differences: nature of the choice scenario (randomized)</i>					
annual payment	0.05424	(3.40)**	0.04447	(3.32)**	
time horizon	-0.007361	(-3.52)**	0.01056	(5.88)**	
<i>Individual-specific characteristics (non-orthogonal)</i>					
log(age)	1.062	(8.05)**	0.8673	(8.10)**	0.62
female	2.063	(4.22)**	1.738	(4.41)**	0.99
female*log(age)	-0.7019	(-4.43)**	-0.5893	(-4.61)**	0.99
educ. attain	-0.1022	(-3.80)**	-0.08455	(-3.79)**	0.33
family income	0.002069	(4.89)**	0.001750	(4.97)**	0.09
capital access	0.002689	(5.09)**	0.002318	(5.47)**	0.14
major = business	-0.07746	(-2.26)**	-0.06122	(-2.13)**	0.12
major = soc.sci.	0.2016	(5.83)**	0.1672	(5.85)**	0.11
econ course	-0.1261	(-2.42)**	-0.1191	(-2.74)**	0.08
conservatism	0.02398	(1.66)*	0.01517	(1.27)	0.04
work	0.03315	(1.07)	0.02446	(0.94)	0.02
<i>...See Appendix III for discussion of additional parameters...</i>					
Max Log L	-15106.699		-15107.472		
Mean rate	0.06056		0.2977		

Table 3
Distributions of Fitted Individual Discount Rates

Percentile	Exponential ($E[r_i]$)	Hyperbolic ($E[\beta_i]$)
min	.02354	.1462
5	.03505	.1902
10	.03830	.2049
50	.05676	.2817
90	.08582	.4058
95	.09980	.4536
max	.21657	1.002

Table 2, continued - Other parameters
Parameter Estimates, Heteroscedastic Exponential and
Hyperbolic Discounting Models (n=2062)

	Exponential		Hyperbolic		
	$r_i = \exp(r'Z_i)$		$\beta_i = \exp(\beta'Z_i)$		
Variables	Estimate	t-ratio	Estimate	t-ratio	Aux. R²
<i>Characteristics of the elicitation format (randomized):</i>					
yes" on left	0.02919	(0.97)	0.03020	(1.20)	-
increasing sums	-0.05917	(-1.96)**	-0.04819	(-1.91)*	-
5 lump sums	-0.09691	(-1.62)	-0.06801	(-1.37)	-
7 lump sums	0.04096	(0.75)	0.04431	(0.98)	-
13 lump sums	0.05712	(1.11)	0.06310	(1.49)	-
3-level answers	-0.3958	(-6.51)**	-0.3113	(-6.19)**	-
4-level answers	-0.06886	(-1.39)	-0.06014	(-1.49)	-
5-level answers	-0.3638	(-6.18)**	-0.3038	(-6.28)**	-
<i>Heteroscedasticity (factors shifting utility-difference error std. dev.):</i>					
lottery (can play)	-0.2933	(-5.80)**	-0.2923	(-5.78)**	0.32
lottery (can't play)	-0.6125	(-8.30)**	-0.6130	(-8.31)**	0.31
times play/yr	-0.003321	(-1.81)*	-0.003302	(-1.80)*	0.02
finished survey?	-0.3723	(-6.29)**	-0.3668	(-6.20)**	0.04
how rushed?	-0.04835	(-1.74)*	-0.04899	(-1.77)*	0.05
"good" duration	-0.1357	(-2.23)**	-0.1426	(-2.34)**	0.28
duration on task	-0.1794	(-7.89)**	-0.1761	(-7.74)**	0.30
# of lump sums	0.0003338	(0.09)	0.0004097	(0.12)	-
increasing bids	0.04048	(1.43)	0.03630	(1.28)	-
econ course	-0.1688	(-3.86)**	-0.1703	(-3.89)**	0.02
<i>Incidental parameters (ordered logit thresholds)</i>					
α_{30}	-0.1970	(-10.45)**	-0.1897	(-10.18)**	
α_{31}	0.007781	(0.43)	0.01494	(0.84)	
α_{40}	-0.3644	(-24.81)**	-0.3651	(-24.84)**	
α_{42}	0.3269	(25.35)**	0.3267	(25.33)**	
α_{50}	-0.4987	(-22.56)**	-0.4981	(-22.84)**	
α_{51}	-0.2125	(-11.80)**	-0.2116	(-11.97)**	
α_{52}	-0.02199	(-1.29)	-0.02090	(-1.25)	
α_{53}	0.2502	(13.61)**	0.2515	(13.87)**	

Table IV.1
Age Distribution in the Sample

Approximate Age in Years ("midpoints")	Age in Years (range)	Frequency	Percent	Cum.
18	20 or less	856	41.51	41.51
23	21-25	938	45.59	87.00
28	26-30	153	7.42	94.42
36	31-40	79	3.83	98.25
46	41-50	30	1.45	99.71
58	51-64	4	0.19	99.90
69	65 or more	2	0.10	100.00
Total		2062	100.00	

Table IV.2
Income Distribution in the Sample

Annual Family Income Now ("midpoints")	Income (range)	Frequency	Percent	Cum.
8	<\$10,000	101	4.90	4.90
15	\$10,000-20,000	157	7.61	12.51
25	\$20,000-30,000	206	9.99	22.50
40	\$30,000-50,000	361	17.51	40.01
62.5	\$50,000-75,000	405	19.64	59.65
87.5	\$75,000-100,000	371	17.99	77.64
125	>\$100,000	461	22.36	100.00
Total		2062	100.00	

Table IV.3
 Multinomial Logit Model to Explain Status of Lottery Winnings
 Responses: "Good" (numeraire) versus "Constant" and "Reversed"

	Constant Responses		"Reversed" Responses	
Variables	Estimate	t-ratio	Estimate	t-ratio
constant	-1.1959	(-1.53)	-2.0608	(-5.95)**
<i>Individual characteristics (non-orthogonal)</i>				
family income	-0.0033	(-2.48)**		
log(age)	0.5806	(2.42)**		
female			4.4318	(2.64)**
female*log(age)			-1.2956	(-2.35)**
educ. attain			-0.1555	(-1.37)
major = business			0.2204	(1.52)
major = soc.sci.	-0.3800	(-3.35)**		
econ course			-0.5002	(-2.63)**
work	0.1718	(1.70)*	-0.2614	(-1.88)*
<i>Contextual differences: Nature of the choice scenario (randomized)</i>				
horizon	-0.0100	(-1.66)*	0.0146	(1.78)*
<i>Characteristics of the stated preference format (randomized)</i>				
yes" on left			-0.2151	(-1.59)
increasing sums	0.1550	(1.55)		
5 lump sums	-0.9129	(-6.44)**		
7 lump sums	-0.7783	(-5.72)**		
13 lump sums	-0.7674	(-5.60)**	0.4887	(3.35)**
3-level answers	-0.2717	(-2.01)**		
4-level answers	-0.6584	(-4.60)**		
5-level answers	-0.8270	(-5.87)**		
Max Log L	-1181.0578		-748.9764	